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Location Choice, Product Proliferation and Entry Deterrence

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Within a slightly modified version of Hotelling's model we reconsider the claim that the threat of entry induces existing firms to produce a larger number of products than they would otherwise. We show that entry deterrence, although optimal, need not be achieved through product proliferation. In some cases the incumbent monopolist resorts to an entry-deterring strategy based on location choice rather than product proliferation. We also show that in some cases the number of products chosen by the incumbent facing the threat of entry is strictly greater than the minimum number required to deter entry.

1. INTRODUCTION

It has often been claimed in the literature (see, especially, Hay (1976), Prescott and Visscher (1977) and Schmalensee (1978)) that the threat of entry induces existing firms to produce a larger number of products than they would otherwise. This "product proliferation" strategy—it is claimed—has the effect of "crowding out" prospective entrants, even though it leads to inefficient production when there are increasing returns to scale in the production of each brand. However, no satisfactory proof of this claim has been given so far.[1] The purpose of this paper to re-examine the notion of product proliferation and to point out the possibility of more profitable forms of entry deterrence.

We provide an example based on a slightly modified version of Hotelling's (1929) model (quadratic transportation costs, positive set-up cost for each "store", finite reservation price). We consider a three-stage game between an incumbent monopolist and a potential entrant. In stage 1 the incumbent decides how many "stores" to open and where to locate them. In stage 2 the potential entrant (having observed the action taken by the incumbent) decides whether to enter or not and—if he decides to enter—where to locate his store. Finally, in stage 3 incumbent and entrant compete in prices. The structure of the game is intended to capture the notion that prices can in practice be changed at will, while entry into the industry requires the construction of one or more plants. We look at the sub-game perfect equilibria (Selten (1975)) of the game.

The assumption that the entrant contemplates entry with only *one* store—although commonly made in the literature—is far from satisfactory, but was dictated by the need to make the computation of the Bertrand-Nash equilibria tractable. However, it is worth stressing that the purpose of this paper is not to provide a very general analysis but to show—by means of an example—that there may be better entry-deterring strategies than product proliferation.

The main result is that entry deterrence is indeed optimal, but it need not be achieved through product proliferation. In fact, for some values of the parameters, the number of stores opened by the incumbent at the unique perfect equilibrium of the game is equal to the number opened by a protected monopolist who does not face the threat of entry, but the *locations* of these stores are different. This can be interpreted as an entry-deterring strategy based on *product specification* (or location choice) as opposed to product proliferation.

One more feature of the model, worth noting here, is the following. The number of stores opened by the incumbent facing the threat of entry may be *strictly greater* than the minimum number of stores required to deter entry.

The paper is organized as follows. In Section 2 we outline the model and in Section 3 we determine the optimum number (and location) of stores for a protected monopolist. In Sections 4 and 5 we determine the Betrand–Nash equilibria for any possible location pattern of two or three stores, owned by two different players. In Section 6 we determine the minimum number of stores required to deter entry and the main results are proved in Section 7.

2. THE MODEL

We consider a linear market of unit length, represented by the interval [0, 1]. There are $n \geqq 1$ stores located on this line, each selling a homogeneous product. We denote by x_i the location of store i $(i = 1, \ldots, n)$. The following assumption is introduced merely in order to simplify the analysis and our results do not depend on it.[2]

Assumption 1. It is not possible for two stores to be located at the same point, that is, $i \neq j$ implies $x_i \neq x_j$.

We number stores in such a way that $x_i < x_{i+1}$. Therefore we have

$$0 \leqq x_1 < x_2 < \cdots < x_{n-1} < x_n \leqq 1. \tag{1}$$

We denote by p_i the mill price of store i.

Assumption 2. Consumers are uniformly distributed along the line and face a quadratic transportation cost given by

$$C(d) = cd^2, \qquad c > 0 \tag{2}$$

where d is distance travelled.

