# GENERAL EQUILIBRIUM THEORY WITH IMPERFECT COMPETITION[1]

Giacomo Bonanno
*University of California, Davis*

**Abstract.** This survey is organized as follows. I. Introduction. II. The main issues. III. Negishi's model. IV. Objective demand in the Cournot-Nash framework. V. Objective demand in the Bertrand-Nash framework. VI. The assumption of quasi-concavity of the profit functions. VII. Compromises between the conjectural and the objective approach. VIII. Insights into the notion of perfect competition. IX. Conclusion.

**Keywords.** general equilibrium theory, imperfect competition, conjectural demand, objective demand, Nash equilibrium.

## 1. Introduction

The first rigorous analysis of the behaviour of firms which do not treat prices as exogenous parameters is due to Cournot (1838), whose book preceded Walras's (1874) by thirty-six years. Cournot studied the case of a homogeneous-product industry where each firm knows the inverse demand function which associates with every value of industry output the common price at which every firm is able to sell. Each firm independently chooses its own output level. The notion of equilibrium put forward by Cournot is a special case of the more general notion of Nash equilibrium introduced later (Nash, 1950, 1951). It consists of a list of output levels, one for each firm, with the property that no firm can increase its profits by *unilaterally* changing its output. We shall refer to this notion of equilibrium as *Cournot-Nash*. On the other hand, when the decision variable of a firm is the price it charges — rather than its own output level — the corresponding notion of equilibrium will be referred to as *Bertrand-Nash*.[2]

The 1930s saw, with the publication of the books by Chamberlin (1933) and Robinson (1933), an upsurge of interest in the theory of imperfect competition. The approach, however, remained one of partial equilibrium and some authors (e.g. Triffin, 1940) expressed the hope that a general theory of monopolistic competition could be developed that would match the scope of the Walrasian theory of general equilibrium with perfect competition. While the latter was fully developed and systematized in the 1950s (Arrow and Debreu, 1954, Debreu, 1959), the first attempt to introduce imperfect competition in a general equilibrium model was made as late as 1961 by Negishi. Since then a large number of contributions have appeared, but unfortunately we are still far from a satisfactory theory of general equilibrium with imperfect competition.

0950–0804/90/04 297–32 \$02.50/0
JOURNAL OF ECONOMIC SURVEYS Vol. 4, No. 4

In this survey I shall attempt to highlight the main problems that arise when the hypothesis of perfect competition is relaxed and discuss the way in which such problems have been dealt with (or set aside) in the literature. After a brief outline of some of the main issues (section 2), I shall take as point of departure (section 3) the pioneering contribution of Negishi (1961), which falls within the Cournot-Nash approach. In section 4 more recent developments within the same framework are reviewed. In section 5 I shall abandon the Cournot-Nash hypothesis and discuss models which fall within the Bertrand-Nash approach. In sections 6 and 7 I deal with issues that apply to both approaches. In section 8 I briefly mention some new insights into the notion of perfect competition which have emerged from the literature on imperfect competition. Section 9 contains some concluding remarks. Due to space limitations, I will be unable to cover all the contributions to the literature on general equilibrium with imperfect competition. In particular, I will not discuss the recent and numerous contributions to the foundations of the theory of unemployment. An excellent and up-to-date survey of this part of the literature can be found in Silvestre (1988, 1989).

## 2. The main issues

There are essentially three reasons why it seems desirable to have a well-developed theory of general equilibrium with imperfect competition.

The first is realism: real-world economies are characterized by the widespread presence of large firms whose behaviour can hardly be captured by the hypothesis of price-taking.

The second reason is that an enormous amount of literature has investigated the cases of monopoly, oligopoly and monopolistic competition from the partial-equilibrium point of view and it seems that a general equilibrium synthesis would be necessary in order to somehow complete the theory and check its consistency.

The third reason is that there seem to be some conceptual difficulties with the notion of *perfect* competition which one may be able to solve by appealing to the notion of *imperfect* competition. As Hahn (1987, p. 321) points out, the notion of Walrasian equilibrium is implicitly based on some hypothesis of what would happen out of equilibrium:

> 'Any textbook will tell you that market equilibrium entails the equality of supply and demand. To be more precise, it entails the equality of what some price-taking agents would optimally demand and some other price-taking agents would optimally supply. Now ask: why has this particular state been singled out as an equilibrium? I think that the answer which would most frequently be given is that in any other state prices would change. But that is a proposition of dynamics by which I mean a theory which explains the temporal movement (as well as the temporary constancy) of an economic variable. In this case the theory is as follows: the rate of change of the price of any good is a sign preserving function of the excess demand for it and a zero excess demand is a critical point of this dynamics. Our willingness to

> accept the text-book equilibrium notion must be contingent on our willingness to accept the dynamics which it entails.'

However, as Arrow (1959) pointed out, it is impossible to reconcile the hypothesis of price-taking agents with the notion of a dynamic adjustment of prices towards a Walrasian equilibrium: if each agent in the economy considers prices to be beyond his control, who changes prices out of equilibrium? The auctioneer, of course, is simply a fictitious device which leaves that basic question unanswered: 'if we apply the methodology of the textbook consistently then we shall want price changes to be the outcome of the calculations of rational optimizing agents and not hand them over to an auctioneer' (Hahn, 1987, p. 322).

What does relaxation of the price-taking hypothesis entail? An agent who is not a price-taker is an agent who realizes that the decisions she makes affect her environment: for example, the output she chooses determines the price at which she will be able to sell, or the price she charges affects the quantity that will be demanded. What behaviour do we expect from a rational agent who has market power (that is, who is not a price taker)? The answer obviously depends on what we mean by rationality. As Arrow (1986, p. 206) points out, the common understanding of the notion of rationality involves not only maximization of a well-specified objective function, but also sound reasoning, the complete exploitation of information, etc. The question of what an imperfect competitor does, or should, know is an important one. While in a perfectly competitive world the individual agent needs to know 'only' the relevant prices in order to choose an optimal action,

> 'the knowledge requirements of the decision maker change radically under monopoly or other forms of imperfect competition. Consider the simplest case, pure monopoly in a one-commodity partial equilibrium model, as originally studied by Cournot in 1838. The firm has to know not only prices but a demand curve. Whatever definition is given to complexity of knowledge, a demand curve is more complex than a price. It involves knowing about the behaviour of others. ... From a general equilibrium point of view, the difficulties are compounded. The demand curve relevant to the monopolist must be understood *mutatis mutandis*, not *ceteris paribus*. A change in the monopolist's price will in general cause a shift in the purchaser's demands for other goods and therefore in the prices of those commodities. These price changes will in turn by more than one channel affect the demand for the monopolist's produce and possibly also the factor prices that the monopolist pays. The monopolist, even in the simple case where there is just one in the entire economy, has to understand all these repercussions. In short, the monopolist has to have a full general equilibrium model of the economy. The informational and computational demands become much stronger in the case of oligopoly or any other system of economic relations where at least some agents have power against each other. There is a qualitatively new aspect to the nature of knowledge, since each agent is assuming the *rationality* of other agents' (Arrow, 1986, pp. 207–208).

In the passage quoted above Arrow highlights two theoretical issues which must be faced when attempting to relax the assumption of price-taking in a general equilibrium model: (i) the notion of demand curve, and (ii) the notion of rational behaviour. The latter is not an issue which is specific to the general equilibrium approach: even at the partial equilibrium level the problem arises as to what constitutes a rational choice of action in a situation where a number of agents realize that their utility is affected not only by their own actions but also by the actions of other agents.[3] The traditional way of dealing with this problem is to identify rational behaviour with the notion of Nash equilibrium. A discussion of the adequacy of this notion is beyond the scope of this paper.[4] We shall therefore concentrate on the first issue.

In the passage quoted above Arrow points out one important problem concerning the introduction of imperfect competition in a general equilibrium model: the 'parameters' of the demand function (such as the prices of other goods and consumers' incomes) which are ignored in a partial equilibrium model (by appealing to the *ceteris paribus* assumption) can no longer be left out. There is, however, another problem:

> 'In the theories of monopolistic competition one talks often and easily about the "demand function" of a firm. One is happy enough not to know what it is so long as one's concern is only with the behavior of the firm, or with the monopolistic competition among several firms having respective demand functions. But happiness fades when one becomes seriously interested in the working of a national economy involving monopolistic competition where all economic agents are mutually interdependent in a completely circular way.
>
> Demand for goods must be effective demand coming from the incomes earned by agents in the national economy. The traditional oligopoly theorist pays little attention to the source of effective demand. He lets a monopolist seek a maximum of its profits calculated in terms of its demand function. Suppose the maximum monopoly profit is distributed among certain agents. The distributed profit will be spent and will result in the effective demand for goods. *Thus, the demand function may have profit as one of its arguments*. How ignorant can the theorist be of the possible inconsistency of profit as one of the arguments in the demand function with that as the firm's maximand calculated in terms of the function?' (Nikaido, 1975, p. 7; emphasis added).

What Nikaido is referring to is the so-called 'feedback effect' or 'Ford effect'[5] and it concerns the circularity between demand, price/quantity decisions and profit. Thus the question arises whether the notion of demand function can be given a coherent meaning in a general equilibrium context.[6]

Finally, there is one more problem that arises when the assumption of price taking is relaxed in a general equilibrium model. It is standard to assume that firms' objective is profit maximization. However, as Hart (1985, pp. 106–107) observes,

> 'In a competitive framework, this is usually accepted without question as the right objective for a firm. Under imperfect competition matters are more complicated, a point noted a long time ago by Marshall (1940, p. 402) and emphasized more recently by Gabszewicz and Vial (1972). The reason is that the owners of a firm are interested not in monetary profits per se, but rather in what this profit can buy. Given that a monopolistically competitive firm can influence prices, the owners may prefer low monetary profit but favourable prices for consumption goods to high monetary profit and unfavourable prices.
>
> This argument suggests that we should substitute owner utility maximization for profit maximization as the firm's goal. Unfortunately, things are not that simple. If owners have different tastes, they will have different trade-offs concerning high monetary profit versus favourable consumption goods prices. That is, each owner will have his own private objective function which he would like the firm to pursue, and the problem then is how to aggregate these into an overall objective function.'

This problem has been noted by a number of authors,[7] but no satisfactory alternative to the assumption of profit maximization has been put forward.

