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Deterrence, Observability and Awareness

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A simple example is used to analyze the issue of imperfect observability of commitment and to highlight the following phenomenon: when a player has the option of taking – at a cost – a (potentially) deterring action, she is less *likely to do so against an opponent who is* aware *of the availability of this option than against an opponent who is not aware.*

1. *Introduction*

We consider a simple deterrence game between two players, the *Incumbent* and the *Potential Intruder*. The Incumbent decides whether or not to take a costly action which – if observed by the Potential Intruder – will deter him from taking an "aggressive" action [1]. For ease of exposition we shall refer to the Incumbent's costly action as commitment [2]. We focus attention on the case where payoffs are such that commitment would indeed be chosen by the Incumbent if it were perfectly observable.

When the incumbent's action is observed with probability less than *1*, we can envision two situations. In one, which we call the *Aware Potential Intruder* case, the Potential Intruder is aware of the possibility of commitment, whether or not he actually observes it. In the other situation, called the *Unaware Potential Intruder* case, the Potential Intruder is initially unaware of the availability of commitment and becomes aware of it only if he actually observes the Incumbent's action. We prove the somewhat counterintuitive result that the Incumbent is *less likely* to commit if she faces a Potential Intruder who is *aware* of the possibility of commitment. The reason is as follows. A Potential Intruder who is aware of the possibility of commitment will tend to be more cautious. This fact enables the Incumbent to "free ride" on the opponent's caution and avoid taking a costly action. In other words, *the mere availability of commitment has a deterrent effect on an opponent who is aware of it and, as a*

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[1] Throughout the paper I shall speak of the Incumbent as female and the Potential Intruder as male.

[2] This term may not be entirely appropriate since the Incumbent is the first-mover and, therefore, all her choices could be labelled as "commitment". However, in general, a deterring action does have the nature of a commitment in that it either reduces the options available to the Incumbent later on or it alters the Incumbent's incentives to choose among different options.

consequence, the need to commit is reduced. We also show that both players are better off in the situation where the Potential Intruder is aware of the possibility of commitment.

Ben Porath and Dekel (1989) and van Damme (1989) have shown that the mere availability of choices, *without their actual use*, may affect the outcome of a game [3]. Thus they highlight a different phenomenon to ours, namely that (if the opponent is rational) the mere fact that an action is available may make it unnecessary to use it. We, on the other hand, bring to light the issue of awareness and show how the opponent's awareness (or lack of it) concerning the availability of certain choices, may influence the behavior of a player in a counterintuitive way [4].

2. *The model*

There are two players, the Incumbent and the Potential Intruder. The Incumbent has to decide whether or not to take a costly pre-emptive action,

Table 1

It is assumed that $a_1 > a_3 > a_4 > a_2$ and $b_4 > b_1 = b_3 > b_2$.

Outcome	Utility of Incumbent	Utility of Potential Intruder
Two-sided passivity (Incumbent: no action Intruder: no action)	a_1	b_1
Unsuccessful attempt to deter (Incumbent: action Intruder: action)	a_2	b_2
Successful deterrence (Incumbent: action Intruder: no action)	a_3	b_3
Acquiescence (Incumbent: no action Intruder: action)	a_4	b_4

[3] For example, if the Battle of the Sexes game is modified by adding an initial stage where player I is given the option of burning some "utils", then the unique "rational" outcome turns out to be the Nash equilibrium outcome of the Battle of the Sexes which is most preferred by player I. [By "rational" outcome we mean the outcome obtained by iterative deletion of weakly dominated strategies; hence it is also the unique stable outcome in the sense of Kohlberg and Mertens (1986)]. Furthermore, this outcome is obtained without any need for player I to actually take the costly action of burning some utils.

[4] In the papers by Ben Porath and Dekel (1989) and van Damme (1989) the fact that each player is aware of the choices available to the other player is implicitly assumed (it is a consequence of the assumption that the structure of the game is common knowledge among the players). There is no comparison between a situation where one player is aware of something and a situation where he is not.

while the Potential Intruder must decide whether or not to take an "aggressive" action. Thus there are four possible outcomes:

1) *Two-sided passivity*: neither player takes the respective action;

2) *Unsuccessful attempt to deter*: both players take their actions;

3) *Successful deterrence*: the Incumbent takes the pre-emptive action, the Potential Intruder does not act;

4) *Acquiescence*: the Incumbent does not act, the Potential Intruder takes the aggressive action.

Table 1 shows the four outcomes and the corresponding utilities for the two players.

