VERTICAL RESTRAINTS IN A MODEL OF VERTICAL DIFFERENTIATION*

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We consider the case of a manufacturer who sells a homogeneous good to retailers who compete in prices and "cum-sales" or "post-sales" services. We show that the optimal linear-price contract is inefficient from the point of view of the vertical structure and that simple forms of vertical restraints, such as resale price maintenance and franchise fees, dominate the optimal linear-price contract, but do not restore vertical efficiency. Our analysis is concluded with the description of an efficient contract.

I. INTRODUCTION

A number of explanations have been given in the literature for the widespread phenomenon of contractual restraints in manufacturer-distributor relations: the need to avoid double marginalization [Spengler, 1950], the need to eliminate free-riding in the provision of costly pre-sales services [Telser, 1960; Mathewson and Winter, 1984; Marvel and McCafferty, 1984], the desire to control the density of retail outlets [Gould and Preston, 1965; Gallini and Winter, 1983; Dixit, 1983], etc.[1] All these explanations have one feature in common: vertical restraints, such as resale-price-maintenance, exclusive territories, quantity fixing, or franchise fees, can be perfect substitutes for vertical integration.

In this paper we consider the case where retailers provide *cum-sales* or *post-sales services* or both. This situation, unlike the one studied by Telser [1960], is not characterized by a free-rider problem, since consumers can benefit from a given retailer's services only if they purchase the good from him. The provision of such services, however, can give rise to a situation of *vertical differentiation*, characterized by the fact that if two distinct products are offered at the same price, then all consumers buy from the retailer offering the product of higher quality. Examples of cum-sales or post-sales services are provision of parking space; waiting

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1. For a more complete list of explanations see the survey by Rey and Tirole [1986a].

time (the time between purchase and delivery of the good); the provision of services such as (free) credit, (free) delivery, (free) installation, (free) repairs; the location of the retail outlet;[2] etc.

It is a well-known result [Shaked and Sutton, 1982] that if the manufacturer sells the good to independent retailers, then the latter will choose different qualities in order to "relax price competition through product differentiation." This gives rise to an obvious vertical inefficiency in those situations where a vertically integrated structure would find it optimal to produce only one quality. The purpose of this paper, however, is to bring to light a less obvious inefficiency that arises also in those situations where a vertically integrated structure would choose to offer products of different qualities (in order to price-discriminate among consumers with different willingness to pay for quality: see Mussa and Rosen [1978] and Gabszewicz et al. [1986]). We show that, in this latter case, vertical restraints are also desirable, but simple forms such as resale-price-maintenance (from now on RPM) or franchise fees are not sufficient to enforce the efficient outcome. The reason is the following. The objective of a vertically integrated structure is the maximum extraction of consumer surplus, and in order to achieve this, two instruments must be used: (i) quality differentiation, and (ii) "high" prices. If a manufacturer dealing with independent retailers uses a linear-price contract, which specifies only the wholesale price (at which retailers can buy any amount they wish), then retailers will have an incentive to differentiate, but competition will still lead to retail prices that are "too low." A franchise fee can only transfer profits from one party to the other, but cannot remedy the "inefficiency" caused by price competition. If, on the other hand, the manufacturer directly controls the retail price, by means of RPM, then the incentives for product differentiation are eliminated. Therefore, unlike the existing models of vertical restraints with no uncertainty,[3] franchise fees or RPM do *not* restore efficiency in our model, although we show that, from the point of view of the manufacturer, they are strictly better than the optimal linear-price contract. We also show that there exist more sophisticated forms of vertical restraint which do restore efficiency.

The paper is organized as follows. Section II presents the model. Section III characterizes the optimal solution for the verti-

2. For a locational model of vertical differentiation, see Bonanno [1986].
3. Rey and Tirole [1986b] have shown that when there is uncertainty about demand or retail costs, simple forms of vertical restraints may no longer be efficient.

cally integrated structure. Section IV compares the optimal linear-price contract with vertical integration. Section V analyzes two standard vertical restraints, namely franchise fees and RPM. Section VI describes an optimal contract, while Section VII offers some concluding comments.

