ECN 103 : ECONOMICS of UNCERTAINTY Professor Giacomo Bonanno

FINAL EXAM: ANSWERS for VERSION 2

1. [Note: this is a small variation on Exercise 10.3 in the Textbook]

(a) Bob's initial total wealth is 860,000 + 250,000 = $1,110,000. Thus

$$EU(x) = \left(0.5 - \frac{x}{400{,}000}\right)\sqrt{1{,}110{,}000 - 700{,}000 - x} + \left(0.5 + \frac{x}{400{,}000}\right)\sqrt{1{,}110{,}000 - x}$$

(b) EU(0) = 134.2189, EU(100,000) = 135.3018 and EU(200,000) = 137.212. Thus **Bob would choose *x* = 200,000**.

(c) Since, for every *x*, a full-insurance contract at premium *h* guarantees a wealth of $1{,}110{,}000 - h - x$, **Bob would choose *x* = 0**.

(d) In this case his expected utility, as a function of *x*, is

$$EU(x) = \left(0.5 - \frac{x}{400{,}000}\right)\sqrt{1{,}110{,}000 - 80{,}000 - 100{,}000 - x} + \left(0.5 + \frac{x}{400{,}000}\right)\sqrt{1{,}110{,}000 - 80{,}000 - x}$$

EU(0) = 137.94, EU(100,000) = 137.145 and EU(200,000) = 136.2918. Thus **Bob would choose *x* = 0.**

(e) From the above calculations we get that under option (1) he would choose *x* = 200,000 and his expected utility would be 137.212, under option (2) he would choose *x* = 0 and his expected utility would be $\ln(1{,}110{,}000 - 150{,}000) = 137.7469$ and under option (3) he would choose *x* = 0 and his expected utility would be 137.94. Thus **Bob will choose the partial-insurance contract and *x* = 0.**

2. (a.1) First we need to figure out what level of effort the Agent will choose. Her expected utility is:

- with $e = L$: $\frac{5}{8}\sqrt{25.5} + \frac{1}{4}\sqrt{28} + \frac{1}{8}\sqrt{39} - 3 = 2.2596$
- with $e = H$: $\frac{1}{4}\sqrt{25.5} + \frac{1}{2}\sqrt{28} + \frac{1}{4}\sqrt{39} - 4 = 1.4694$

Thus the Agent will choose $e = L$ and her expected utility will be 2.2596.

(a.2) Hence the Principal's expected utility is $\frac{5}{8}\sqrt{76.5} + \frac{1}{4}\sqrt{84} + \frac{1}{8}\sqrt{117} = 9.1099$

(b.1) First we need to figure out what level of effort the Agent will choose. Her expected utility is:

- with $e = L$: $\frac{5}{8}\sqrt{0} + \frac{1}{4}\sqrt{37.33} + \frac{1}{8}\sqrt{52} - 3 = -0.5711$
- with $e = H$: $\frac{1}{4}\sqrt{0} + \frac{1}{2}\sqrt{37.33} + \frac{1}{4}\sqrt{52} - 4 = 0.8578$

Thus the Agent will choose $e = H$ and her expected utility will be 0.8578.

(b.2) Hence the Principal's expected utility is $\frac{1}{4}\sqrt{102} + \frac{1}{2}\sqrt{74.66} + \frac{1}{4}\sqrt{104} = 9.3949$

(c) No, because the Principal prefers *B* to *A* while the Agent prefers *A* to *B*.

3. (a) *y* =10 is worse than *y* = 8 (same wage but higher cost) and, similarly, *y* =18 is worse than *y* = 14, so each person only needs to consider $y \in \{8, 14, 21\}$.

Decision problem for person of productivity 18,000

y	gross wage	cost	net income
8	18,000	0	18,000
14	28,000	18,000	10,000
21	36,000	39,000	-3,000

Hence the best choice is y = 8.

Decision problem for person of productivity 28,000

y	gross wage	cost	net income
8	18,000	0	18,000
14	28,000	9000	19,000
21	36,000	19500	16,500

Hence the best choice is y = 14.

Decision problem for person of productivity 36,000

y	gross wage	cost	net income
8	18,000	0	18,000
14	28,000	6000	22,000
21	36,000	13000	23,000

Hence the best choice is y = 21.

Thus there is a signaling equilibrium, which is as follows. All people with productivity 18,000 will choose y = 8 and be paid $18,000, which is their true productivity. All people with productivity 28,000 will choose y = 14 and be paid $28,000, which is their true productivity. Finally, all people with productivity 36,000 will choose y = 21 and be paid $36,000, which is their true productivity. The employers' beliefs that more education correlates with higher productivity are confirmed.

(b) If schools beyond 8th grade were eliminated, the employers would no longer have education as a signal of productivity. Hiring an employee would then be the same as playing the lottery $\left(\begin{array}{c|ccc} \text{productivity} & 18,000 & 28,000 & 36,000 \\ \hline \text{probability} & 0.5 & 0.4 & 0.1 \end{array}\right)$ which has an expected value of 23,800. Thus employers (being risk-neutral) would pay everybody $23,800. Every employee would be better off (while the employers would be as well off). Hence closing down schools beyond 8th grade would lead to a Pareto superior situation.

4. The consumers with the thicker and steeper indifference curve choose contract *C* while the others choose *D*. Thus the expected profits are:

$$40\left[\underbrace{(900-520)}_{h_C}-\frac{1}{15}\left(700-\left(\underbrace{(520-300)}_{d_C}\right)\right)\right]+40\left[\underbrace{(900-880)}_{h_D}-\frac{1}{25}\left(700-\left(\underbrace{(880-220)}_{d_D}\right)\right)\right]$$

$$=13,920+736=14,656.$$