Having outlined the main issues involved, we can now turn to a direct examination of the main contributions to the subject. It is worth noting from the very beginning that the literature on imperfect competition in general equilibrium has mainly focused on the question of existence of an equilibrium. While general equilibrium theory with perfect competition has gone beyond mere existence results, by establishing (under suitable conditions) the Pareto efficiency of Walrasian equilibria and the possibility of decentralizing any Pareto efficient allocation, no such results are available in a world of imperfect competition. Indeed, Nash equilibria are typically Pareto inefficient (see Grote, 1974, for a precise statement) and the efficiency of Walrasian equilibria is a feature which quickly disappears as soon as we relax any one of the crucial assumptions of the Walrasian model (price taking, complete markets, etc.).

## 3. Negishi's model

Since Negishi's contribution was the first attempt to deal with imperfect competition at the general equilibrium level, it represents a natural starting point. Negishi's model is along the lines of Debreu's (1959) characterization of a private-ownership economy: there are $m$ goods, $n$ consumers (each consumer is a price taker) and $r$ firms, of which $r'$ are perfectly competitive (with $0 \leqslant r' < r$). Let $F_P = \{1, 2, \ldots, r'\}$ be the set of perfectly competitive firms and $F_M = \{r' + 1, \ldots, r\}$ be the set of monopolistically competitive firms. We let $\mathbf{p} \in \mathbb{R}^m$ denote a price vector. Each consumer $i (i = 1, \ldots, n)$ is characterized by a consumption set $X_i$ (a subset of the commodity space $\mathbb{R}^m$), a utility function $U_i : X_i \to \mathbb{R}$, an initial endowment of goods $e_i \in \mathbb{R}^m$ and a share $\theta_{ik} \in [0, 1]$ of the profits of firm $k$ ($k = 1, \ldots, r$; for each $k$, the sum of the $\theta_{ik}$'s over the set of consumers is equal to 1). Each firm $k$ is characterized by a production set $Y_k$ (a subset of the commodity space $\mathbb{R}^m$). A production decision for firm $k$ is a vector $y_k = (y_{k1}, \ldots, y_{km}) \in Y_k$ and the convention is that if the $j$th component of $y_k$, $y_{kj}$, is negative, then good $j$ is an input for firm $k$, while if it is positive, it is an output. For each monopolistic firm $k \in F_M$ we denote by $J^k$ the set of goods for which the firm has market power (monopoly or monopsony power). Negishi assumes that if firm $k$ is an imperfect competitor with respect to good $j$, then no other firm is: for every $k, k' \in F_M$, $J^k \cap J^{k'} = \emptyset$. This assumption rules out homogeneous-product oligopoly and bilateral monopoly (it does allow for multi-product monopolistic firms, however). It is also assumed that no firm is an imperfect competitor for all goods, that is, $J^k$ is a proper subset of the set of goods, for every $k \in F_M$.

A *state of the market* (or *status quo*) is a pair $(\hat{\mathbf{p}}, \hat{\mathbf{w}})$, where $\hat{\mathbf{p}}$ is a price vector and $\hat{\mathbf{w}} = (\hat{x}_1, \ldots, \hat{x}_n;\ \hat{y}_1, \ldots, \hat{y}_r)$ is a consumption-production allocation [$\hat{\mathbf{x}}_i = (\hat{x}_{i1}, \ldots, \hat{x}_{im})$ is consumer $i$'s consumption decision and $\hat{\mathbf{y}}_k = (\hat{y}_{k1}, \ldots, \hat{y}_{km})$ is firm $k$'s production decision]. Negishi assumes that each monopolistic firm $k \in F_M$ in each state of the market $(\hat{\mathbf{p}}, \hat{\mathbf{w}})$, has — for each $j \in J^k$, that is, for each good for which it has market power — some *conjectures*

$$p_j(y_{kj};\ \hat{\mathbf{p}}, \hat{\mathbf{w}})$$

concerning the price the firm could charge (would have to pay) if its output (input) were $y_{kj}$. Negishi imposes two restrictions on these conjectures. The first is a consistency condition: if $y_{kj}$ is equal to $\hat{y}_{kj}$ (recall that $\hat{y}_{kj}$ is the value of $y_{kj}$ at the status quo), then the conjectured price $p_j$ must coincide with the observed price $\hat{p}_j$, that is,

$$p_j(\hat{y}_{kj};\ \hat{\mathbf{p}}, \hat{\mathbf{w}}) = \hat{p}_j.$$

This means that the conjectural demand curve must pass through the observed *status quo*.[8] The second restriction on conjectures imposed by Negishi is that in each state of the market $(\hat{\mathbf{p}}, \hat{\mathbf{w}})$ the graph of the conjectural (inverse) demand function $p_j(y_{kj};\ \hat{\mathbf{p}}, \hat{\mathbf{w}})$ be a decreasing straight line (that is, the function be affine and decreasing).

To illustrate the conjectures postulated by Negishi, suppose that firm $k$ is an imperfect competitor for goods 3, 5 and 9, which it produces. Given a state of the market $(\hat{\mathbf{p}}, \hat{\mathbf{w}})$, the firm conjectures the inverse demand functions:

$$\begin{aligned} p_3(y_{k3};\ \hat{\mathbf{p}}, \hat{\mathbf{w}}) &= a - by_{k3} && \text{(with } b > 0) \\ p_5(y_{k5};\ \hat{\mathbf{p}}, \hat{\mathbf{w}}) &= \alpha - \beta y_{k5} && \text{(with } \beta > 0) \\ p_9(y_{k9};\ \hat{\mathbf{p}}, \hat{\mathbf{w}}) &= c - dy_{k9} && \text{(with } d > 0) \end{aligned}$$

where the parameters $a, b, \alpha, \beta, c, d$ are allowed to vary (in a continuous, but otherwise arbitrary, way) with the state of the market $(\hat{\mathbf{p}}, \hat{\mathbf{w}})$. The consistency condition requires that

$$\begin{aligned} a - b\hat{y}_{k3} &= \hat{p}_3 \\ \alpha - \beta\hat{y}_{k5} &= \hat{p}_5 \\ c - d\hat{y}_{k9} &= \hat{p}_9 \end{aligned}$$

Thus in the *status quo* $(\hat{\mathbf{p}}, \hat{\mathbf{w}})$, firm $k$ conjectures that if it changes its output of, say, good 5 from $\hat{y}_{k5}$ to $\tilde{y}_{k5}$, the price at which it will be able to sell it will change from $\hat{p}_5$ to $\tilde{p}_5 = \alpha - \beta\tilde{y}_{k5}$. Note that these conjectures are particularly simple, since firm $k$ believes that if, *ceteris paribus*, it changes its output of good 5 then only the price of good 5 will change, while all the other prices will remain constant.[9] This way of modelling conjectures gives a partial equilibrium flavour to the analysis, but it also provides a numeraire in terms of which firm $k$ measures its profits (the set of goods which are not in $J^k$: recall the assumption that $J^k$ is a proper subset of the set of goods).

Negishi then defines an equilibrium — which we shall call a *conjectural Cournot-Nash-Walras equilibrium* — as a state of the market where demand equals supply in each market and furthermore every consumer is maximizing utility (within her budget set), every competitive firm is maximizing profits as price-taker, and every monopolistically competitive firm is making a production decision which — on the basis of the state of the market and the firm's conjectural demand — is a profit-maximizing one. Formally, a conjectural Cournot-Nash-Walras equilibrium is a state of the market $(\hat{\mathbf{p}}, \hat{\mathbf{w}})$ such that:[10]

(3.1) for every consumer $i(i = 1, \ldots, n)$, $\hat{\mathbf{x}}_i$ belongs to the budget set

$$B = \{\mathbf{x}_i \in X_i \mid \hat{\mathbf{p}} \cdot \mathbf{x}_i \leqslant \hat{\mathbf{p}} \cdot \mathbf{e}_i + \sum_{k=1}^{r} \theta_{ik} \ (\hat{\mathbf{p}} \cdot \hat{\mathbf{y}}_k)\}$$

and maximizes the consumer's utility function $U_i$ in $B$;

(3.2) for every competitive firm $k \in F_p$, $\hat{\mathbf{y}}_k$ maximizes the firm's profit function $\hat{\mathbf{p}} \cdot \mathbf{y}_k$ in the production set $Y_k$;

(3.3) for every monopolistically competitive firm $k \in F_M$, $\hat{\mathbf{y}}_k$ maximizes the firm's profit function

$$\sum_{j \notin J^k} \hat{\mathrm{p}}_j y_{kj} + \sum_{j \in J^k} p_j(y_{kj};\ \hat{\mathbf{p}}, \hat{\mathbf{w}}) y_{kj}$$

in the production set $Y_k$;

(3.4) $$\sum_{i=1}^{n} \hat{\mathbf{x}}_i - \sum_{i=1}^{n} \mathbf{e}_i - \sum_{k+1}^{r} \hat{\mathbf{y}}_k = 0$$

(that is, there is zero excess demand in each market).

Negishi's main concern was to prove the existence of an equilibrium. He was able to do so by making the usual assumptions about preferences, endowments and production sets which are used to prove the existence of a Walrasian equilibrium (cf. Debreu, 1959). One of these assumptions is that the production set $Y_k$ of every firm is convex and contains the origin, which implies decreasing or constant returns to scale. Negishi (1961, p. 199) himself felt uncomfortable about making this assumption 'considering the fact that monopolistic competition has much to do with so-called increasing returns (Sraffa, 1926)'. Silvestre (1977a) in a less general version of Negishi's model, where each firm is a price taker for its inputs (thus monopsony is ruled out), and each imperfect competitor produces a single good, was able to somewhat relax the assumption of convexity of the production sets and still prove the existence of a conjectural Cournot-Nash-Walras equilibrium.[11]

As Negishi (1961, p. 198) himself points out, the assumption that conjectural inverse demand curves be straight lines is not necessary in order to prove the existence of an equilibrium. What is crucial is that the profit function of firm $k$ be quasi-concave in $y_k$ (for every $k = 1, \ldots, r$). Thus any other class of conjectural demand functions which guarantee quasi-concavity of the profit functions can be allowed. We shall discuss the plausibility of the assumption of quasi-concavity of the profit functions in section 6.

## 4. Objective demand in the Cournot-Nash framework

The main criticism which can be raised concerning Negishi's approach is that there is an element of arbitrariness in the conjectures of monopolistic firms. As Hart (1985, p. 107) puts it,

> 'The problem is that the very generality of the model gives it very little predictive power. Given particular subjective demand functions or conjectures ... the model will of course generate a small number of equilibria (possibly only one). However, the model does not tell us how these conjectures are formed. To an outside observer who is asked to predict the market outcome but who does not know what conjectures are, almost anything could be an equilibrium. ... To put it slightly differently, an economy with given demand conditions, sizes of firms, etc., could end up in a highly monopolistic state or a highly competitive one, depending on whether firms conjecture that they face low elasticities of demand or high ones. ... Negishi's theory does not tell us which is more likely.'

