Economics 103

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PRACTICE EXAM FOR THE FINAL: ANSWERS

1. **(a)** $\mathbb{E}[U(A)] = \frac{2}{5}\sqrt{25} + \frac{3}{5}\sqrt{100} = 8$ **(b)** $\mathbb{E}[U(B)] = \frac{2}{5}\sqrt{100} + \frac{3}{5}\sqrt{25} = 7$

(c) The slope of the indifference curve through point *A*, at point *A*, is

$$-\frac{p}{1-p}\left(\frac{U'(25)}{U'(100)}\right) = -\frac{\frac{2}{5}}{\frac{3}{5}}\left(\frac{U'(25)}{U'(100)}\right) = -\frac{2}{3}\left(\frac{\frac{1}{2\sqrt{25}}}{\frac{1}{2\sqrt{100}}}\right) = -\frac{4}{3}$$

(d) The slope of the indifference curve through point *B*, at point *B*, is

$$-\frac{p}{1-p}\left(\frac{U'(100)}{U'(25)}\right) = -\frac{2}{3}\left(\frac{\frac{1}{2\sqrt{100}}}{\frac{1}{2\sqrt{25}}}\right) = -\frac{1}{3}$$

(e) It is given by the equation: $\frac{2}{5}\sqrt{x} + \frac{3}{5}\sqrt{y} = 8$. Solving for *y* we get $y = \left(\frac{2\sqrt{x}}{3} - \frac{40}{3}\right)^2$.

(f) It is given by the equation: $\frac{2}{5}\sqrt{x} + \frac{3}{5}\sqrt{y} = 7$. Solving for *y* we get $y = \left(\frac{2\sqrt{x}}{3} - \frac{35}{3}\right)^2$.

2. **(a)** $\underbrace{3,600 - W_2}_{h} = 1,200 - \frac{2}{5}\underbrace{(W_2 - W_1)}_{d}$. Solving for W_2 we get $W_2 = 4,000 - \frac{2}{3}W_1$.

(b) The slope of any isoprofit line is $-\frac{\frac{15}{100}}{1-\frac{15}{100}} = -\frac{15}{85} = -\frac{3}{17}$. Since the slope of the insurance budget line is $-\frac{2}{3} \neq -\frac{3}{17}$, the equation of Part (a) does **not** correspond to an isoprofit line.

(c) Replacing W_1 with (3,600 − 2,700) = 900 in the equation $W_2 = 4,000 - \frac{2}{3}W_1$ we get 3,400 which is less than the initial wealth (which is 3,600). Thus the insurance budget line does not go through the no-insurance point.

(d) First of all let us compute the reservation level of utility:

$$\mathbb{E}[U(NI)] = \frac{15}{100}\sqrt{900} + \frac{85}{100}\sqrt{3,600} = 55.5$$

The existence of contracts on the insurance budget line that yield a utility greater than 55.5 requires that the insurance budget line cross the reservation indifference curve, that is, there needs to be a solution to the following equations (within the range $W_1 \in [900, 3600]$):

$$W_2 = 4,000 - \frac{2}{3}W_1 \quad \text{and} \quad \frac{15}{100}\sqrt{W_1} + \frac{85}{100}\sqrt{W_2} = 55.5$$

[There is no solution to the above equations.]

(e) $\underbrace{3,600 - W_2}_{h} = 1,080 - \frac{2}{5}\underbrace{(W_2 - W_1)}_{d}$. Solving for W_2 we get $W_2 = 4,200 - \frac{2}{3}W_1$.

(f) Replacing W_1 with (3,600 − 2,700) = 900 in the equation $W_2 = 4,200 - \frac{2}{3}W_1$ we get 3,600 which is the initial wealth. Thus the insurance budget line does go through the no-insurance point.

(g) We need to compare the slope of the reservation indifference curve at *NI* to the slope of the insurance budget line. The slope of the reservation indifference curve at *NI* is

$$-\frac{p}{1-p}\left(\frac{U'(900)}{U'(3,600)}\right)=-\frac{3}{17}\left(\frac{\sqrt{3600}}{\sqrt{900}}\right)=-\frac{3}{34}$$

Thus the insurance budget line is steeper at *NI* than the reservation indifference curve; it follows that insurance budget line is entirely below the reservation indifference curve (except at point *NI*), that is, there are no contracts on the insurance budget line that Anna prefers to no insurance

3. At a signaling equilibrium the employer's beliefs must be confirmed. Thus Group I workers must choose $y < a$ (in which case they would choose $y = 0$) and Group II workers must choose $y \geq a$ (in which case they would choose $y = a$). For Group I this requires: $6 > 10 + \frac{1}{2}a - 4a$, while for Group II this requires: $10 + \frac{1}{2}a - 2a > 6$. Both inequalities are satisfied if and only if $\frac{8}{7} < a < \frac{8}{3}$.

4. **(a)** For the H type expected utility of no insurance is $\frac{1}{3}\ln(6)+\frac{2}{3}\ln(15)=2.4026$. The maximum premium that the H type would be willing to pay for full insurance is the solution to $\ln\left(\frac{15,000-h}{1,000}\right)=2.4026$ which is \$3,947.91. The monopolist would offer such a contract and its expected profit would be:

per contract: $3,947.91-\frac{1}{3}(9,000)=\$947.91$, total profits: 947.91(1,800)=\$1,706,238.