II. The Model

We shall use a well-known model of consumer choice due to Gabszewicz and Thisse [1979]. There is a continuum of consumers represented by the unit interval [0,1]. Consumers have identical tastes, but different incomes. The income of consumer $t \in [0,1]$ is given by $E(t)$, where

$$E(t) = Et, \qquad E > 0. \tag{1}$$

For our purposes there is no loss of generality in assuming that $E = 1$.[4] Consumers are assumed to buy at most one indivisible unit of the good sold by retailers. We shall introduce the simplifying assumption[5] that there are only two retail outlets and that only two different levels of quality can be offered: a low level L (corresponding to the case where no services are provided) and a high level H (corresponding to the case where services are provided). When consumer t does not purchase the commodity, her utility is given by

$$V(0,E(t)) = U_o t, \qquad U_o > 0, \tag{2}$$

while if she buys one unit of a good of quality $k \in \{L, H\}$ at price p_k, her utility is given by

$$V(k,E(t) - p_k) = U(k)\,(t - p_k), \tag{3}$$

where

$$U_o < y \equiv U(L) < x \equiv U(H), \tag{4}$$

It can be shown[6] that if both retailers offer the same quality $k \in \{L,H\}$ at price p, demand is given by

$$D(p,k) = 1 - [p\,U(k)]/[U(k) - U_o], \tag{5}$$

while if they offer different qualities at prices p_H and p_L ($p_H > p_L$),

4. Thus, incomes are uniformly distributed in the interval [0,1].
5. We shall argue in Section VII that this assumption involves no loss of generality and discuss the consequences of relaxing it.
6. A derivation of the demand functions can be found in Bolton and Bonanno [1987].

demand is given by

(6a) $D_H(p_H,p_L) = 1 - (xp_H)/(x - y) + (yp_L)/(x - y)$

(6b) $D_L(p_H,p_L) = (xp_H)/(x - y)$
$$- [yp_L(x - U_o)]/[(x - y)(y - U_o)].$$

Throughout the paper, prices will be taken to be net of unit cost of supply, which is assumed to be constant and includes retailing expenses other than the cost of providing services (i.e., the cost of offering high quality). As far as these are concerned, it is standard in this type of model to assume that high quality can be produced at the same cost as low quality (see Shaked and Sutton [1982]).[7] We shall make the same assumption here. The justification for it is that, if in the presence of price competition one retailer will refrain from increasing the quality of his product—even though the higher quality could be produced at zero additional cost—then, a fortiori, he will refrain from increasing the quality of his product if the higher quality is more expensive to produce. There is, therefore, no loss of generality in this assumption. A similar argument shows that this assumption is also innocuous with respect to the optimal solution of the vertically integrated structure.[8]

III. THE VERTICALLY INTEGRATED STRUCTURE

A vertically integrated manufacturer could offer one quality (in which case he would choose p and k so as to maximize $pD(p,k)$, where D is given by (5)), or he could offer both qualities (in which case he would choose p_H and p_L so as to maximize $[p_HD_H(p_H,p_L) + p_LD_L(p_H,p_L)]$, where D_H and D_L are given by (6)).

LEMMA 1. A vertically integrated structure would offer both qualities at prices which we denote by $\hat{p}_H$ and $\hat{p}_L$.

This is a well-known result (a proof can be found in Bolton and Bonanno [1987]): by offering both qualities, the monopolist can

7. In a subsequent paper Shaked and Sutton [1983] showed that the introduction of costs of quality improvements can have important consequences from the point of view of the equilibrium market structure. This, however, is of no relevance to our model, since we have taken the number of retailers as exogenous and therefore we are not concerned with the endogenous determination of market structure at the retail level. The assumption of an exogenously given number of retail outlets is common to the majority of papers in the literature.

8. If a vertically integrated structure finds it profitable to offer two different qualities, even though there is no saving involved in the production of the low quality, then a fortiori it will find it profitable to offer differentiated products if there is such a saving.

discriminate between consumers with a high willingness to pay for services (that is, consumers with high income) and consumers with a low willingness to pay (see Mussa and Rosen [1978] and Gabszewicz et al. [1986]).[9]

IV. The Optimal Linear-Price Contract

We now look at the case where the manufacturer deals with independent retailers. The manufacturer faces many different contractual possibilities. The simplest contract is one where he fixes the wholesale price and sells whatever amount is demanded by retailers at that price. We shall show in this section that such a *linear-price contract* is inefficient from the point of view of the vertical structure.

