

**ECN 103 : Economics of Uncertainty and Information**

**MATH QUIZ to test your readiness for this class**

If you have any difficulties answering the questions below, then you should not take this class. Calculus will be used extensively in ECN 103

**The answers are given at the end of this file.**

First some notation. Let  $f(x)$  be a real-valued function; then its first derivative is denoted by  $f'(x)$  or  $\frac{d}{dx}f(x)$  and its second derivative by  $f''(x)$  or  $\frac{d^2}{dx^2}f(x)$ .

- 1.** Let  $f(x) = 120 - \left(\frac{x}{4}\right)^2$ . (a) Calculate  $f(20)$ . (b) Calculate  $f'(x)$ . (c) Calculate  $f'(20)$ .  
(d) Calculate  $f''(x)$ . (e) Calculate  $f''(20)$ .
- 2.** Let  $f(x) = 6x^3$ . (a) Calculate  $f(3)$ . (b) Calculate  $f'(x)$ . (c) Calculate  $f'(3)$ .  
(d) Calculate  $f''(x)$ . (e) Calculate  $f''(3)$ .
- 3.** Let  $f(x) = 12\sqrt{x}$ . (a) Calculate  $\frac{d}{dx}f(x)$ . (b) Calculate  $\frac{d^2}{dx^2}f(x)$ .
- 4.** Let  $f(x) = 2\ln\left(\frac{x}{2}\right)$ , where  $\ln$  denotes the natural logarithm (that is, the logarithm to the base  $e$ ).  
(a) Calculate  $\frac{d}{dx}f(x)$ . (b) Calculate  $\frac{d^2}{dx^2}f(x)$ .
- 5.** Let  $f(x) = x(60 - 2x) - 4x$ . Find the value of  $x$  that maximizes the function  $f$ .
- 6.** Let  $f(x) = 200 - \left(\frac{x}{2}\right)^2$ . (a) Draw the graph of the function  $f$  for  $x \in [0, 400]$ .  
(b) Write the equation of the straight line that is tangent to the graph of  $f$  at the point  $x = 100$ .

## ECN 103 : Economics of Uncertainty and Information

## ANSWERS to MATH QUIZ

1. Let  $f(x) = 120 - \left(\frac{x}{4}\right)^2$ . (a)  $f(20) = 95$ . (b)  $f'(x) = -\frac{x}{8}$ . (c)  $f'(20) = -\frac{5}{2} = -2.5$ .

(d)  $f''(x) = -\frac{1}{8}$ . (e)  $f''(20) = -\frac{1}{8}$ .

2. Let  $f(x) = 6x^3$ . (a)  $f(3) = 162$ . (b)  $f'(x) = 18x^2$ . (c)  $f'(3) = 162$ .

(d)  $f''(x) = 36x$ . (e)  $f''(3) = 108$ .

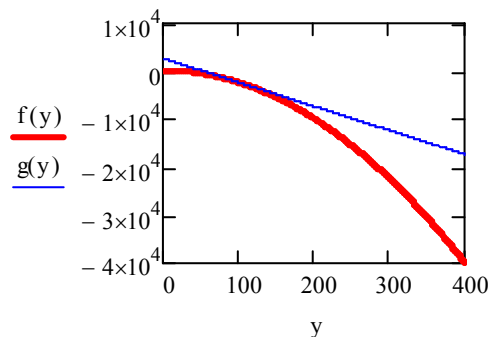
3. Let  $f(x) = 12\sqrt{x}$ . (a)  $\frac{d}{dx} f(x) = \frac{6}{\sqrt{x}}$ . (b)  $\frac{d^2}{dx^2} f(x) = -\frac{3}{x^{\frac{3}{2}}} = -\frac{3}{\sqrt{x^3}}$ .

4. Let  $f(x) = 2\ln\left(\frac{x}{2}\right)$ , where  $\ln$  denotes the natural logarithm (that is, the logarithm to the base  $e$ ).

(a)  $\frac{d}{dx} f(x) = \frac{2}{x}$ . (b)  $\frac{d^2}{dx^2} f(x) = -\frac{2}{x^2}$ .

5. Let  $f(x) = x(60 - 2x) - 4x$ . The value of  $x$  that maximizes the function  $f$  is given by the solution to  $f'(x) = 0$ , that is, the solution to  $56 - 4x = 0$ . Hence  $x = 14$ .

6. Let  $f(x) = 200 - \left(\frac{x}{2}\right)^2$ . (a) The graph of the function  $f$  for  $x \in [0, 400]$  is shown below as the thick curve:



(b) The equation of the straight line that is tangent to the graph of  $f$  at the point  $x = 100$  is given by  $y = f(100) + f'(100)(x - 100) = 2,700 - 50x$  (its graph is shown above as the thin straight line).