Indeed, it was shown by Gary-Bobo (1986) that every feasible allocation such that the production of each firm is different from zero and yields non-negative profits is a conjectural Cournot-Nash-Walras equilibrium, that is, there exist subjective demand curves for which this allocation is an equilibrium as defined by Negishi.[12]

The need was therefore felt for an analysis based not on conjectural demand functions but on the 'true' or 'objective' demand functions faced by the imperfect competitors. In the Cournot-Nash framework this approach was pioneered by Gabszewicz and Vial (1972). The authors summarize the methodology they employ as follows (pp. 381–382):

> 'The institutional organization of this economy can be described as follows. The consumers provide firms with labor and other nonconsumable resources, like primary factors. With these resources, the firms choose production plans which consist only of bundles of consumption goods. The various forms of labor and other primary factors are not "marketable"; rather, the firms distribute "real wages" to the consumers — who have provided them with these factors and labor — in terms of preassigned shares

of their output. At the end of the production process, each consumer is thus endowed with the sum of his shares in the various firms, namely, with some bundle of consumption goods. Exchange markets are then organized, where the consumers aim at improving their consumption through trade. The institutional rule of exchange consists in using a price mechanism. The prices on the exchange markets then serve as an information for the firms to adjust eventually their production plans according to some preassigned rule.'

Thus in Gabszewicz and Vial's model the existence of intermediary goods produced by some firms for other firms is excluded: all inputs to production come from the initial endowments of consumers. Despite this very strong simplifying assumption, there remain some conceptual problems in the construction of the objective inverse demand curves facing firms (each firm is an imperfect competitor in Gabszewicz and Vial's model). The methodology employed is as follows.[13] Fix a production decision $\mathbf{y}_k \in Y_k$ for each firm $k$ $(k = 1, \ldots, r)$. We are now in a pure-exchange economy where consumer $i (i = 1, \ldots, n)$ has a *modified endowment* given by

$$\mathbf{e}_i + \sum_{k=1}^{r} \theta_{ik} \mathbf{y}_k$$

Let $(\mathbf{p}^*, \mathbf{x}^*)$ be a Walrasian equilibrium of this pure-exchange economy (where $\mathbf{p}^* = (p_1^*, \ldots, p_m^*)$ is a price vector and $\mathbf{x}^* = (x_1^*, \ldots, x_n^*)$ is a consumption allocation). The aim is to define a mapping

$$(\mathbf{y}_1, \ldots, \mathbf{y}_r) \mapsto P(\mathbf{y}_1, \ldots, \mathbf{y}_r)$$

which associates with every production allocation $(\mathbf{y}_1, \ldots, \mathbf{y}_r)$ a price vector $\mathbf{p}^* = P(\mathbf{y}_1, \ldots, \mathbf{y}_r)$ corresponding to a Walrasian equilibrium of the pure exchange economy resulting from the production allocation $(\mathbf{y}_1, \ldots, \mathbf{y}_r)$. This mapping represents the objective inverse demand function facing each firm. We can then define an *objective Cournot-Nash-Walras equilibrium* as a $(r + n + 1)$-tuple $(\mathbf{y}_1^*, \ldots, \mathbf{y}_r^*; \mathbf{x}_1^*, \ldots, \mathbf{x}_n^*; \mathbf{p}^*)$ such that:

(4.1) $\mathbf{p}^* = P(\mathbf{y}_1^*, \ldots, \mathbf{y}_r^*)$;

(4.2) $\mathbf{x}_i^*$ maximizes consumer $i$'s utility function $U_i$ in the budget set

$$B = \{\mathbf{x}_i \in X_i \mid \mathbf{p} \cdot \mathbf{x}_i \leqslant \mathbf{p}^* \cdot (\mathbf{e}_i + \sum_{k=1}^{r} \theta_{ik} \mathbf{y}_k^*)\}$$

(4.3) $$\sum_{i=1}^{n} \mathbf{x}_i^* - \sum_{i=1}^{n} \mathbf{e}_i - \sum_{k=1}^{r} \mathbf{y}_k^* = 0$$

(Conditions (4.1)–(4.3) mean that $(\mathbf{p}^*, \mathbf{x}^*)$ is a Walrasian equilibrium of the exchange economy resulting from fixing firm $k$'s production decision at $\mathbf{y}_k^*$)

(4.4) for every firm $k$,

$$\mathbf{y}_k^* \cdot \mathbf{p}^* \geqslant \mathbf{y}_k \cdot P(\mathbf{y}_1^*, \ldots, \mathbf{y}_{k-1}^*, \mathbf{y}_k, \mathbf{y}_{k+1}^*, \ldots, \mathbf{y}_r^*) \text{ for every } \mathbf{y}_k \in Y_k$$

(Condition (4.4) means that $(\mathbf{y}_1^*, \ldots, \mathbf{y}_r^*)$ is a Cournot-Nash equilibrium of the game between the $r$ firms, where each firm's strategy space is its production set $Y_k$ and its payoff function is the profit function $\mathbf{y}_k \cdot P(\mathbf{y}_1, \ldots, \mathbf{y}_r)$.

There are problems, however, with this approach:

(i) Given a production allocation $(\mathbf{y}_1, \ldots, \mathbf{y}_r)$, the corresponding exchange economy may have no Walrasian equilibria, even if we make standard assumptions on consumer's preferences and endowments.[14]

(ii) Even if we only allow firms to choose production plans for which the corresponding exchange economy has a Walrasian equilibrium, there may be many Walrasian equilibria with different prices, in which case we need an *exogenous* selection rule in order to construct the mapping $P(\mathbf{y}_1, \ldots, \mathbf{y}_r)$.[15]

(iii) Even if we adopt an exogenous selection rule, the resulting function $P(\mathbf{y}_1, \ldots, \mathbf{y}_r)$ may be unavoidably discontinuous, as shown in Figure 1, with the consequence that a Cournot-Nash equilibrium of the game among firms may not exist.

(iv) Even if the function $P(\mathbf{y}_1, \ldots, \mathbf{y}_r)$ turns out to be continuous, the profit function of each firm need not be quasi-concave in the firm's own decision variable, with the consequence that a Cournot-Nash equilibrium of the game among firms may not exist (see section 6).

In order to prove the existence of an objective Cournot-Nash-Walras equi-

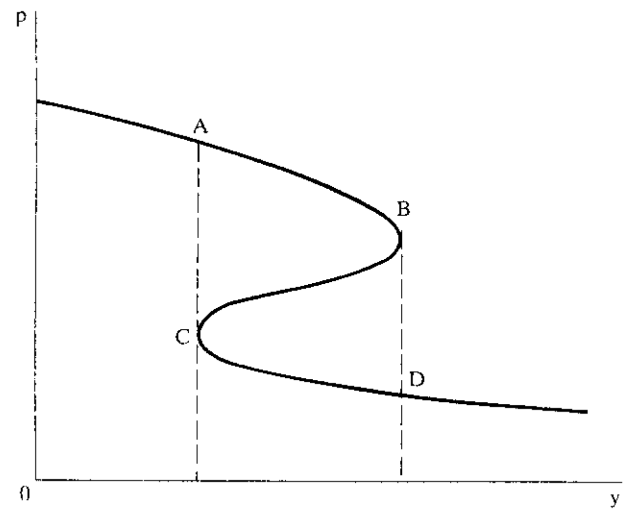


**Figure 1.** The graph of the correspondence which associates with every production allocation $\mathbf{y} = (\mathbf{y}_1, \ldots, \mathbf{y}_r)$ the price vectors $\mathbf{p}$ of the Walrasian equilibria of the resulting exchange economy. (Since $\mathbf{y}$ belongs to an $mr$-dimensional space and $\mathbf{p}$ belongs to an $m$-dimensional space, the curve shown is a two-dimensional section of the graph of the correspondence.) Starting from the upper portion of the curve, as $\mathbf{y}$ increases a jump to the lower portion must occur at a point between A and B. Similarly, starting from the lower portion of the curve, as $\mathbf{y}$ decreases a jump to the upper part must occur at a point between C and D.

librium Gabszewicz and Vial assume that for every $(\mathbf{y}_1, \ldots, \mathbf{y}_r)$ the corresponding exchange economy has a unique Walrasian equilibrium and that for each firm $k$ and for every $(\mathbf{y}_1, \ldots, \mathbf{y}_{k-1}, \mathbf{y}_{k+1}, \ldots, \mathbf{y}^r)$ the profit function

$$\mathbf{y}_k \cdot P(\mathbf{y}_1, \ldots, \mathbf{y}_r)$$

is quasi-concave in $\mathbf{y}_k$. We shall discuss the plausibility of the latter assumption in section 6. The assumption of existence and uniqueness of Walrasian equilibria is obviously very strong.

To the list of conceptual difficulties given above, two more items should be added:

(v) In the Cournot-Nash approach each firm considers the effect on prices of a change in its own production plan, *ceteris paribus*, that is, while everything else — in particular the production plans of the other firms — remains constant. However, if firm $k$ uses as input a commodity produced by firm $k'$, how can an increase in the output of firm $k$ be obtained without at the same time increasing the output of firm $k'$? Gabszewicz and Vial do not address this problem and simply assume that all inputs come from the endowments of consumers. This is clearly not a satisfactory assumption.[16]

(vi) As Gabszewicz and Vial emphasize, the construction of the price function $P(\mathbf{y}_1, \ldots, \mathbf{y}_r)$ requires the specification of a normalization rule, that is, the choice of a *numeraire*. In an example (pp. 398–399) they show that the objective Cournot-Nash-Walras equilibria are not invariant to the normalization rule chosen. In other words, an equilibrium for a given choice of *numeraire* ceases to be an equilibrium if a different good (or set of goods) is chosen as *numeraire*. This is to be contrasted with the fact that Walrasian equilibria are independent of the chosen normalization rule. We discussed this problem in section 2 when we mentioned the difficulties associated with the assumption of profit maximization. It may be helpful to illustrate this with a very simple example. Suppose that there are only three goods ($m = 3$), plus money, and that when firm $k$ changes its output of good 1, $y_{k1}$, from 1 to 2 units, the vector of monetary prices changes from $\mathbf{p} = (2, 2, 1)$ to $\mathbf{p}' = (1, 1, 2)$ (we shall also assume zero production cost). Monetary profits remain, therefore, constant (equal to 2). If we take good 2 as *numeraire*, the inverse demand curve looks like Figure 2a and the corresponding profit function like Figure 2b, and the move from $y_{k1} = 1$ to $y_{k1} = 2$ is profitable. If, instead, we choose good 3 as *numeraire*, the inverse demand curve looks like Figure 2c and the corresponding profit function like Figure 2d, and the move from $y_{k1} = 1$ to $y_{k1} = 2$ is not profitable.[17]

Concerning this problem Gabszewicz and Vial (1972, p. 400) observe:

> 'On this basis some readers could accordingly be tempted to reject our theory as a whole; but they should be aware that they would simultaneously reject the whole theory of imperfect competition in partial analysis. By a similar argument, it can be shown indeed, that the graph of the classical demand function in the price-quantity coordinates is not invariant on the set of normalization rules of the whole price system in the economy!'