Assumption 3. Each consumer buys exactly one unit of the good if and only if there is at least one store which offers him a delivered price (that is, mill price plus transportation cost) which is less than r, where r is a positive number which we call the *reservation price* (common to all consumers). If a consumer buys the good, he buys it from the store which offers the least delivered price. Throughout the paper we shall assume that r is relatively large (by this we mean that $r \geqq 3c/4$ (see Section 3) and r is large enough for all consumers to buy at the equilibrium (see Sections 4 and 5)).

Finally, we introduce the following assumption about costs.

Assumption 4. There is a positive set-up cost for each store, which we denote by K, while the marginal cost of production is zero.[3]

3. THE OPTIMUM NUMBER OF STORES FOR A PROTECTED MONOPOLIST

In this Section we consider the case of a protected monopolist, that is, of a monopolist who does not face the threat of entry. A proof of (a more general version of) the following Proposition can be found in Bonanno (1985*a*, *b*).[4]

Proposition 1. *Let* $r \geqq 3c/4$. *Then the maximum revenue which a protected monopolist can obtain from* $n \geqq 1$ *stores is given by*

$$R^*(n) = r - c/(4n^2) \tag{3}$$

and there is a unique revenue-maximizing vector of locations and prices given by

$$x_i = (2i-1)/(2n), \qquad i = 1, \ldots, n. \tag{4}$$

and

$$p_i = r - c/(4n^2). \tag{5}$$

Thus the optimal strategy for a protected monopolist selling n products is to choose regularly spaced varieties at a uniform price.

The function $R^*(n)$ given by (3) is strictly increasing and concave in n. It follows that the optimal number of stores for a protected monopolist will be that unique number n^* satisfying

$$\begin{cases} R^*(n^*) - R^*(n^*-1) \geqq K \\ R^*(n^*+1) - R^*(n^*) < K \end{cases} \tag{6}$$

The optimal number of stores n^* as a function of K is illustrated in Figure 1.

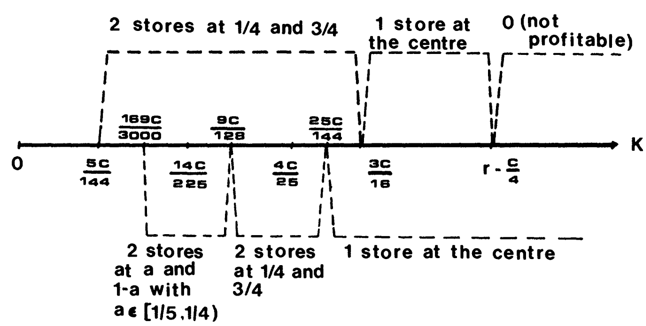

FIGURE 1

The upper part gives the number and location of stores for a protected monopolist (when $r \geqq 3c/4$). The lower part gives the minimum number of stores required to deter entry, with the respective locations

In order to determine the perfect equilibria of the game explained in the Introduction, we first need to compute the Bertrand–Nash equilibria of the last-stage game. This is done in the following two Sections.

4. BERTRAND–NASH EQUILIBRIUM WITH TWO STORES

Let there be two stores, 1 and 2, owned by two different players. The statements of the following Proposition and Corollary can be found, without proof, in D'Aspremont *et al.* (1979) (however, the authors follow Hotelling (1929) more closely and use a different notation to ours). A proof can be found in Bonanno (1985*a*, *b*).

Proposition 2. *Assume that players maximize revenue. Then for every pair of locations* (x_1, x_2), *with* $0 \leqq x_1 < x_2 \leqq 1$, *there exists a unique Bertrand-Nash equilibrium (BNE) with corresponding revenues*

$$\hat{R}_1 = (c/18)(x_2 - x_1)(2 + x_1 + x_2)^2 > 0 \tag{7}$$

$$\hat{R}_2 = (c/18)(x_2 - x_1)(4 - x_1 - x_2)^2 > 0. \tag{8}$$

Corollary 3. *For every pair of locations* (x_1, x_2), *with* $0 \leqq x_1 < x_2 \leqq 1$ *we have that*

$$\partial \hat{R}_1 / \partial x_1 < 0 \quad \text{and} \quad \partial \hat{R}_2 / \partial x_2 > 0. \tag{9}$$

Therefore, given the location of the competing store, each player maximizes his BNE revenue by locating as far as possible from the competing store (i.e. at one extreme of the market).