We assume that, given the wholesale price charged by the manufacturer, which we denote by w, the retailers play a two-stage game as follows. First, they simultaneously decide which quality they want to offer. Then, having observed each other's quality, they simultaneously choose retail prices (that is, they simultaneously decide what markup to charge above the wholesale price). This is a standard way of modeling competition in quality and prices (see Shaked and Sutton [1982]), and is motivated by the observation that, typically, prices can be changed much more quickly and easily than qualities. For example, the quality produced by a retailer may depend on what type of store he sets up. If a retailer wants to change the quality of his product, he must "set up a new store" (e.g., change the location of his store; hire/lay off sales assistants; hire/lay off extra personnel for delivery, installation, repairs; increase his storing capacity in order to reduce waiting time for customers; increase parking space; etc.).

Given the wholesale price set by the manufacturer, w, if the two retailers choose the same quality, then, by Bertrand's theorem, their profits at the Nash equilibrium in prices will be zero. If, on the other hand, they differentiate their products, then their demand functions will be given by $D_H(m_H,m_L;w)$ and $D_L(m_H,m_L;w)$, obtained

9. Gabszewicz et al. [1986] have shown that in this model if we denote the range of incomes by $[a,b]$ then Lemma 1 is true if and only if $[b - a)/a] > (y/x)$. This condition is clearly satisfied in our case, where $a = 0$ and $b = 1$. If the opposite inequality holds, then the vertically integrated structure would offer only one quality. As was explained in the Introduction, we have concentrated on the case where Lemma 1 holds, because in this case the causes of vertical inefficiency are less obvious, since both the vertically integrated structure and independent retailers would choose to differentiate. In the alternative case it is easy to see that RPM is efficient.

from (6) by replacing p_H with $(m_H + w)$ and p_L with $(m_L + w)$, where m_H and m_L are the markups chosen by the high- and low-quality retailer, respectively. Their profit functions will be given by $\pi_H = m_H D_H(m_H, m_L; w)$ and $\pi_L = m_L D_L(m_H, m_L; w)$, respectively. We can now determine the subgame-perfect equilibria of the two-stage game for every possible wholesale price charged by the manufacturer.

LEMMA 2. If $w < w_o \equiv (y - U_o)/(y + U_o)$, there exists a unique[10] (pure-strategy) perfect equilibrium of the two-stage game at which the two retailers offer different qualities and the manufacturer's profits are given by

$$\pi_M(w) = \frac{3(x - U_o)}{4x - y - 3U_o} w - \frac{3xy - xU_o - 2yU_o}{(4x - y - 3U_o)(y - U_o)} w^2. \tag{7}$$

Lemma 2 states the well-known result (see Shaked and Sutton [1982]) that—provided the wholesale price is not too high—retailers use product differentiation in order to relax price competition (for a proof of Lemma 2 see Bolton and Bonanno [1987]).

When the manufacturer charges a very high wholesale price ($w \geq w_o$), retail prices are forced to be so high that it is now in the interest of the high-quality retailer to reduce his price sufficiently to force the low-quality retailer out of the market (a proof of this result can be found in Bolton and Bonanno [1987]). In this case the latter will be indifferent between facing zero demand, by offering low quality, and facing Bertrand competition, by offering high quality. We assume that he would choose the second option. All this is summarized in the following lemma.

LEMMA 3. If $w \geq w_o$, both retailers choose the high-quality level; their profits are zero; and the manufacturer's profits are given by

$$\hat{\pi}_M(w) = w - [x/(x - U_o)]w^2. \tag{8}$$

The question now arises of what the optimal linear-price contract is for the manufacturer. The next lemma shows that depending on the values of the parameters x, y, and U_o, the optimal wholesale price may or may not induce product differentiation. A proof is given in the Appendix.

10. The pure strategy equilibrium is unique from the point of view of the manufacturer (which is the point of view we are interested in) since it does not matter to him which retailer chooses high quality and which chooses low quality. There is also a symmetric mixed-strategy equilibrium, which is characterized in Bolton and Bonanno [1987].