We shall make the following assumptions about the Potential Intruder's preferences: $b_4 > b_1 = b_3 > b_2$. That is,

(i) if the Potential Intruder is passive, he does not care whether the Incumbent did or did not take the pre-emptive action $(b_1 = b_3)$;

(ii) if the Incumbent takes the pre-emptive action, then the Potential Intruder prefers not to act $(b_3 > b_2)$;

(iii) the Potential Intruder's most preferred outcome is acquiescence.

For the Incumbent we shall assume that $a_1 > a_3 > a_4 > a_2$. That is,

(iv) the pre-emptive action is costly, so that – if the Potential Intruder is passive – the Incumbent prefers to be passive herself $(a_1 > a_3)$;

(v) successful deterrence is worthwhile for the Incumbent $(a_3 > a_4)$;

Table 2

The normalized utility functions

Outcome	Utility of Incumbent	Utility of Potential Intruder
Two-sided passivity (Incumbent: no action Intruder: no action)	$a > 1$	0
Unsuccessful attempt to deter (Incumbent: action Intruder: action)	$-b$ $(b > 0)$	$-c$ $(c > 0)$
Successful deterrence (Incumbent: action Intruder: no action)	1	0
Acquiescence (Incumbent: no action Intruder: action)	0	1

(vi) acquiescence is better than an unsuccessful attempt to deter ($a_4 > a_2$).

We assume that these are von Neumann-Morgenstern utility functions. We can therefore normalize them as shown in Table 2 [5].

Given the above preference structure, it is clear that if the pre-emptive action is perfectly observable, the Incumbent will decide to take it and the Potential Intruder will remain passive [6]. Suppose, however, that the pre-emptive action, if taken, would only be observed with probability q, where $0 < q < 1$. Then, as said in the introduction, we can envision two situations. In the situation with an *Aware* Potential Intruder, the latter knows that the Incumbent had the option to commit, even if he does not actually observe the commitment. In the situation with an *Unaware* Potential Intruder, the latter is not aware of the possibility of pre-emptive actions: he only becomes aware of it if he sees one [7].

The game with an Aware Potential Intruder is represented in Figure 1, where the top number in each column represents the utility of the Incumbent and the bottom number the utility of the Potential Intruder. First Nature selects one of two possibilities: with probability q the incumbent's action, if taken, will be observed by the Potential Intruder, while with probability $(1 - q)$ it

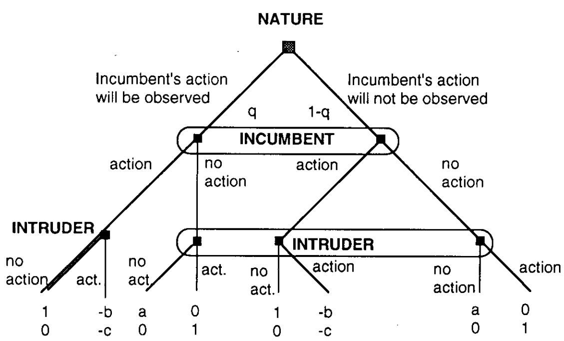

Figure 1 - The game with an aware Potential Intruder

[5] The normalization is the result of the following linear transformations: $a'_i = \dfrac{a_i - a_4}{a_3 - a_4}$ for the Incumbent and $b'_i = \dfrac{b_i - b_1}{b_4 - b_1}$ for the Potential Intruder.

[6] This is the unique subgame-perfect equilibrium (cf. Selten, 1975) of the corresponding perfect information game.

[7] It is perhaps worth repeating that the unaware Potential Intruder does not know that the pre-emptive action is available to the Incumbent. Thus he will act under the assumption that the only possible outcomes are what we called "two-sided passivity" and "acquiescence". However, he may accidentally observe the Incumbent's commitment (this will happen with probability q), in which case he will suddenly become aware of the pre-emptive action and react optimally be refraining from taking his aggressive action.

will not be observed. The Incumbent, not knowing what Nature chose, has to decide whether or not to take a pre-emptive action. The Potential Intruder can find himself in two different situations: in one he has learnt that the incumbent took the pre-emptive action, while in the other he does not know whether the pre-emptive action was taken (and he did not receive any signals) or it was not taken. In both situations the Potential Intruder has to decide whether or not to take an aggressive action.

The players' preferences are such that if the Incumbent takes her pre-emptive action and the Potential Intruder observes it, then the latter will decide to be passive: this is denoted by a double edge in Figure 1. Thus we shall restrict our attention to Nash equilibria that are consistent with this behavior on the part of the Potential Intruder (that is, to subgame-perfect equilibria: cf. Selten, 1975). As a consequence, the game reduces to one whose normal form is shown in Table 3[8].