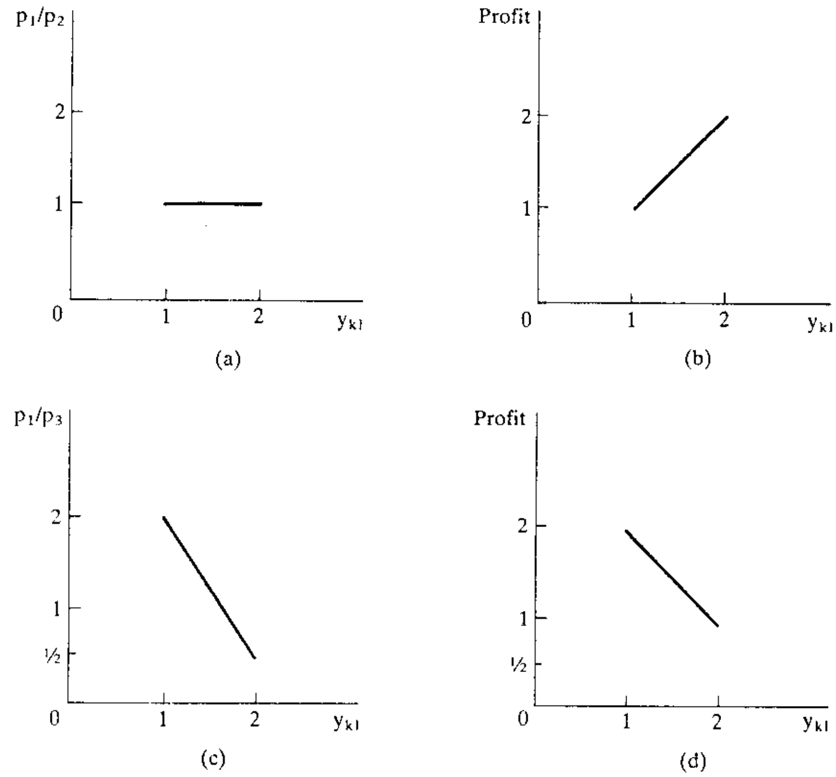


**Figure 2.**

## 5. Objective demand in the Bertrand-Nash framework

In the Cournot-Nash framework imperfect competitors are assumed to treat output levels as their strategic variables. Even though, from a purely formal point of view, the corresponding prices can be obtained from the objective inverse demand functions, the question remains as to what mechanism leads to the determination of prices. This was Bertrand's original criticism of Cournot's approach and it still remains a valid objection.[18]

A number of authors (Marshak and Selten, 1974, Nikaido, 1975, Benassy, 1988) have therefore followed the Bertrand-Nash approach, where firms' strategies are prices rather than output levels. Marshak and Selten's (1974) model shares with Gabszewicz and Vial's model the feature that all inputs to production are assumed to come from consumers' endowments, so that no imperfect competitor buys from or sells to another imperfect competitor, that is, there are no intermediary goods. Nikaido's (1975) model does not suffer from this limitation, although it suffers from lack of generality in another respect, namely the assumption of a Leontief technology with single-product firms. We shall here describe Nikaido's model.[19]

Nikaido assumes that each firm is an imperfect competitor and produces a single good and that labour is the only non-produced good. Let $m$ be the number of produced goods, which, therefore, is also the number of firms. Let $A = (a_{jk})_{j,k=1,\dots,m}$ be a Leontief matrix, where $a_{jk}$ is the amount of good $j$ needed by firm $k$ in order to produce one unit of its own output (firm $k$ produces good $k$). Let $\mathbf{y} = (y_1, \dots, y_m)$ denote the vector of gross outputs, $\mathbf{l} = (l_1, \dots, l_m)$ the (positive) vector of labour inputs, $\mathbf{p} = (p_1, \dots, p_m)$ be the price vector, $w$ the wage rate and $\boldsymbol{\pi} = (\pi_1, \dots, \pi_m)$ be the vector of total profits. Nikaido assumes that the labour force consists of many atomistic workers who behave as price takers in supplying labour and demanding goods. Workers do not get any share of the firms' profits and therefore their supply of labour and demand for goods depend only on $(\mathbf{p}, w)$. Nikaido takes labour as *numeraire*, that is, he chooses the normalization rule $w = 1$. From now on, therefore, prices will be interpreted as prices in terms of labour. Let $L(\mathbf{p})$ denote the workers' aggregate supply-of-labour function and $F(\mathbf{p}) = (F_1(\mathbf{p}), \dots, F_m(\mathbf{p}))$ the workers' aggregate demand-for-goods function.[20] Firms are owned by capitalists, who — as producers — are price makers, but — as consumers — are price takers. Let $G(\mathbf{p}, \boldsymbol{\pi}) = (G_1(\mathbf{p}, \boldsymbol{\pi}), \dots, G_m(\mathbf{p}, \boldsymbol{\pi}))$ be the aggregate demand-for-goods function from the capitalist class.[21]

Given a vector of prices in terms of labour, $\mathbf{p}$, the aggregate supply of labour will be $L(\mathbf{p})$. The labour market will therefore be in equilibrium if the vector of gross outputs $\mathbf{y}$ is such that $\mathbf{l} \cdot \mathbf{y} = L(\mathbf{p})$. Workers' aggregate demand for goods is given by $F(\mathbf{p})$. The vector of net outputs will be $(I - A)\mathbf{y}$ (where $I$ is the identity matrix). If the net output of each firm is sold, the total profit of firm $k$ will be

$$\pi_k = \left(p_k - \sum_{j=1}^{m} p_j a_{jk} - l_k\right) y_k$$

Let $\boldsymbol{\pi} = (\pi_1, \dots, \pi_m)$. Then, if the net output of each firm is sold, capitalists' demand will be $G(\mathbf{p}, \boldsymbol{\pi})$, and the net output will indeed be sold and the market for each good will be in equilibrium if and only if

$$G(\mathbf{p}, \boldsymbol{\pi}) + F(\mathbf{p}) = (I - A)\mathbf{y}.$$

Thus the question is whether, given $\mathbf{p}$, there exists a vector of gross outputs $\mathbf{y}$ such that:

(5.1) $$\mathbf{l} \cdot \mathbf{y} = L(\mathbf{p});$$

(5.2) $$\boldsymbol{\pi} = (\pi_1, \dots, \pi_m) \text{ and}$$

$$\pi_k = \left(p_k - \sum_{j=1}^{m} p_j a_{jk} - l_k\right) y_k;$$

(5.3) $$G(\mathbf{p}, \boldsymbol{\pi}) + F(\mathbf{p}) = (I - A)\mathbf{y}.$$

(Nikaido calls such a $\mathbf{y}$ *a* 'competitive choice'). Nikaido shows that under standard conditions such a $\mathbf{y}$ exists for every positive $\mathbf{p}$.[22] Furthermore, if $G(\mathbf{p}, \pi)$ is differentiable and no good is inferior for the capitalist class (that is, the aggregate demand function $G(\mathbf{p}, \boldsymbol{\pi})$ is monotonically non-decreasing with respect to the arguments $\pi_1, \ldots, \pi_m$ for each fixed $\mathbf{p}$), then such a $\mathbf{y}$ is unique. Thus we can define a function

$$\mathbf{p} \mapsto \mathbf{y}(\mathbf{p})$$

which associates with every price vector $\mathbf{p}$ the unique vector of gross outputs which satisfies (5.1)–(5.3) above. Nikaido calls this function an *objective gross demand function*. It is worth emphasizing that the objective demand function $\mathbf{y}(\mathbf{p})$ is constructed in such a way as to incorporate the circularity of profits and demand, referred to in section 2. We can now define an *objective Bertrand-Nash-Walras equilibrium* as a pair $(\mathbf{p}^*, \mathbf{y}^*)$ such that:

(5.4) $\mathbf{y}^* = \mathbf{y}(\mathbf{p}^*)$ (this condition guarantees, in virtue of (5.1)–(5.3) above, that every consumer is maximizing utility and in every market supply equals demand); and

$$(5.5) \qquad \left(p_k^* - \sum_{j=1}^{m} p_j^* a_{jk} - l_k\right) y_k^* \geqslant$$
$$\left(p_k - \sum_{\substack{j=1 \\ j \neq k}}^{m} p_j^* a_{jk} - p_k a_{kk} - l_k\right) y_k(p_1^*, \ldots, p_{k-1}^*, p_k, p_{k+1}^*, \ldots, p_m^*)$$

for all (admissible) $p_k$ (this condition means that $\mathbf{p}^*$ is a Nash equilibrium of the game between firms where strategies are prices and the payoff functions are the profit functions).

Nikaido did not define such an equilibrium explicitly, nor did he investigate conditions under which such an equilibrium exists. The reason is that, as he demonstrates in a simple example (pp. 53–56), the objective demand functions 'are much different from those which the traditional oligopoly theorist has in mind', indeed 'they need not be downward-sloping even with respect to the price of the good in question'.[23] In order to prove the existence of an objective Bertrand-Nash-Walras equilibrium one would need to impose a condition of the form:

for every $(\hat{p}_1, \ldots, \hat{p}_{k-1}, \hat{p}_{k+1}, \ldots, \hat{p}_m)$ the profit function of firm $k (k = 1, \ldots, m)$,

$$f(p_k) \equiv \left(p_k - \sum_{\substack{j=1 \\ j \neq k}}^{m} \hat{p}_j a_{jk} - p_k a_{kk} - l_k\right) y_k(\hat{p}_1, \ldots, \hat{p}_{k-1}, p_k, \hat{p}_{k+1}, \ldots, \hat{p}_m)$$

is quasi-concave in $p_k$.[24]

Such an assumption, however, implies restrictions on the shape of the objective demand function which, as we said above, are unlikely to be met. This topic is taken up in the following section.