LEMMA 4. The optimal wholesale price is less than w_o—and, therefore, it induces product differentiation—if and only if y is sufficiently close to x; more precisely, if and only if the following inequalities are simultaneously satisfied:

$$(9a)\qquad (x - U_o)(y + U_o) < 2x(y - U_o)$$

$$(9b)\qquad 9(x - U_o)^3(y + U_o)^2 > 4U_o(2x - y - U_o)(4x - y - 3U_o) \times (3xy - xU_o - 2yU_o).$$

The intuition behind Lemma 4 is as follows. If the manufacturer sets $w < w_o$, he induces differentiation, which in principle is "good" because it involves price discrimination, but retailers appropriate some of the profits. If he sets $w \geq w_o$, there will be no loss of profits to retailers, but no differentiation either. There is therefore a tradeoff. Now, if y is close to x, that is, if products are not too different in the eyes of consumers, price competition will be fierce; and thus $w < w_o$ will be better because it induces price discrimination, high sales, and little loss of profits to retailers.

Given lemmas 1 to 4, it is straightforward to establish the following proposition, which is proved in the Appendix.

PROPOSITION 1. With the optimal linear-price contract the sum of the profits of manufacturer and retailers is less than the profit of a vertically integrated structure.

Proposition 1 is obvious when (9) is not satisfied, for in this case retailers do not differentiate, and we know from Lemma 1 that it is not optimal for a vertically integrated structure to produce only one quality. When (9) is satisfied, however, retailers do differentiate (and make positive profits). The intuition behind Proposition 1 in this case is as follows: the "horizontal externality" due to price competition between retailers outweighs the "vertical externality" due to double marginalization. As a consequence, retail prices end up being "too low."

V. RESALE-PRICE-MAINTENANCE AND FRANCHISE FEES

Proposition 1 suggests that there is a role for vertical restraints in this model. We shall assume that the manufacturer has all the bargaining power; thus, he sets the contract, and retailers will accept any contract which gives them nonnegative profits (this assumption is common to all the existing literature).

The most obvious choice of vertical restraint for the manufacturer, in this model, would be one where each retailer is told what combination of quality and price to choose. We assume, however, that the contract between the manufacturer and a retailer cannot be made contingent on the quality (services) chosen by the latter. This will be the case, for example, if quality is not verifiable by a court. Alternatively, it may be too difficult or too costly to fully describe the quality that a retailer is supposed to supply.[11] The fact that the manufacturer cannot sign contracts which are contingent on quality would be of no consequence if he distributed his output to only one retailer (in which case a zero wholesale price plus a franchise fee would be sufficient to achieve efficiency). There are situations, however, where this is not in the manufacturer's interest. For example, it could be the case that each retailer has a captive market, as well as a share of a market which is in common with the other retailer (we have only formalized the common market above). Then, if the manufacturer supplies only one retailer, he will lose one of the captive markets.[12] In this section we examine two simple and very common forms of vertical restraint, namely RPM and franchise fees (for other restraints see footnote 14), and show that, *although they are superior to the optimal linear-price contract from the point of view of the manufacturer, they are not equivalent to vertical integration.* RPM is a provision in the contract that dictates the choice of the final price to the retailer,[13] while a franchise fee is a fixed payment from the retailer to the manufacturer (therefore the combination of a franchise fee and a constant wholesale price gives rise to the simplest form of nonlinear pricing). A proof of the following proposition, which is given in the Appendix.

PROPOSITION 2. Both RPM and franchise fee dominate the optimal linear-price contract from the point of view of the manufacturer.

The proof for the franchise fee is straightforward: whenever retailers make positive profits with a linear-price contract, the manufac-

11. In our model, where there are only two quality levels, this may be a strong assumption. In a more general model, however, where quality can vary over a continuum or involve many dimensions, this would not be a strong assumption.

12. This problem would disappear if the manufacturer could monitor in which market each retailer sold the commodity, for then he could give the whole common market to one retailer and let the other retailer supply his own captive market. We are therefore assuming that the monopolist cannot monitor where each retailer sells the commodity.