5. BETRAND-NASH EQUILIBRIUM WITH THREE STORES, TWO OF WHICH ARE OWNED BY THE SAME PLAYER

We shall now consider the case where there are three stores in the market, owned by two different players. A proof of the following Propositions can be found in Bonanno (1985*a*, *b*).

Proposition 4. *Let player I own store* 1 *and player II own stores* 2 *and* 3 *and assume that players maximize revenue. Then for every triple of locations* (x_1, x_2, x_3), *with* $0 \leqq x_1 < x_2 < x_3 \leqq 1$, *there exists a unique BNE at which player I's revenue is given by*

$$\bar{R}_I = (c/18)(x_2 - x_1)(2 + x_1 + x_2)^2 > 0. \tag{10}$$

Comparing (10) with (7) we can conclude that from the point of view of the owner of store 1 it is immaterial whether the owner of store 2 owns that store only or also another store further away: the equilibrium revenue of store 1 will be the same in both cases. Note, again, that

$$\partial \bar{R}_I / \partial x_1 < 0 \tag{11}$$

and therefore the best location for store 1 is $x_1 = 0$.

Given the symmetry of the model, Proposition 4—with the necessary amendments—covers also the case where player I owns store 3, while player II owns stores 1 and 2.

Proposition 5. *Let player I own store* 2 *and player II own stores* 1 *and* 3 *and assume that players maximize revenue. Then for every triple of locations* (x_1, x_2, x_3), *with* $0 \leqq x_1 < x_2 < x_3 \leqq 1$, *there exists a unique BNE at which player I's revenue is given by*

$$R_I^0 = c(x_2 - x_1)(x_3 - x_2)(2 + x_3 - x_1)^2 / (18(x_3 - x_1)) > 0. \tag{12}$$

Corollary 6. *For every pair of locations* (x_1, x_3) *of stores* 1 *and* 3, *with* $0 \leqq x_1 < x_3 \leqq 1$, *the equilibrium revenue of player I (store 2),* R_I^0, *is maximized when*

$$x_2 = (x_1 + x_3)/2 \tag{13}$$

that is, when player I locates his store exactly half-way between the two stores of player II.

We can now turn to the question of entry deterrence.

6. THE MINIMUM NUMBER OF STORES REQUIRED TO DETER ENTRY

In this Section we determine the minimum number of stores required to deter entry for a range of values of the set-up cost K.

By Proposition 2 and Corollary 3 we know that if the incumbent has opened *one* store, the entrant would locate his store as far as possible from the incumbent's store and his revenue would be given by the maximum between $\hat{R}_1|_{x_1=0}$ and $\hat{R}_2|_{x_2=1}$ (where $\hat{R}_1$ and $\hat{R}_2$ are given by (7) and (8), respectively). The *minimum* of the max $\{\hat{R}_1|_{x_1=0}, \hat{R}_2|_{x_2=1}\}$ is $25c/144$, which corresponds to the case where the incumbent's store is located at the centre of the market. Therefore, if

$$K > \tfrac{25}{144}c$$

one store located at the centre of the market is sufficient to deter entry.