In a very recent contribution Benassy (1988) generalized Nikaido's model in two directions. First of all, he relaxed the assumption of a Leontief technology and introduced more general production possibility sets; secondly, he relaxed the (implicit) assumption that price makers serve whatever demand or supply is addressed to them. Even though it may seem counterintuitive, an imperfect competitor may find it in his interest to ration his customers and a truly general model should allow for this possibility.[25] Benassy (1988) makes use of some of the techniques and definitions introduced in the literature on fix-price equilibria (Benassy, 1976, 1982, Silvestre, 1987) and gives sufficient conditions for the existence of an equilibrium.[26] One of the conditions is, again, that the profit functions of the imperfect competitors are quasi-concave. We can now turn to a discussion of the plausibility of this assumption.

## 6. The assumption of quasi-concavity of the profit functions

Almost every model of imperfect competition — whether it is a partial equilibrium or a general equilibrium one, whether it makes use of subjective or objective demand functions, whether it follows the Cournot-Nash or the Bertrand-Nash approach — contains the assumption that the profit function of each imperfectly competitive firm is quasi-concave in the firm's decision variable.[27]

Imposing restrictions on the shape of the profit function implies imposing restrictions on the shape of the demand curve. In fact, in general, if the demand curve is not concave the profit function will turn out to be multimodal and, therefore, not quasi-concave. The following example illustrates this fact in a simple partial equilibrium model where the failure of the profit functions to be quasi-concave leads to non-existence of a Bertrand-Nash equilibrium.[28]

There are two firms which produce differentiated products at zero cost. Let $p_i$ be the price of firm $i$ $(i = 1, 2)$. Firm 1's demand function, illustrated in Figure 3a, is given by

$$D_1(p_1, p_2) = -p_1^3 + 12p_1^2 - 52p_1 + 93 + p_2$$

while firm 2's demand function, illustrated in Figure 3b, is given by

$$D_2(p_1, p_2) = \begin{cases} 3 + 0.74p_1 - p_2 & \text{if } 0 \leqslant p_2 \leqslant 1 + 0.68p_1 \\ 2.5 + 0.4p_1 - 0.5p_2 & \text{if } p_2 \geqslant 1 + 0.68p_1 \end{cases}$$

As can be seen from Figure 3, neither demand curve is concave. Since we assumed zero production costs, the profit function of firm $i$ $(i = 1,2)$ is given by

$$P_i(p_1, p_2) = p_i D_i(p_1, p_2).$$

A Bertrand-Nash equilibrium is a pair of prices $(p_1^*, p_2^*)$ such that

$$P_1(p_1^*, p_2^*) \geqslant P_1(p_1, p_2^*) \qquad \text{for all } p_1 \geqslant 0$$

and

$$P_2(p_1^*, p_2^*) \geqslant P_2(p_1^*, p_2) \qquad \text{for all } p_2 \geqslant 0.$$

Figure 4 shows the graph of the profit function of firm 1, as a function of $p_1$, for different values of the 'parameter' $p_2$. It can be seen that the function is multimodal and, therefore, not quasi-concave. With every $p_2$ we can associate the

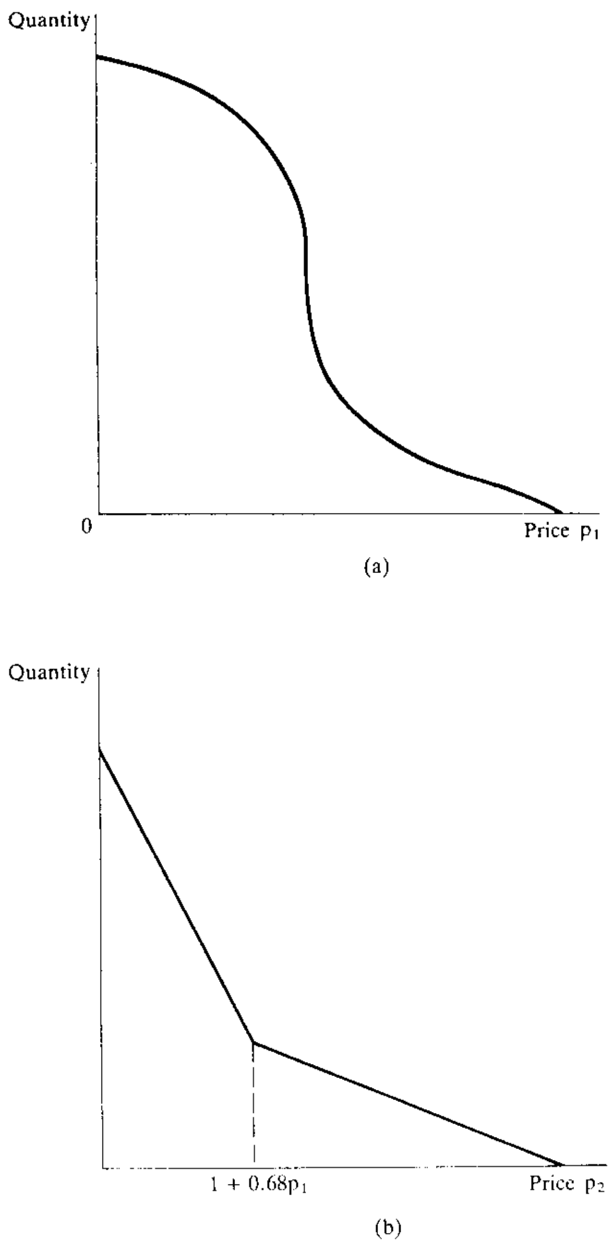


**Figure 3.** (a) The demand function of firm 1, $D_1\ (p_1, p_2)$, for a given $p_2$. (b) The demand function of firm 2, $D_2\ (p_1, p_2)$, for a given $p_1$.

price $p_1$ which maximizes $P_1(p_1, p_2)$. In general, this $p_1$ is unique, but — as Figure 4 shows — there may be a critical value of $p_2$ at which the profit function $P_1(p_1, p_2)$ has two global maxima at the same level (Figure 4b). As this critical value of $p_2$ is crossed, the profit-maximizing price for firm 1 jumps discontinously from a value close to $\hat{p}_1$ to a 'distant' value close to $\tilde{p}_1$ (cf. Figure 4b).

Thus the reaction curve of firm 1, which associates with every $p_2$ the price (or prices) $p_1$ which maximizes $P_1(p_1, p_2)$ is not connected.[29] Figure 5 shows the reaction curves of the two firms. Since a Nash equilibrium exists if and only if the reaction curves intersect, it can be seen from Figure 5 that in this simple duopoly model no Bertrand-Nash equilibrium exists.

Of course, the demand curves of this example are rather artificial and it may very well be that behind them lie some unusual assumptions about consumers' preferences. Roberts and Sonnenschein (1977), however, have shown that it is possible to construct general equilibrium models with standard assumptions on consumers' preferences and technology (e.g. convexity) where the demand curves turn out to have convex regions and, as a consequence, the corresponding profit functions are not quasi-concave and no equilibrium exists.[30]

Thus there is an asymmetry between the general equilibrium theory with perfect competition and the various general equilibrium theories with imperfect competition put forward so far: in the former the existence of an equilibrium is proved starting from simple assumptions on the data of the theory, namely endowments, preferences and technology; in the latter a further assumption is added — the quasi-concavity of the profit functions — which cannot be obtained from simple assumptions on endowments, preferences and technology.

A possible way out of the problem of non-existence of equilibria due to the lack of quasi-concavity of the profit functions is to resort to the more general notion of Nash equilibrium in mixed strategies.[31] Indeed, Dasgupta and Maskin (1986a,b) have shown that, in general, an equilibrium in mixed strategies exists, even if the profit functions are not quasi-concave and, in some cases, even if they are not continuous. Two objections can be raised concerning this 'solution' to the problem of non-existence of equilibria. First of all, the notion of mixed strategy, although widely used in game theory, is far from being satisfactory.[32] Secondly, Dierker and Grodal (1986) have shown that there are standard general equilibrium models where not even mixed-strategy equilibria exist. In their example, which is in the Cournot-Nash framework with objective demand functions, there are two firms with convex production sets, and each firm's best reply in pure strategies to the strategy of the other firm consists of at most one production plan. Thus the non-existence of equilibria — which they prove — cannot be attributed to the fact that the profit functions are not quasi-concave.[33] The reason lies in the fact that for a given production allocation the corresponding pure-exchange economy has many Walrasian equilibria (cf. section 4) and any selection rule is necessarily discontinuous (cf. Figure 1). Dierker and Grodal (1986) also show that the non-existence of equilibria (even in mixed strategies) is independent of the fact that firms are assumed to maximize profits and that the demand functions depend on the normalization rule chosen (cf. section 4).

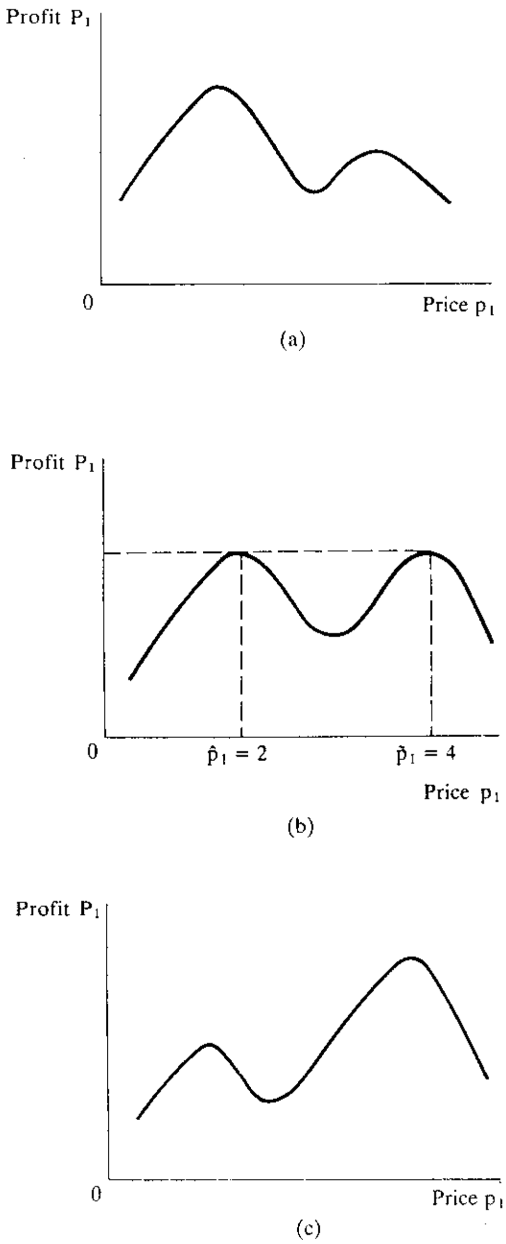


**Figure 4.** The profit function of firm 1, $P_1(p_1, p_2)$, for different values of the parameter $p_2$: (a) for $p_2 < 3$, (b) for $p_2 = 3$, (c) for $p_2 > 3$.