Table 3

		POTENTIAL INTRUDER	
		aggressive action (r)	no action (1 − r)
INCUMBENT	pre-emptive action (p)	$q - b(1-q)$, $-c(1-q)$	1, 0
	no action (1 − p)	0, 1	a, 0

If $[q - b(1-q)] \leqslant 0$[9], that is, if the probability that the pre-emptive action is observed is sufficiently small, then 'no action' is a dominant strategy for the Incumbent and there is a unique Nash equilibrium where the Incumbent does not act and the Potential Intruder does. Thus:

Result 1: In the Aware Potential Intruder case, if $q \leqslant \frac{b}{1+b}$ the Incumbent does not take her pre-emptive action, the Potential Intruder takes his aggressive action and the outcome is acquiescence, with corresponding utility of 0 for the Incumbent and 1 for the Potential Intruder.

Consider now the case $\frac{b}{1+b} < q < 1$ [that is, $q - b(1-q) > 0$]. Then the normal-form game of Table 3 does not have a Nash equilibrium in pure strategies. There exists, however, a unique Nash equilibrium in mixed strategies.

[8] Note that the normal form of Table 3 is the agent normal form of the extensive game of Figure 1 when the Potential Intruder who observes the pre-emptive action follows the equilibrium strategy.

Note also that if $q = 0$, then the game of Figure 1 reduces to a simultaneous game, in which not taking the pre-emptive action is a strictly dominant strategy for the Incumbent, so that the outcome would be acquiescence. On the other hand, if $q = 1$ we have a game of perfect information where the unique subgame-perfect equilibrium outcome is successful deterrence.

[9] Equivalently, if $q \leqslant \frac{b}{1+b}$. Recall that our assumptions imply that $0 < \frac{b}{1+b} < 1$.

Result 2: In the Aware Potential Intruder case, if $\frac{b}{1+b} < q < 1$ the Incumbent takes her pre-emptive action with probability

$$0 < p = \frac{1}{1 + c(1 - q)} < 1,$$

the Potential Intruder does not take his aggressive action if the observes the Incumbent's commitment, and takes it with probability

$$0 < r = \frac{a - 1}{a - 1 + [q - b(1 - q)]} < 1$$

if he does not know whether the Incumbent did or did not take her pre-emptive action. Furthermore, the expected utility for the players is $\frac{a[q - b(1 - q)]}{a - 1 + [q - b(1 - q)]} > 0$ for the Incumbent and 0 for the Potential Intruder.

It follows from results 1 and 2 that the probability with which the Incumbent chooses to commit is an increasing function of q (the probability that commitment is observed by the Potential Intruder). Furthermore, the Incumbent's equilibrium payoff is also an increasing function of q, while the Potential Intruder's equilibrium payoff decreases with q.

We now consider the situation with an Unaware Potential Intruder. Here the Potential Intruder is not aware of the fact that the Incumbent has the option of taking a pre-emptive action and therefore his perception is that it pays to take the aggressive action. If, however, the Potential Intruder happens to observe the Incumbent's action then he becomes aware of it and reacts optimally by not taking his aggressive action. Implicit in game theory is the assumption that the structure of the game is common knowledge among the players. Hence game theory does not allow us to model lack of awareness directly. An indirect way of representing the Unaware Potential Intruder situation is by eliminating the passive choice for the Intruder at his large information set, as shown in Figure 2. It should be stressed that for our purposes all that is needed is a representation of this situation *as seen by the Incumbent*, because from the Incumbent's point of view the Unaware Potential Intruder case is a simple one-person decision problem: she knows that if she takes her pre-emptive action there is a probability q that it will be observed by the Potential Intruder and that, as a consequence, it will induce him not to act, while in every other case the Potential Intruder will thoughtlessly take his aggressive action [10]. However, it is useful to represent this situation by means of the game shown in Figure 2, since it allows us to compare the two cases by comparing the subgame-perfect equilibria of two games with a similar structure.

[10] Thus the Incumbent's expected utility is $[q - b(1 - q)]$ if she takes her pre-emptive action and 0 if she does not.

The game of Figure 2 has a unique subgame-perfect equilibrium where the Potential Intruder is passive whenever he observes the Incumbent's action, while the Incumbent's strategy varies with the parameter q as follows.