Now consider the case where the incumbent has opened *two* stores. By Propositions 4 and 5 and Corollary 6, the entrant would locate his store either half-way between the incumbent's stores or at one extreme of the market, depending on where he would obtain the highest revenue. Without loss of generality we can assume that the incumbent's stores are located at

$$x_1 = a \quad \text{and} \quad x_3 = 1 - a, \quad \text{with } a \in [0, \tfrac{1}{2}). \tag{14}$$

Then entry at one extreme of the market would yield a revenue of

$$\bar{R} = \frac{a(2+a)^2 c}{18} \tag{15}$$

(using (10) and (14)), while entry half-way between the incumbent's stores would yield a revenue of

$$R^0 = \frac{(1-2a)(3-2a)^2 c}{72} \tag{16}$$

(using (12), (13) and (14)). The functions $\bar{R}$ and R^0 are shown in Figure 2. The minimum of the max $\{\bar{R}, R^0\}$ is given by $169c/3000$, achieved when $a = \frac{1}{5}$. Therefore, if

$$\tfrac{169}{3000}c < K < \tfrac{25}{144}c \tag{17}$$

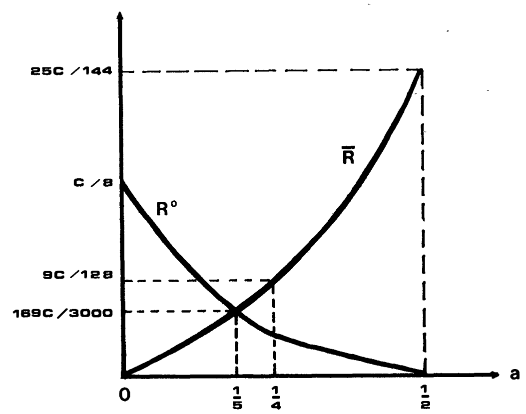

FIGURE 2

The entrant's revenue if he enters at one extreme ($\bar{R}$) or half-way between the incumbent's stores (R^0), when the latter are located at $x_1 = a$ and $x_2 = 1 - a$, with $a \in [0, \frac{1}{2})$

two stores located at $x_1 = a$ and $x_2 = 1 - a$ with $a \in [\frac{1}{5}, \frac{1}{4}]$ (depending on the value of K) are sufficient to deter entry. The bottom part of Figure 1 illustrates these results. We can now prove our main results.

7. PERFECT EQUILIBRIA AND ENTRY DETERRENCE

We now determine the perfect equilibria of the game for a range of values of K. The first Proposition shows that when

$$\tfrac{9}{128}c < K < r - \frac{c}{4} \tag{18}$$

the *number and locations* of the stores opened by the incumbent facing the threat of entry coincide with the number and locations chosen by a protected monopolist. Therefore, in Bain's terminology (Bain (1956)), entry is blockaded. In other words, in the range given by (18) the fixed cost K acts as an "innocent" barrier to entry (Salop (1979)).

Proposition 7. *If*

$$\tfrac{3}{16}c < K < r - \frac{c}{4} \tag{19}$$

the game has a unique perfect equilibrium at which the incumbent opens one store at the centre of the market and makes positive profits, while entry is blockaded. If

$$\tfrac{9}{128}c < K < \tfrac{3}{16}c \tag{20}$$

the game has a unique perfect equilibrium at which the incumbent opens two stores at $\frac{1}{4}$ and $\frac{3}{4}$ and makes positive profits, while entry is blockaded.

Comparing the result of the second part of Proposition 7 with Figure 1 we can draw an interesting conclusion. When

$$\tfrac{25}{144}c < K < \tfrac{3}{16}c \tag{21}$$

the number of stores opened by the incumbent at the unique perfect equilibrium of the game (two) is *strictly greater* than the minimum number of stores required to deter entry (one).

The main result of the paper, given in Proposition 8, is that for some values of the parameters the number of stores opened by the incumbent facing the threat of entry coincides with the number of stores chosen by a protected monopolist, but the *locations* of these stores are *different*. As said in the Introduction, this can be interpreted as an entry-deterring strategy based on *product specification* (or location choice) as opposed to product proliferation (thus in this case the set-up costs ($2K$) appear as a strategic barrier to entry (Salop (1979))).[5]

Proposition 8. *If*

$$\tfrac{14}{225}c < K < \tfrac{9}{128}c \tag{22}$$

the game has a unique perfect equilibrium at which the incumbent opens two stores, makes positive profits and entry is deterred. The locations of these stores are different from the locations chosen by a protected monopolist.