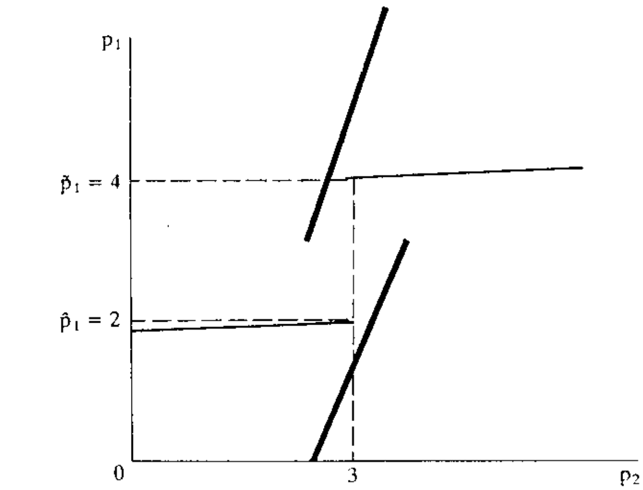


**Figure 5.** The thick lines represent the reaction curve of firm 2, while the thin lines represent the reaction curve of firm 1.

In fact, they also give an example where each firm has exactly one owner and, therefore, the firm has a natural objective, namely choosing a production plan so as to maximize the indirect utility function of its owner (this behaviour is, of course, independent of the normalization rule). Also in this example there are no (pure- or) mixed-strategy equilibria.

## 7. Compromises between the conjectural and the objective approach

We saw above (section 4) that the conjectural (or subjective) approach of Negishi can be criticized on the grounds that conjectures are somewhat arbitrary and can be 'very wrong', that is, very different from the underlying objective functional relations (the only objective constraint imposed by Negishi is that the conjectural demand curve be consistent with the observed state of the market). The objective approach, on the other hand, can be criticized for being extremely unrealistic: in order to calculate the objective demand functions, firms need to have a general equilibrium model of the economy, a huge amount of information available and almost unlimited processing capabilities. It is for this reason that a number of authors (Bonanno and Zeeman, 1985, Bonanno, 1988, Gary-Bobo, 1987, Hart, 1985, Silvestre, 1977b) have suggested some sort of compromise between the two approaches. The first contribution along these lines is the one by Silvestre (1977b). Silvestre suggests that firms may try to gain some knowledge of their 'true' or objective demand curves by performing small price experiments (by 'small' we mean 'within a small interval of prices'). They thus collect data and use it to estimate their demand function. Suppose that the estimated demand function is linear (or, to be more precise, affine). Then we can say that it is a 'correct' extrapolation, at a given state of the market, if the estimated demand curve coincides with the linear approximation to the 'true' demand curve (see Figure 6). Thus it is not only the *value* of the estimated demand function which is required to be consistent with the objective demand function (as in Negishi, 1961), but also its *slope*. An equilibrium is then defined as in Negishi (1961) but with the added constraint concerning the slope of the estimated demand curve. Silvestre's (1977b) model is in the Bertrand-Nash framework (all firms are price setters, consumers are price takers) and the assumptions needed to prove the existence of an equilibrium are very weak.[34] Silvestre's idea has been generalized, at the partial equilibrium level, by Bonanno and Zeeman (1985), who show that an equilibrium exists always, that is, with *arbitrary* objective demand functions.[35] In other words, no assumptions need to be made concerning the shape of the demand and profit functions.

The drawback with this notion of equilibrium is that at equilibrium a firm may be 'considerably wrong', that is, it may believe that it is maximizing profits and yet be only at a local — but not global — maximum of its 'true' profit function. Indeed, it can even be at a local minimum of its 'true' profit function, as Figure 6 shows.[36]

One could take the above idea one step further and assume that, through their small price experiments, firms manage to acquire complete and correct knowledge of their 'true' demand curves within a small neighbourhood of prices (rather than a linear estimate of it). One would then define an equilibrium as a *local Nash equilibrium*, that is, a point where each firm is at a local maximum of its 'true' profit function, although it may not be at a global maximum of it.

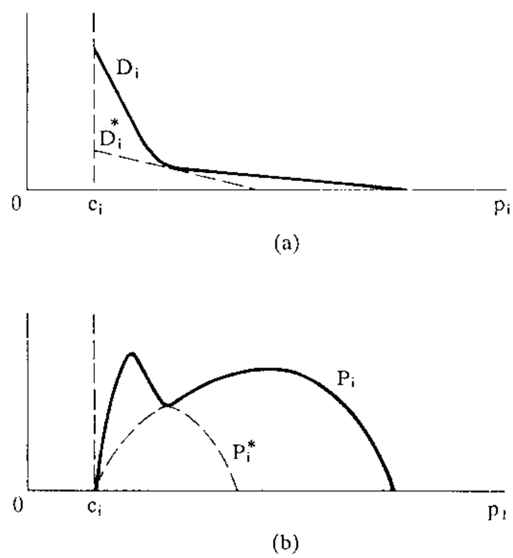


**Figure 6.** The demand function $D_i$ and the conjectural demand function $D_i^*$. (b) The profit function $P_i$ and the conjectural profit $P_i^*$.

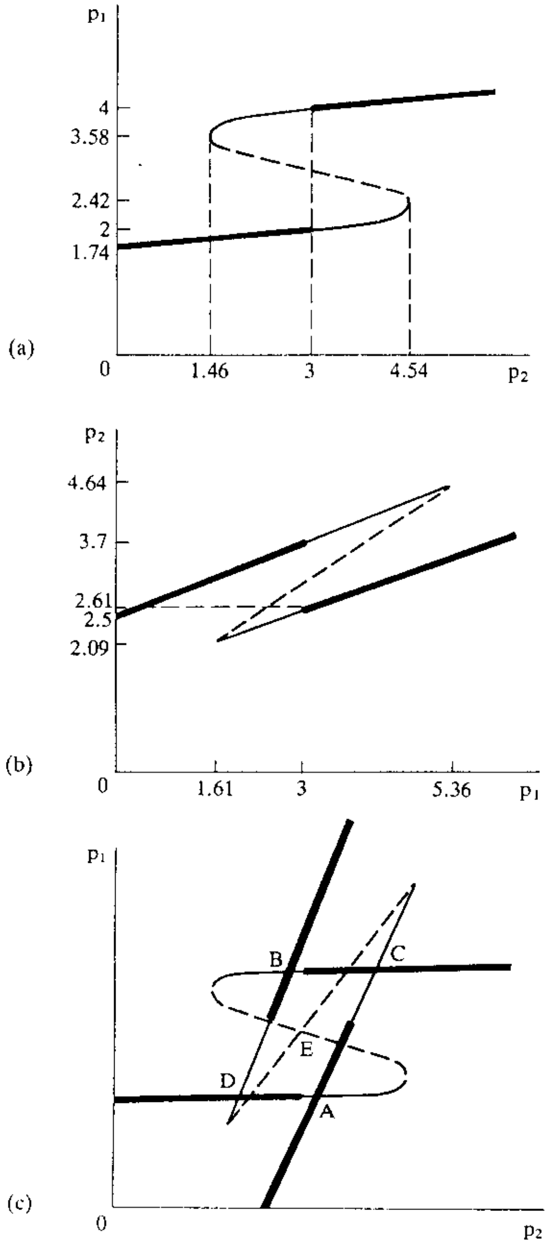


**Figure 7.** (a) The reaction curve of firm 1. (b) The reaction curve of firm 2. (c) The two curves together.

This notion of equilibrium is weaker than that of Nash equilibrium: there are games where a Nash equilibrium does not exist and yet a *local* Nash equilibrium exists. The example of section 6 can be used to illustrate this. The S-shaped curve of Figure 7a is the graph of the correspondence which associates with every $p_2$ the set of prices of firm 1, $p_1$, which satisfy the first-order condition for profit maximization.[37] The heavy lines denote global maxima of $P_1$ (as a function of $p_1$, parameterized by $p_2$), the thin continuous lines local maxima and the dashed lines local minima (thus the reaction curve of firm 1 is given by the two heavy lines and, in fact, it is only this portion of the S-shaped curve which was shown in Figure 5). Similarly, the (inverted) Z-shaped curve of Figure 7b is the set of points where the first-order condition for profit maximization is satisfied for firm 2.[38] Again, the heavy lines denote global maxima of $P_2$ (as a function of $p_2$, parameterized by $p_1$), the thin continuous lines local maxima and the dashed lines local minima (thus the reaction curve of firm 2 is given by the two heavy lines and, in fact, it is only this portion of the (inverted) Z-shaped curve which was shown in Figure 5).

Figure 7c shows the two curves together. It was shown in Figure 5 that no Nash equilibrium exists in this game. It can be seen from Figure 7c that there are four *local* Bertrand-Nash equilibria, $A$, $B$, $C$, and $D$. At points $A$ and B firm 2 is at a global maximum of its 'true' profit function, while firm 1 is at a local — but not global — maximum of its 'true' profit function. The situation is reversed at points $C$ and $D$.[39]

The existence of local Bertrand-Nash equilibria has been investigated at the partial equilibrium level by Bonanno (1988). The conditions for the existence of a local equilibrium are rather weak and do not require the quasi-concavity of the profit functions. However, they do require some restrictions on the shape of the 'true' or objective demand functions and, as we know (cf. Roberts and Sonnenschein, 1977), there is no guarantee that standard conditions on consumers' preferences give rise to demand functions with the desired shape.[40]

We saw in section 2 that at the general equilibrium level a problem arises, which is usually ignored in partial equilibrium analyses by appealing to the *ceteris paribus* clause, namely the feedback effect or Ford effect, that is, the fact that the demand function, which enters the definition of profit function, may have profit as one of its arguments, giving rise to some sort of circularity. We also saw, however, that the feedback effect can be incorporated in an objective demand function in a coherent way. However, it has been observed that it doesn't seem to be realistic to assume that firms take into account the feedback effect:

> '...while it may be inappropriate to rule out the feedback effect in the case of very large imperfect competitors (e.g. General Motors), it may be quite realistic to do so in the case of small imperfect competitors (e.g. the local supermarket)' (Hart, 1985, p. 121).