13. Thus, as we have defined it, RPM consists in a price floor and a price ceiling which coincide. It can be shown that, in this model, if the two were to be different, the same results would hold. The reason is that it is the price of the *high-quality* product which is "too low" (cf. the proof of Proposition 1).

turer can use a franchise fee to extract part or all of these profits. Less obvious is the result concerning RPM. When the retail price is fixed by the manufacturer, the incentives for product differentiation are eliminated. *By preventing price competition, the manufacturer exacerbates quality competition.* Consequently, both retailers will offer the same (high) quality (the manufacturer can then set the wholesale price equal to the retail price and extract all the retailers' profits). The absence of product differentiation, however, implies that there is no price discrimination. The reason why the manufacturer prefers RPM is that the loss of profits to retailers and the fact that price competition keeps prices too low outweigh the benefit from price discrimination.

In most of the existing models of vertical restraints where the number of retailers is fixed exogenously, RPM or franchise fees achieve the same outcome as vertical integration. This is not the case in our model.

PROPOSITION 3. Franchise fee and RPM contracts are inefficient from the point of view of the vertical structure.

The proof of Proposition 3 is straightforward: with RPM, retailers do not differentiate, while a vertically integrated structure would (Lemma 1); franchise fees, on the other hand, merely entail a transfer of profits from retailers to manufacturer, and we know from Proposition 1 that the total sum of profits is less than the profit of a vertically integrated structure. It is worth recalling, however, that one can find configurations of demand for which it is optimal for the vertically integrated manufacturer not to price discriminate (see footnote 9). For those configurations of demand RPM *is* efficient.

VI. AN OPTIMAL CONTRACT

The above discussion suggests that an optimal contract ought to restrict the set of prices which can be chosen by retailers. This can be done by means of a price-dependent franchise fee, as shown in the following proposition, which is proved in the Appendix.

PROPOSITION 4. The manufacturer can approximate the outcome of vertical integration arbitrarily closely by fixing the wholesale price

$$w = \hat{p}_L, \tag{10}$$

(where $\hat{p}_L$ is defined in Lemma 1) and the following franchise fee:

$$
(11) \qquad \begin{aligned} F &= +\infty && \text{if retail price} \neq \hat{p}_H \text{ or } w \\ &\ 0 && \text{if both retail prices are equal to } w, \end{aligned}
$$

and otherwise

$$
(12) \qquad \begin{aligned} F &= -\epsilon && \text{if retail price} = w \\ &\ (\hat{p}_H - w)D_H(\hat{p}_H,\hat{p}_L) - \epsilon && \text{if retail price} = \hat{p}_H, \end{aligned}
$$

where $\hat{p}_H$ is defined in Lemma 1, D_H is given by (6a), and $\epsilon > 0$ is arbitrarily small. The intuition behind Proposition 4 is as follows. The clause "$F = +\infty$ if the retail price is different from $\hat{p}_H$ or w" has the purpose of restricting the set of retail prices that can be chosen by retailers to $\{\hat{p}_H,\hat{p}_L\}$, that is, to the prices which would be chosen by a vertically integrated structure (thereby eliminating the problem discussed above, namely that the horizontal externality between retailers leads to retail prices which are "too low"). Second, the contract must ensure that retailers prefer to differentiate rather than produce the same quality. This is achieved by (12), which ensures them an equal profit of ϵ if they differentiate and negative profits if they both choose high quality. Finally, the clause "$F = 0$ if both retail prices are equal to w" ensures that retailers will not make positive profits if they both choose low quality.[14]

The contract described in Proposition 4—and any contract in which *all* retail prices are taken into account—may be extremely costly to enforce, especially when the number of retailers is large (which is usually the case). Therefore, a simpler—although suboptimal—contract may be preferable. The following proposition shows that when the degree of product differentiation is small, RPM is a good substitute for an optimal contract (for a proof see Bolton and Bonanno [1987]).

PROPOSITION 5. When y is close to x, RPM yields an outcome which is close to that of vertical integration.

The intuition behind this result is clear: when y is close to x,

14. There are a number of other contracts that can approximate the outcome of vertical integration. For example, the manufacturer could use RPM to force one retailer to charge $\hat{p}_H$ and the other retailer to charge $\hat{p}_L$ and then use *quantity fixing* to force one retailer to buy $D_H(\hat{p}_H,\hat{p}_L)$ and the other to buy $D_L(\hat{p}_H,\hat{p}_L)$. Such a contract, however, is inferior to the one described in Proposition 4 whenever demand is subject to random fluctuations that are observed by the retailers but not by the manufacturer.

consumers' willingness to pay for quality does not vary much with income; and therefore the extra consumer surplus extracted by means of price discrimination (through quality differentiation) is small. It is also worth recalling that when y is close to x the optimal linear-price contract induces product differentiation (Lemma 4). Finally, as was pointed out before, RPM is sufficient to fully restore vertical efficiency in those cases where the configuration of demand is such that the vertically integrated structure would not differentiate (see footnote 9).