The intuition behind Proposition 8 is as follows. When K belongs to the interval (22), a protected monopolist would open two stores at $\frac{1}{4}$ and $\frac{3}{4}$. Given such a regular spacing, the entrant's revenue would be greater if he entered at one extreme of the market than if he entered half-way between the incumbent's stores, the reason being that in the first case he would virtually be competing with only one store (cf. Proposition 4) while in the second case he would be competing with both stores (cf. Proposition 5). When K belongs to the interval (22), the entrant's profits—if he enters at one extreme of the market—will be positive. The incumbent can reduce those profits from positive to negative by moving his stores towards the extremes, thereby increasing the degree of competition faced by the entrant at one extreme (however, this process must stop at the point where entry at the centre becomes more profitable than entry at one extreme). The loss of profits implied by this sub-optimal (cf. Proposition 1) location of two stores is less than the loss of profits due to the opening of an extra store at an additional cost of K (product proliferation strategy) and, of course, is also less than the loss of profits the incumbent would face if he allowed entry to take place.

Proof of Proposition 8. We shall prove Proposition 8 in two steps. We shall first prove that the "location choice" strategy is better than any product-proliferation-entry-deterring-strategy (PPDES) and then we shall show that it is indeed better for the incumbent to deter entry rather than allow entry to take place.

Let (s, x) by any PPDES, where s is the number of stores (hence, $s \geqq 3$) and x is the corresponding vector of locations. Let $\pi(s, x)$ be the corresponding monopoly profits of the incumbent. Then by Proposition 1

$$\pi(s, x) \leqq \pi^*(s) \tag{23}$$

where, for any n, $\pi^*(n) = R^*(n) - nK$ and $R^*(n)$ is given by (3). Furthermore, by the results of Section 3

$$\pi^*(s) \leqq \pi^*(3) \tag{24}$$

(since $n = 3$ is the maximum of $\pi^*(n)$ for K in the range given by (22) and $\pi^*(n)$ is strictly concave). Therefore it is sufficient to prove that

$$\pi^*(3) < \pi_2 \tag{25}$$

where π_2 is the monopoly profit of the incumbent corresponding to the location choice strategy, which is given as follows. For each K in the range (22) there is a unique $a^* \in (\frac{1}{5}, \frac{1}{4})$ such that $\max\{\bar{R}, R^0\} = \bar{R} = K$ (see Section 6 and Figure 2) and therefore two stores located at $x_1^* = a^*$ and $x_2^* = 1 - a^*$ are sufficient to deter entry.[6] It can be seen from Figure 2 that a^* is strictly increasing in K. Given this choice of locations, the incumbent can set

$$p_1 = p_2 = r - c(1 - 2a^*)^2/4 \tag{26}$$

and serve the whole market, with corresponding profits

$$\pi^0 = r - c(1 - 2a^*)^2/4 - 2K. \tag{27}$$

Hence

$$\pi_2 \geqq \pi^0. \tag{28}$$

Furthermore, using (3), we have

$$\pi^*(3) = r - c/36 - 3K. \tag{29}$$

Therefore, a sufficient condition for (25) to be satisfied is $\pi^0 > \pi^*(3)$, which, using (27) and (29), is equivalent to

$$K > cf(a^*) \tag{30}$$

where

$$f(a^*) = (2 + 9a^{*2} - 9a^*)/9. \tag{31}$$

Now, f is strictly decreasing in a^* on $[\frac{1}{5}, \frac{1}{4}]$ and $f(\frac{1}{5}) = \frac{14}{225}$. Hence (22) is a sufficient condition for (25). We have therefore proved that location choice is better than product proliferation. We now show that it is indeed in the interest of the incumbent to deter entry.