Indeed, a number of authors (Marshak and Selten, 1974, Silvestre, 1977b, Hart

1985) have studied models where there is no feedback effect.[41] Hart (1985, p. 121) notes that, apart from realism,

> 'There seem to be two advantages of considering a model ... in which there is no feedback effect. First the feedback effect is likely to cause the firm's demand function to be less well-behaved than the usual Marshallian demand function (as Nikaido, 1975, points out, it may be upward-sloping, it is unlikely to satisfy gross substitutability, etc.) ... Second if there is a feedback effect arising, say, from the fact that the owners of the firm also consume the firm's products, then, as we have noted, profit maximization is no longer a natural objective function for the firm, since the owners will be interested in low consumption-goods prices as well as high monetary profit.'

Although it certainly seems realistic to assume that firms ignore the feedback effect, one may wonder when firms are indeed justified in doing so. Hart (1985) develops a model (based on Hart, 1982a) where each firm's action has a negligible effect on the wealth of its customers and therefore the firm is approximately correct in taking the latter as given.

## 8. Insights into the notion of perfect competition

A perfectly competitive firm takes prices as given. How should we interpret this assumption? One possible answer is that perfectly competitive firms simply conjecture that the prices of all products, including their own, would not be affected by a change in their own output.[42] This is not a satisfactory interpretation, however, because it does not tie conjectures to real effects (for example, it would not be rational for a large firm like, say, General Motors, to have such conjectures). A large number of contributions have emerged in the past twenty years aimed at providing an 'objective' explanation for price taking, that is, at building models where firms are indeed justified in treating prices parametrically because their actions have a negligible effect on all prices, including the prices of their own products.[43] A survey of this part of the literature would require a considerable amount of space[44] and we shall only briefly mention the issues which are more closely related to the topics discussed in this paper.

The pioneering contribution is the paper by Gabszewicz and Vial (1972) which was partly discussed in section 4. Gabszewicz and Vial posed the following problem: given an economy as described in section 3, define a sequence of larger and larger economies based on it, where each agent — in particular each firm — becomes smaller and smaller relative to its market. Consider the objective Cournot-Nash-Walras equilibria of these economies. It is the case that if the economy becomes sufficiently large, that is, if each firm becomes sufficiently small relative to its market, the Cournot-Nash-Walras equilibria become arbitrarily close to a Walrasian equilibrium? If the answer is affirmative, then we can interpret price taking behaviour (that is, the assumption of perfect competition) as a sufficiently good approximation of imperfectly competitive behaviour in economies where agents are small relative to their markets.

In order to 'enlarge' the economy, Gabszewicz and Vial (1972) follow a procedure first introduced by Edgeworth. Call the original economy $E_1$. Recall (cf. section 3) that in $E_1$ there are $m$ goods, $n$ consumers (each consumer $i$ is characterized by a consumption set $X_i$, a utility function $U_i$, an initial endowment of goods $\mathbf{e}_i$ and a share ownership $\theta_{ik}$) and $r$ firms (each firm $k$ is characterized by a production set $Y_k$). Now define an $s$-replica $E_s$ of $E_1$ ($s \geqslant 1$) as an economy where there still are $n$ goods, but the number of consumers is $sn$ and the number of firms is $sr$. For each consumer $i$ in $E_1$ there are now $s$ such consumers (each characterized by $X_i$, $U_i$, $\mathbf{e}_i$ and a share ownership $(1/s)\theta_{ik}$). For each firm $k$ in $E_1$ there are now $sk$ firms (each characterized by $Y_k$).

Gabszewicz and Vial assume that the initial economy $E_1$ has a unique Walrasian equilibrium and prove that any objective Cournot-Nash-Walras equilibrium of $E_s$ can be made arbitrarily close to the Walrasian equilibrium of $E_1$ by choosing $s$ sufficiently large.

The assumptions made by Gabszewicz and Vial were rather strong (strict convexity of preferences and production sets, unique Walrasian equilibrium) and a number of authors have later investigated the problem of convergence of Cournot-Nash-Walras equilibria to Walrasian equilibria under weaker assumptions. The questions that have been studied in great detail are:

(1) does a sequence of Cournot-Nash-Walras equilibria necessarily converge?
(2) when a sequence of Cournot-Nash-Walras equilibria converges, does it necessarily converge to a Walrasian equilibrium?
(3) given a Walrasian equilibrium, does there exist a sequence of Cournot-Nash-Walras equilibria that converges to it?

The interested reader can find a detailed discussion of these issues in MasColell (1982).

## 9. Conclusion

In this paper we have discussed the various attempts that were made to introduce the hypothesis of imperfect competition in general equilibrium. We have seen that the contributions can be divided into four categories, according to whether conjectural or objective demand functions were postulated and whether the Cournot-Nash or the Bertrand-Nash approach was followed. One of the main achievements of general equilibrium theory with perfect competition was to establish the existence of an equilibrium starting from simple assumptions on the data of the theory, namely preferences, endowments and technology. No comparable existence theorem can be found in the various general equilibrium models with imperfect competition put forward so far. In fact, the existence theorems proved so far make use of an assumption — the quasi-concavity of the profit functions — which is not derived from basic assumptions on the data of the theory. Furthermore, once the doors of general equilibrium theory were opened to the hypothesis of imperfect competition, some hitherto unchallenged behavioural assumptions — such as the hypothesis that firms' objective is profit maximization — appeared to be questionable.

Moreover, the difficulties one faces in trying to introduce imperfect competition in a general equilibrium model are not confined to the ones listed above. The recent growth of the Industrial Organization literature has pointed to a number of important aspects of imperfect competition which go beyond the simple modelling of price and quantity decisions. The strategic behaviour of firms includes such phenomena as entry-deterring efforts, reputation-building policies, self-enforcing collusive agreements, product differentiation, etc.[45] A general equilibrium theory of imperfect competition which is truly general ought to incorporate these phenomena. It does not seem likely, at this stage, that such a theory can be constructed.

## Notes

1. The literature on general equilibrium with imperfect competition has already been reviewed by Hart (1985) and Gary-Bobo (1988). I have tried to make this survey complementary to theirs, by sometimes focusing on different contributions and issues and by refraining from a very technical presentation. I also discuss some recent contributions which were not covered by those two surveys.
2. The French mathematician Joseph Bertrand (1883) criticized Cournot for assuming that firms choose output levels. He claimed that the natural decision variable for a firm is the price of its own product and reformulated the notion of equilibrium proposed by Cournot in terms of prices (obtaining results which were at variance with those of Cournot). Thus a Bertrand-Nash equilibrium is a list of prices — one for each firm — with the property that, if the other firms do not change their prices, no firm can increase its profits by changing its own price.
3. This is what characterizes a game-theoretic situation. Oligopoly is the most conspicuous example. However, there may be interdependencies which may not be recognized at the partial equilibrium level and become apparent only when we take a general equilibrium point of view. For example, two monopolists who operate in completely different markets are not, in general, independent: the actions taken by one have an (indirect) effect on the profits of the other, through the changes produced in the budget sets and incomes of consumers.
4. For a critical examination of the notion of Nash equilibrium in game theory see Bacharach (1987), Binmore (1987), Bonanno (1987b), Reny (1985).
5. The expression 'Ford effect' is due — as far as I know — to Gabszewicz (1985, p. 155): 'Henry Ford once explained that paying higher wages to his workers would lead them to buy a larger number of cars, increasing the receipts of Ford Corporation, perhaps beyond the increase in costs entailed by the wage increments!'.
6. There is one more observation which is worth making. As Nikaido (1975, pp. 10–11) points out, 'the very familiar concept of demand function as such more or less presupposes the presence of competitive atomistic agents, who behave as price-takers. If no competitive atomistic price taker is involved in the national economy as a closed system, so that it is composed solely of nonatomistic price setters, no demand function can be conceivable.' The contributions reviewed in this paper are those where there are agents who are price takers (typically consumers). General equilibrium models where no agent is a price taker have also been studied, see, for example, Shubik (1973), Shapley and Shubik (1977), Dubey and Shubik (1977), Roberts (1986).

7. Cornwall (1977) also noticed this problem and illustrated it in an example (pp. 56–58) which is worth reading. He also remarked: 'of course it is not realistic to assume that firms actually recognize that their production choices influence the consumption possibilities which are feasible for the firms' owners and that the firms consequently choose non-profit-maximizing plans. However, it is equally clear that it is not enough to say that there are a lot of firms in a real world economy and that therefore the assumption of profit-maximizing behavior gives a good approximation. This is not enough of a justification because it is not clear what or how profit maximization approximates.'
8. Negishi (1961, p. 197) gives credit to Bushaw and Clower (1957) for this way of modelling conjectural demand functions.
9. In a later contribution Negishi (1972, chapter 7) allowed for a change in, say, $y_{k5}$ to affect $p_3$, $p_5$ and $p_9$ — rather than $p_5$ only — but the prices of all the other goods (that is, the goods not in $J^k$) are assumed to remain constant (of course, this is a statement about conjectures, not about real effects).
10. Given two vectors $\mathbf{u}, \mathbf{v} \in \mathbb{R}^q$, we denote by $\mathbf{u} \cdot \mathbf{v}$ their inner product: $\mathbf{u} \cdot \mathbf{v} = u_1v_1 + u_2v_2 + \ldots + u_qv_q$.
11. Fix an imperfectly competitive firm $k$ and let $j$ be the good it produces. For any given price vector $\mathbf{p}$, let $C_k^T\ (y_{kj};\ \mathbf{p})$ denote the minimum cost of producing $y_{kj}$ units of output (recall that firm $k$ is a price taker with respect to its inputs). Silvestre assumes that the function $C_k^T\ (y_{kj};\ \mathbf{p})$ is differentiable and that $C_k^T\ (0;\ \mathbf{p}) = 0$, so that fixed costs are ruled out. Let $C_k^M\ (y_{kj};\ \mathbf{p})$ denote marginal cost. Silvestre further assumes that $C_k^M\ (0;\ \mathbf{p})$ is finite and that $C_k^M\ (y_{kj};\ \mathbf{p})$ is convex in $y_{kj}$ for every $\mathbf{p}$. Finally, Silvestre assumes that (as in Negishi, conjectural demand curves are decreasing straight lines and) the marginal revenue curve is steeper than the marginal cost curve at the origin and that the input isoquants are convex to the origin for each value of $y_{kj}$. These assumptions are made in order to guarantee that the solutions to the equation 'marginal cost = marginal revenue' form a convex set, so that firm $k$'s reaction correspondence is convex-valued. The importance of the convexity of the reaction correspondences will be discussed in section 6.
12. The conjectural approach was further developed by Hahn (1977, 1978) and a criticism concerning the arbitrariness of conjectures was raised by John (1985).
13. Throughout the paper we shall maintain the notation of section 3.
14. One of the assumptions needed in order to prove the existence of a Walrasian equilibrium is that the initial endowment of consumer $i$, $\mathbf{e}_i$, be in the relative interior of her consumption set $X_i$ (cf. Debreu, 1959, p. 84). Such an assumption is no longer sufficient in this context, since the modified endowment corresponding to $(\mathbf{y}_1, \ldots, \mathbf{y}_r)$ may lie on the boundary of (or even outside) the consumption set. Gabszewicz and Vial (1972, p. 386, footnote 5) suggest that one could 'solve' this problem by not allowing firms to choose production vectors for which the corresponding exchange economy does not have a Walrasian equilibrium.
15. Gabszewicz and Vial (1972, p. 384) assume this problem away by introducing the assumption that Walrasian equilibria are always unique.
16. For a discussion of other, similar, problems, see Hart (1985, pp. 113–114).
17. Of course, this example is artificial and incomplete. A complete example can be found in Gabszewicz and Vial (1972, pp. 398–399).
18. A possible answer to this objection was provided by Kreps and Sheinkman (1983) in a partial equilibrium framework. In their model the Cournot-Nash equilibria emerge as the perfect equilibria (cf. Selten, 1975) of a two-stage game where firms first choose capacity levels and then compete in prices with the constraint that output levels in the second stage cannot exceed the capacity levels chosen in the first stage.
19. For a simplified account of Marshak and Selten's (1974) model, see Hart (1985, pp. 115–117) and Benassy (1988).
20. Given $\mathbf{p}$, each worker $i$ chooses her consumption $\hat{x}_{ij}$ of good $j$ ($j = 1, \ldots, m$) and $\hat{x}_{io}$