(b) First we need to calculate the average probability of loss $\bar{p}$:

$$\bar{p}=\frac{1}{3}\left(\frac{N_H}{N_H+N_L}\right)+\frac{1}{12}\left(\frac{N_L}{N_H+N_L}\right)=\frac{2}{15}$$

Option 2 is profitable if and only if the reservation indifference curve of the L type is steeper at the no-insurance point than the average zero-profit line, that is, if and only if

$$\frac{p_L}{1-p_L}\left(\frac{U'(6,000)}{U'(15,000)}\right)>\frac{\bar{p}}{1-\bar{p}} \quad \text{that is} \quad \frac{5}{22}>\frac{2}{13} \quad \text{which is true.}$$

(c) The profit-maximizing contract under Option 2 is given by the solution to the following equations (the first says that the *L* types are indifferent between insuring and not insuring and the second equation says that, at the offered contract, the slope of the *L*-type indifference curve is equal to the slope of the average isoprofit line; note that $U'(x)=\frac{1}{x}$):

$$\frac{1}{12}\ln\left(\frac{15,000-h-d}{1,000}\right)+\frac{11}{12}\ln\left(\frac{15,000-h}{1,000}\right)=\frac{1}{12}\ln\left(\frac{15,000-9,000}{1,000}\right)+\frac{11}{12}\ln\left(\frac{15,000}{1,000}\right)$$

$$\frac{\frac{1}{12}}{\frac{11}{12}}\left(\frac{15,000-h}{15,000-h-d}\right)=\underbrace{\frac{2}{13}}_{=\frac{\bar{p}}{1-\bar{p}}}$$

(d) First calculate the expected utility from now insurance for each type:

Type H: $\mathbb{E}[U_H(NI)]=\frac{1}{3}\ln(6)+\frac{2}{3}\ln(15)=2.4026$ (this was calculated in part (a))

Type L: $\mathbb{E}[U_L(NI)] = \frac{1}{12}\ln(6) + \frac{11}{12}\ln(15) = 2.6317$

- $\mathbb{E}[U_H(C_H)] = \frac{1}{3}\ln(10.8) + \frac{2}{3}\ln(11.4) = 2.4156 > \mathbb{E}[U_H(NI)]$ and thus IR_H is satisfied.
- $\mathbb{E}[U_L(C_L)] = \frac{1}{12}\ln(12.8) + \frac{11}{12}\ln(13) = 2.6825 > \mathbb{E}[U_L(NI)]$ and thus IR_L is satisfied.
- $\mathbb{E}[U_H(C_L)] = \frac{1}{3}\ln(12.8) + \frac{2}{3}\ln(13) = 2.6462 > \mathbb{E}[U_H(C_H)]$ and thus IC_H **fails**.
- $\mathbb{E}[U_L(C_H)] = \frac{1}{12}\ln(10.8) + \frac{11}{12}\ln(11.4) = 2.4291$ and thus IC_L is satisfied.

(e) From Part (d) we deduce that both types would choose contract $C_L = (h = 200, d = 2000)$. Thus the expected profit per contract is (recall from Part (b) that the average probability of loss is $\frac{2}{15}$):

$$200 - \tfrac{2}{15}(9,000 - 2,000) = \$-733.33.$$

Thus total expected profits are $(-733.33)(N_H + N_L) = (-733.33)(9,000) = \$-6,599,970$: a huge loss!

(f) The contract $C_L = (h_L, d_L)$ targeted to the L types should be such that (1) the L type is indifferent between contract $C_L = (h_L, d_L)$ and no insurance (this is the first of the two equations below) and (2) the H type is indifferent between C_H and C_L (this is the second of the two equations below)

$$\tfrac{1}{12}\ln\left(\frac{15,000 - h_L - d_L}{1,000}\right) + \tfrac{11}{12}\ln\left(\frac{15,000 - h_L}{1,000}\right) = 2.6317$$

$$\tfrac{1}{3}\ln\left(\frac{15,000 - 3,600 - 600}{1,000}\right) + \tfrac{2}{3}\ln\left(\frac{15,000 - 3,600}{1,000}\right) = \tfrac{1}{3}\ln\left(\frac{15,000 - h_L - d_L}{1,000}\right) + \tfrac{2}{3}\ln\left(\frac{15,000 - h_L}{1,000}\right)$$

5. **(a)** $\frac{2}{5}\sqrt{900 - 275} + \frac{3}{5}\sqrt{900} = 28$. **(b)** $\frac{1}{10}\sqrt{900 - 275 - 78} + \frac{9}{10}\sqrt{900 - 78} = 28.14$.

(c) With full insurance he will not spend money on prevention: $\sqrt{900 - 90} = 28.46$.

(d) First determine what Albert would do. Expected utility without prevention: $\frac{2}{5}\sqrt{900 - 50 - 150} + \frac{3}{5}\sqrt{900 - 50} = 28.08$. Expected utility without prevention: $\frac{1}{10}\sqrt{900 - 50 - 150 - 78} + \frac{9}{10}\sqrt{900 - 50 - 78} = 27.5$. Thus he would prefer no insurance (with prevention). Hence expected profits are zero (because Albert will not buy insurance).

6. **(a)** First determine what Bill will do. Expected utility with no effort: $\frac{1}{2}\sqrt{324} + \frac{1}{2}\sqrt{900} = 24$. Expected utility with effort: $\frac{1}{6}\left(\sqrt{324} - 2\right) + \frac{5}{6}\left(\sqrt{900} - 2\right) = 26$. Thus the answer is: 26.

(b) $\frac{1}{6}(1,300 - 324) + \frac{5}{6}(1,900 - 900) = 996$. **(c)** First determine what Bill will do. Expected utility with no effort: $\frac{1}{2}\sqrt{400} + \frac{1}{2}\sqrt{484} = 21$. Expected utility with effort: $\frac{1}{6}\left(\sqrt{400} - 2\right) + \frac{5}{6}\left(\sqrt{484} - 2\right) = 19.6$. Thus the answer is: 21.

(d) $\frac{1}{2}(1,300 - 400) + \frac{1}{2}(1,900 - 484) = 1,158$.