VII. Concluding Comments

The model used in this paper makes the simplifying assumption that there are only two retail outlets and only two quality levels. A more general model would allow for n retailers and a continuum of qualities. The results we have proved, however, would hold also in such a model. In fact, the incentives for retailers to differentiate are present whatever the number of firms (see Shaked and Sutton [1983]) and, similarly, a vertically integrated structure would find it profitable to offer all possible qualities (see Gabszewicz et al. [1986]). By allowing for a continuum of quality levels, one extra result can be obtained: the qualities chosen by a vertically integrated structure differ from the qualities chosen by retailers (for a proof of this result in the case of two retailers, see Bolton and Bonanno [1987]). As remarked in the Introduction, however, this vertical inefficiency can be captured even more clearly in a specification of our model where a vertically integrated structure would want to offer only one quality (cf. footnote 9).

In this paper we have considered the case where the manufacturer's product can be differentiated at the retail level on the basis of cum-sales or post-sales services. There is ample evidence that manufacturers of a wide variety of products, from beer to bicycles, are worried about retailers' incentives to use such services as an instrument of vertical differentiation. Coors, for example, pointed out how refrigeration and product rotation services could affect the quality of its beer and specifically complained about distributors who would not offer these services and sell the beer at a discount (see McLaughlin [1979]). Raleigh insisted on how the quality of its bicycles depended both on pre-delivery services (pre-sales inspection, final assembly, and adjustments) and post-delivery services (repairs, technical advice, and the stocking by retailers of an adequate range of spare parts for their own use and for sale to customers). Again, Raleigh's main concern was with discount stores that would not offer these services and sell at a lower price (see Monopolies Commission Report [1981]).[15] These two examples, however, are complicated by the fact that the manufacturer was competing with other manufacturers. In this paper we have followed the existing literature on vertical restraints and restricted ourselves to the case of *one* manufacturer dealing with many retailers. Allowing competition between manufacturers complicates the analysis considerably. Some of the issues arising in this context have been analyzed by Bonanno and Vickers [1987].

APPENDIX

Proof of Lemma 4. Let w^* be the (unique) maximum of the function $\pi_M(w)$ (given by (7)). Then

$$w^* = \frac{3(x - U_o)(y - U_o)}{2(3xy - xU_o - 2yU_o)}, \tag{A.1}$$

and

$$\pi_M(w^*) = \frac{9(x - U_o)^2(y - U_o)}{4(4x - y - 3U_o)(3xy - xU_o - 2yU_o)}. \tag{A.2}$$

Let $\hat{w}^*$ be the (unique) maximum of the function $\hat{\pi}_M(w)$ (given by (8)). Then

$$\hat{w}^* = (x - U_o)/(2x), \tag{A.3}$$

and

$$\hat{\pi}_M(\hat{w}^*) = (x - U_o)/(4x). \tag{A.4}$$

The following facts can be checked easily:

$$w^* < \hat{w}^* \tag{A.5}$$

$$\pi_M(w^*) < \hat{\pi}_M(\hat{w}^*). \tag{A.6}$$

It follows that as soon as $\hat{w}^* \geq w_o$, the manufacturer will want to set $w = \hat{w}^*$, thereby inducing retailers *not* to differentiate. Now,

$$\hat{w}^* \geq w_o \quad \text{if and only if} \quad (x - U_o)(y + U_o) \geq 2x(y - U_o) \tag{A.7}$$

(a necessary condition for this is that y *not* be close to x). On the

15. Both these examples can be accommodated in a specification of our model where a vertically integrated structure would find it optimal to offer only one quality.

other hand, if

$$(x - U_o)(y + U_o) < 2x(y - U_o) \qquad \text{(that is, if } \hat{w}^* < w_o) \tag{A.8}$$