of leisure so as to maximize her utility function $U_i$ in the budget set

$$B = \left\{ \left( x_{io}, x_{i1}, \dots, x_{im} \right) \,\middle|\, \sum_{j=1}^{m} p_j x_{ij} = e_i - x_{io} \right\}$$

where $e_i$ is her initial endowment of leisure (recall the normalization $w = 1$). If $U_i$ is strictly quasi-concave, the solution to this maximization problem is unique. Then

$$L(\mathbf{p}) = \sum_{i \in W} (e_i - \hat{x}_{io}(\mathbf{p})) \text{ and } F_j(\mathbf{p}) = \sum_{i \in W} \hat{x}_{ij}(\mathbf{p})$$

where $W$ is the set of workers.

21. Let $\theta_{ik}$ be capitalist $i$'s share of the profits of firm $k$. Given $(\mathbf{p}, \boldsymbol{\pi})$, consumer-capitalist $i$ chooses her consumption $\hat{x}_{ij}$ of good $j (j = 1, \dots, m)$ so as to maximize her utility function $U_i$ in the budget set

$$B = \left\{ (x_{i1}, \dots, x_{im}) \,\middle|\, \sum_{j=1}^{m} p_j x_{ij} = \sum_{k=1}^{m} \theta_{ik} \pi_k \right\}$$

Again, if $U_i$ is strictly quasi-concave the solution to this maximization problem is unique and

$G_j(\mathbf{p}, \boldsymbol{\pi}) = \sum_{i \in C} \hat{x}_{ij} (\mathbf{p}, \boldsymbol{\pi})$, where $C$ is the set of capitalists.

22. The conditions are: the matrix $(I - A)$ is non-singular and its inverse is non-negative (so that for every non-negative vector $\mathbf{c}$ there exists a unique non-negative $\mathbf{y}$ such that $(I - A)\mathbf{y} = \mathbf{c}$), the functions $L(\mathbf{p})$, $F(\mathbf{p})$ and $G(\mathbf{p}, \boldsymbol{\pi})$ are non-negative and continuous and the limit of $L(\mathbf{p})$ as all prices simultaneously tend to infinity is equal to zero.
23. To this we should add the observation that, in general, the demand function is not independent of the choice of numeraire.
24. Nikaido (1975, pp. 68–73) also defines a subjective equilibrium á la Negishi and finds conditions under which it exists.
25. Boehm *et al.* (1983) have indeed shown, in a partial equilibrium model, that, if consumers are heterogeneous, a monopolist may find it profitable to ration them. For a general equilibrium model where rationing may occur in the labour market see Roberts (1987b).
26. For a brief outline of Benassy's (1988) model see Gary-Bobo (1988, pp. 65–67).
27. There are a few exceptions where existence results are proved without the assumption of quasi-concavity (e.g. McManus, 1962, 1964, Roberts and Sonnenschein, 1976). These models, however, are rather special and the only truly general theorems on the existence of Nash equilibria make use of the assumption of quasi-concavity. Recall that a function $f: \mathbb{R}^n \to \mathbb{R}$ is quasi-concave if for every $\mathbf{x}, \mathbf{y} \in \mathbb{R}^n$ and for every $\alpha \in [0, 1]$, $f(\alpha\mathbf{x} + (1 - \alpha)\mathbf{y}) \geqslant \min \{f(\mathbf{x}), f(\mathbf{y})\}$. Quasi-concavity is a weaker property than concavity, the latter requiring that $f(\alpha\mathbf{x} + (1 - \alpha)\mathbf{y}) \geqslant \alpha f(\mathbf{x}) + (1 - \alpha) f(\mathbf{y})$.
28. The example is taken from Bonanno (1988). For more insights into the issue of quasi-concavity see Bonanno (1987a).
29. That is, the reaction correspondence, of which the reaction curve is the graph, is not convex-valued.
30. This is easily understood as follows: demand functions are related to the first derivatives of the utility functions. Thus the second derivatives of the demand functions — which determine their shape — are related to the third derivatives of the utility functions, on which the assumption of convexity of preferences (quasi-concavity of the utility functions) puts no restrictions at all. Roberts and Sonnenschein (1977) show that this problem arises both in the Bertrand-Nash and in the Cournot-Nash approach.
31. A mixed strategy is a probability distribution over the set of pure strategies. A pure strategy for a firm in the Cournot-Nash approach is an output level, while a pure

strategy in the Bertrand-Nash approach is a price. Thus the equilibria defined so far were pure-strategy equilibria. When a firm chooses a mixed strategy, it decides to delegate the choice of pure strategy to a random mechanism, e.g. the toss of a coin.

32. For a recent criticism see Rubinstein (1988).
33. The reaction correspondences are functions and therefore convex-valued.
34. The main assumption is that the objective demand function be decreasing and its derivative be bounded away from zero. As we saw in section 5, however, in general the objective demand functions need not satisfy these properties. On the production side Silvestre assumes a Leontief technology, as in Nikaido (1975).
35. Bonanno and Zeeman (1985), unlike Silvestre (1977b), do not require the demand curves to be downward-sloping. The only requirements are that demand becomes zero at some (possibly very large) price and that the market is viable, in the sense that when prices equal unit costs demand is positive.
36. Figure 6 is taken from Bonanno and Zeeman (1985). For a numerical example of an equilibrium where all firms are at a local minimum of their 'true' profit functions and yet believe that they are maximizing profits, see Bonanno (1988). It may be worth stressing that if the *estimated* demand function is linear and coincides with the linear approximation to the 'true' demand function at, say, price $\hat{p}$, then the slope of the 'true' profit function at $\hat{p}$ is equal to the slope of the estimated profit function at $\hat{p}$ (and this is true whatever assumptions are made about the cost function).
37. Thus the S-shaped curve of Figure 7a is the set of $(p_1, p_2)$ such that $(\partial P_1/\partial p_1)(p_1, p_2) = 0$.
38. Thus the (inverted) Z-shaped curve of Figure 7b is the set of $(p_1, p_2)$ such that $(\partial P_2/\partial p_2)(p_1, p_2) = 0$.
39. At point $E$ both firms are at a local minimum of their 'true' profit functions. If they only know a linear approximation to their demand curves at that point, they will believe that they are maximizing profits (see Bonanno and Zeeman, 1985, and Bonanno, 1988; Figure 7 is taken from the latter).
40. At the general equilibrium level and in the Cournot-Nash framework Gary-Bobo (1987) has introduced the notion of $k$-th order locally consistent equilibrium, which 'is a general imperfectly competitive equilibrium of the subjective type, at which firms correctly perceive the $k$-th order Taylor expansion of their true demand curves' (Gary-Bobo, 1987, p. 217). Thus the case $k = 0$ corresponds to Negishi's (1961) conjectural equilibrium, the case $k = 1$ to Silvestre's (1977) and Bonanno-Zeeman's (1985) first-order equilibrium, the case $k = 2$ is closely related to Bonanno's (1988) local Nash equilibrium. Gary-Bobo shows that, if the true profit functions are *strictly quasi-concave* and have no points of inflection, 'when $k \geqslant 2$ the firms' equilibrium decisions are 'perfectly rational', leading the economy to states that would also have been attained under perfect knowledge of the demand conditions (Gabszewicz and Vial's (1972) Cournot-Walras equilibria)' (Gary-Bobo, 1987, p. 217). It is worth noting that the strict quasi-concavity of the profit functions and the absence of points of inflection guarantee that if there is a point where the first-order condition for profit maximization is satisfied, then that point is necessarily a global maximum. Hence when the profit function is strictly quasi-concave there cannot be a point which is a local but not global maximum.
41. Marshak and Selten (1974) assume that the profits of monopolistically competitive firms, but not the profits of all firms, are taken as given by each monopolistically competitive firm.
42. From this point of view Negishi's (1961) conjectural approach can be seen as a generalization of the notion of perfect competition: perfectly competitive firms are those whose conjectural demand curves (cf. section 3) are perfectly elastic.
43. See for example: Benassy (1989), Dasgupta and Ushio (1981), Gabszewicz and Vial (1972), Gabszewicz and Mertens (1971), Gabszewicz and Shitowitz (1988), Green (1980), Hart (1979, 1982b, 1985), Makowski (1980a,b), MasColell (1980, 1982, 1983),

McManus (1962, 1964), Novshek, (1980), Novshek and Sonnenschein (1978, 1980, 1983, 1986, 1987), Ostroy (1980), Roberts and Postlewaite (1976), Roberts (1980), Shitowitz (1973), Simon (1984), Ushio (1983).

44. A detailed account of one strand of this literature can be found in MasColell (1980, 1982). Novshek and Sonnenschein (1987) review another strand.
45. For an up-to-date account see Tirole, (1988), and Bonanno and Brandolini (1990).

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