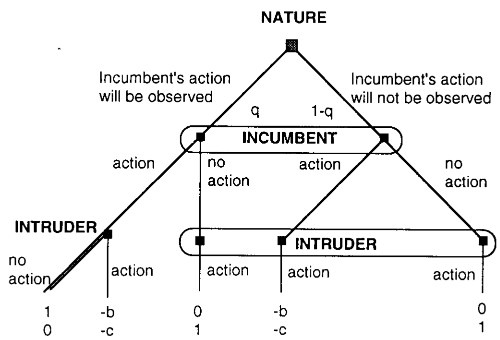

Figure 2 - The game with an unaware Potential Intruder

Result 3: In the Unaware Potential Intruder case, if $q \leqslant \frac{b}{1+b}$ [that is, if $q - b(1-q) \leqslant 0$] the Incumbent chooses not to take the pre-emptive action and the outcome is acquiescence with payoffs of 0 for the Incumbent and 1 for the Potential Intruder.

Result 4: In the Unaware Potential Intruder situation, if $q > \frac{b}{1+b}$ [that is, if $q - b(1-q) > 0$] the Incumbent takes the pre-emptive action and the outcome is successful deterrence with probability q and unsuccessful attempt to deter with probability $(1-q)$.

The payoffs are $[q - b(1-q)]$ for the Incumbent and $[-c(1-q)]$ for the Potential Intruder.

We can now compare the two situations. This is done in Table 4.

It can be seen that if $q \leqslant \frac{b}{1+b}$ or if $q = 1$, then there is no difference between the two situations: the Incumbent's behavior and the utility of both players are the same in both. If, on the other hand, $\frac{b}{1+b} < b < 1$ [which is equivalent to $0 < [q - b(1-q)] < 1$] then we can see that *the Incumbent is less likely to take the pre-emptive action in the Aware Potential Intruder situation than in the Unaware Potential Intruder one.* In fact, in the former the Incumbent takes her pre-emptive action with probability less than one, while in the latter she takes her action with probability 1. Furthermore, since for this range of values of the parameter q we have that $-c(1-q) < 0$

and $\dfrac{a\,[q - b\,(1 - q)]}{a - 1 + [q - b\,(1 - q)]} > [q - b\,(1 - q)]$, *both Incumbent and Potential Intruder are better off in the Aware Potential Intruder situation than in the Unaware Potential Intruder one.*

Table 4

The two SituationS Compared for all Possible Values of q.

	$0 \leqslant q \leqslant \dfrac{b}{1+b}$	$\dfrac{b}{1+b} < q < 1$	$q = 1$
AWARE Potential Intruder	Incumbent does *not* take pre-emptive action	Incumbent takes action with probability $0 < p < 1$ (with $\dfrac{dp}{dq} > 0$)	Incumbent takes action
	Incumbent's payoff: 0	Incumbent's payoff: $\dfrac{a\,[q - b\,(1 - q)]}{a - 1 + [q - b\,(1 - q)]}$	Incumbent's payoff: 1
	Intruder's payoff: 1	Intruder's payoff: 0	Intruder's payoff: 0
UNAWARE Potential Intruder	Incumbent does *not* take pre-emptive action	Incumbent takes pre-emptive action	
	Incembent's payoff: 0	Incumbent's payoff: $q - b\,(1 - q)$	
	Intruder's payoff: 1	Intruder's payoff: $- c\,(1 - q)$	

As explained in the Introduction, the intuition behind this result is that when the Potential Intruder is aware of the possibility of commitment he will tend to be more cautious whenever he does not observe the Incumbent's pre-emptive action, that is, he will take his aggressive action with lower probability. The Incumbent can thus "free ride" on the Potential Intruder's caution and reduce the frequency of a costly commitment. In other words, the mere fact that the opponent is aware of the possibility of commitment reduces the need to actually commit.

3. *Concluding Remarks*

We examined the issue of observability and awareness of commitment in a general two-person model of deterrence. We noticed two phenomena.

First of all, as intuition suggests, when the Potential Intruder is aware of the possibility of commitment, the commitment is more likely the higher the probability that it is observed by the Potential Intruder [11].

[11] Schelling (1960, 1966) pointed out the fact that in order for commitment to be attractive it is necessary that it be observable (with sufficiently high probability).

The second, less intuitive phenomenon, is that when commitment is imperfectly observable, then the Incumbent is *more likely* to commit in the case where the Potential Intruder is *unaware* of the possibility of commitment (he only becomes aware of it if he sees it), than in the case where the Potential Intruder is aware of the possibility.

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