(which is the case if y is close to x, since when $y = x$, (A.8) becomes $2x > x + U_o$, which is obviously true), then by (A.5) also $w^* < w_o$, and the manufacturer's choice will depend on whether $\pi_M(w^*)$ or $\hat{\pi}_M(w_o)$ is larger. Note that $\hat{\pi}_M(w)$ is strictly concave, and therefore, given that $\hat{w}^* < w_o$, the best wholesale price in $[w_o, +\infty)$ is w_o. Now,

$$\hat{\pi}_M(w_o) = \frac{U_o(y - U_o)(2x - y - U_o)}{(x - U_o)(y + U_o)^2}. \tag{A.9}$$

Thus,

$$\pi_M(w^*) > \hat{\pi}_m(w_o) \tag{A.10}$$

if and only if

(A.11)

$$\left\{\frac{9(x - U_o)^2}{4(4x - y - 3U_o)(3xy - xU_o - 2yU_o)}\right\} > \left\{\frac{U_o(2x - y - U_o)}{(x - U_o)(y + U_o)^2}\right\},$$

which is the case if y is close to x, since when $y = x$, (A.11) becomes $(x - U_o)^2 > 0$, which is obviously true. (A.8) and (A.11) prove (9).

To sum up, if (9) is satisfied, the optimal wholesale price is w^* (given by (A.2)) which is less than w_o and therefore, by Lemma 2, induces product differentiation. If (9) is not satisfied, then the optimal price will be

(A.12)

$$\begin{aligned} \hat{w}^* &= (x - U_o)/(2x) && \text{if } (x - U_o)(y + U_o) \geq 2x(y - U_o) \\ w_o &= (y - U_o)/(y + U_o) && \text{if } (x - U_o)(y + U_o) < 2x(y - U_o) \end{aligned}$$

and since in both cases $w \geq w_o$, retailers will not differentiate.

Proof of Proposition 1. When the optimal linear-price contract (from now on OLPC) induces retailers to choose the same quality, Proposition 1 follows directly from Lemma 1. Therefore, we shall concentrate on the case where the OLPC induces differentiation. The prices chosen by a vertically integrated structure (cf. Lemma 1) are given by

$$\hat{p}_H = 2y(x - U_o)/(3xy + xU_o + y^2 - yU_o) \tag{A.13}$$

$$\hat{p}_L = (x + y)(y - U_o)/(3xy + xU_o + y^2 - yU_o). \tag{A.14}$$

The markup chosen by the retailers at the Nash equilibrium in prices when products are differentiated are given by (see Bolton and Bonanno [1987])

$$\text{(A.15)} \qquad m_H^*(w) = \frac{2(x - U_o)(x - y) - w(2x - U_o)(x - y)}{x(4x - y - 3U_o)}$$

$$\text{(A.16)} \qquad m_L^*(w) = \frac{(x - y)(y - U_o) - w(y + U_o)(x - y)}{y(4x - y - 3U_o)}.$$

Now, it can be shown that both $[m_H^*(w) + w]$ and $[m_L^*(w) + w]$ are increasing in w and that there is a value of w, call it $\tilde{w}$, at which

$$\text{(A.17)} \qquad m_L^*(\tilde{w}) + \tilde{w} = \hat{p}_L \quad \text{but} \quad m_H^*(\tilde{w}) + \tilde{w} < \hat{p}_H.$$

Therefore, there does not exist a w such that

$$\text{(A.18)} \qquad m_L^*(w) + w = \hat{p}_L \quad \text{and} \quad m_H^*(w) + w = \hat{p}_H.$$

This implies that with the OLPC retail prices differ from the prices chosen by a vertically integrated structure, and since the latter's profit function is strictly concave, this completes the proof of Proposition 1.

Proof of Proposition 2. We only need to give the proof for RPM. By Lemma 4 and (A.6), if (9) is satisfied, the manufacturer would be better off by using RPM and fixing the retail price (and the wholesale price) equal to $\hat{w}^*$. By (A.6), (A.7), and (A.12), this is true also if (9) is not satisfied but $(x - U_o)(y + U_o) < [2x(y - U_o)]$. Finally, by (A.12), if (9) is not satisfied and $(x - U_o)(y + U_o) \geq [2x(y - U_o)]$, then RPM and the OLPC are equivalent.

Proof of Proposition 4. The proof will be based on the following lemmas.