The highest profit the incumbent can make if he allows entry to occur is

$$\pi^e = c/2 - K \tag{32}$$

which corresponds to the case where he opens one store at one extreme of the market (in which case the entrant would locate his store at the other extreme: cf. Corollary 3). (32) is obtained from (7) by setting $x_1 = 0$ and $x_2 = 1$. Therefore, entry deterrence is optimal if

$$\pi^e < \pi_2. \tag{33}$$

Now using (28) and the fact that π^0 is strictly increasing in a^* on $[\frac{1}{5}, \frac{1}{4}]$ and that

$$\pi^0|_{a^*=1/5} = r - 9c/100 - 2K, \tag{34}$$

if

$$r - 9c/100 - 2K > c/2 - K \tag{35}$$

then (33) is satisfied. Now, (35) is equivalent to

$$K < r - 59c/100 \tag{36}$$

and since we have assumed thar $r \geqq 3c/4$, the RHS of (36) is greater than or equal to $4c/25$. Since K belonging to the range (22) implies $K < 4c/25$ the proof is complete. ||

For smaller values of K, in particular, for K in the range

$$5c/144 < K < 14c/225 \tag{37}$$

the product specification strategy described above will no longer have the effect of deterring entry and the incumbent will have to resort to a product proliferation strategy.

8. CONCLUSION

The purpose of this paper was to point out the possibility—so far unexplored—of entry-deterring strategies based on product specification and to show that in some cases such an entry-deterring strategy is more profitable than product proliferation. Furthermore, unlike the previous papers in the literature, we explicitly allowed for price competition in the post-entry game. The model used was very specific and the analysis confined to a small subset of the parameter values. However, in defence of the paper it could be argued that, in the present state of the art, it is difficult to work with a much more general model. Finally, it should be noted that we only looked at the case where there is a positive set-up cost for each product, that is, where the cost function is linear in the number of products. It may be possible that different cost functions would yield different results (in particular, when there are decreasing or increasing returns to scale in the total number of products).

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NOTES

1. In Schmalensee's model the product space is represented by a circle and each brand is produced under conditions of decreasing costs. The author restricts himself to the very special case where there are N brands, located at distances $1/N$ apart and all charging the same price p. He then introduces the *ad hoc* assumption that any new entrant would also charge that price. Hay's model is flawed in a similar way. Prescott and Visscher are concerned with "vertical" differentiation and provide a series of examples rather than a general result. The paper by Eaton and Lipsey (1979) can also be mentioned in this context, even though the authors are concerned with a somewhat different problem, namely that of a *growing* spatial market.

2. In fact, no player would want to locate two or more stores at the same point (since there is a positive set-up cost for each store) and, on the other hand, if two players opened one store each at the same location then, by Bertrand's theorem, those stores would yield zero revenue and therefore negative profits.

3. The assumption of zero marginal cost is introduced only in order to simplify the analysis and our results do not depend on it.

4. The results of Section 3 were also independently proved by Neven (1985).

5. It is trivial to show that there does indeed exist a product-proliferation-entry-deterring-strategy (PPEDS). In fact, let $\hat{n}$ be the *smallest* integer for which $\pi^*(n)$ is negative (recall that $\pi^*(n)$ is strictly concave and tends to $-\infty$ as n goes to $+\infty$). Then a PPDES which enables the incumbent to make positive profits is given by $(\hat{n}-1)$ stores located according to (4).

6. It can be seen from Figure 2 that for each K in the range (22) there exists a continuum of points a^* (a compact interval containing the point 1/5) such that two stores located at $x_1^*=a^*$ and $x_2^*=1-a^*$ are sufficient to deter entry. However, using the same argument which led to (4)—which gives the optimal location of n stores for a protected monopolist—(see Bonanno (1985*a*) pp. 11-13 and 32-36), it is possible to show that, of all these values of a^*, only the largest maximizes the monopoly profits of the incumbent (subject to the constraint of entry deterrence).

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