LEMMA (i). If the two retailers choose different qualities, there is a unique Nash equilibrium (N.E.) in prices at which the high-quality retailer's price is $\hat{p}_H$ and the low-quality retailer's price is $\hat{p}_L$ and both retailers make positive profits (given by ϵ).

Proof. First we show that $(\hat{p}_H, \hat{p}_L)$ is a N.E.: the low-quality retailer's profits are $\epsilon > 0$; while if he switched to $\hat{p}_H$, he would face zero demand and pay a positive franchise fee (if ϵ is sufficiently small). The high-quality retailer's profits are ϵ; while if he switched to $\hat{p}_L$, his profits would be zero. Next we show that $(\hat{p}_H, \hat{p}_H)$ is not a N.E.: the low-quality retailer faces zero demand and pays a positive franchise fee; while he can increase his profits to ϵ by switching to $\hat{p}_L$. Similarly, $(\hat{p}_L,\hat{p}_L)$ is not a N.E. because the high-quality retailer can increase his profits from zero to ϵ by switching to $\hat{p}_H$.

LEMMA (ii). If both retailers choose high quality, there is a unique N.E. where they both charge the same retail price $\hat{p}_L$ and make zero profits.

Proof. $(\hat{p}_L,\hat{p}_L)$ is a N.E. because both retailers make zero profits, and if either retailer switched to $\hat{p}_H$, he would face zero demand and pay a positive franchise fee (if ϵ is sufficiently small). $(\hat{p}_H,\hat{p}_L)$ is not a N.E. because the retailer with the higher price pays a positive franchise fee and faces zero demand; while he can make zero profits by switching to $\hat{p}_L$. $(\hat{p}_H,\hat{p}_H)$ is not a N.E. because each retailer's profits are

$$\text{(A.19)}\quad (1/2)(\hat{p}_H - w)D(\hat{p}_H,H) - F = (1/2)(\hat{p}_H - w)D(\hat{p}_H,H) - (\hat{p}_H - w)D_H(\hat{p}_H,\hat{p}_L) + \epsilon.$$

Now, using (5) and (A.13), we have

$$\text{(A.20)}\quad (1/2)D(\hat{p}_H,H) = (1/2) - [xy(x - U_o)]/[(x - U_o)(3xy + xU_o + y^2 - yU_o)],$$

while (see Bolton and Bonanno [1987])

$$\text{(A.21)}\quad D_H(\hat{p}_H,\hat{p}_L) = x(y + U_o)/(3xy + xU_o + y^2 - yU_o).$$

It is easy to show that (A.21) > (A.20) and therefore if ϵ is sufficiently small, both retailers are making negative profits, while either of them could make zero profits by switching to $\hat{p}_L$.

LEMMA (iii). If both retailers choose low quality, there is a unique N.E. where they both charge the same retail price $\hat{p}_L$, and make zero profits.

Proof. $(\hat{p}_L,\hat{p}_L)$ is a N.E. because if either retailer switched to $\hat{p}_H$, he would make negative profits. $(\hat{p}_H,\hat{p}_L)$ is not a N.E. because the retailer with the higher price makes negative profits. Finally, $(\hat{p}_H,\hat{p}_H)$ is not a N.E. because each retailer's profit is given by

$$\text{(A.22)}\quad (1/2)(p_H - w)D(p_H,L) - F = (1/2)(p_H - w)D(p_H,L) - (p_H - w)D_H(p_H,p_L) + \epsilon.$$

Now, using (5) and (A.13), we have

$$\text{(A.23)}\quad (1/2)D(\hat{p}_H,L) = (1/2) - \frac{y^2(x - U_o)}{(y - U_o)(3xy + xU_o + y^2 - yU_o)},$$

while $D_H(\hat{p}_H,\hat{p}_L)$ is given by (A.21). Again, it is easy to show that (A.21) > (A.23), and therefore if ϵ is sufficiently small, each retailer makes negative profits.

By lemmas (i)–(iii) we can conclude that there is a unique perfect equilibrium of the two-stage game at which one retailer chooses high quality and charges $\hat{p}_H$ and the other retailer chooses low quality and charges $\hat{p}_L$ and each retailer's profits are $\epsilon > 0$. This is the choice of qualities and prices of the vertically integrated structure, and therefore by choosing ϵ arbitrarily small, the manufacturer can approximate the outcome of vertical integration.

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