Sudden and Surprising Changes of Attitude During Negotiations*

1. *Introduction*

The purpose of this paper is to try and formalize — in the context of a simple example — phenomena which are often observed in negotiations, namely sudden (and sometimes surprising) changes of attitude in one of the parties (or both). It is not uncommon for negotiations to end abruptly, without any warning, or to take sudden turns, as when one party — after having gradually «softened» its position and shown increasing willingness to compromise — suddenly reverts to an extreme position and becomes intransigent, much to the surprise of its opponent.

We shall consider the case of a pay dispute between a firm and a union. Our objective is not to formalize the negotiation game but to show how an *a priori* «good» handling of the negotiations by one party (in this case the firm) may be successful at first and then suddenly produce an unexpected and undesired result. In particular, we show that even though the firm has been successfully convincing the union that the requested pay rise is «unreasonable», the response of the union may be to reduce its request at first and then suddenly revert to its original, extreme request. Therefore the firm has to be careful not to «overdo it», that is, not to go too far in trying to convince the union that the pay rise which the firm can afford to give is very small.

On the other hand, we show that sometimes the desired change of attitude in the opponent comes suddenly and all at once, after an initial and persistent lack of reaction (despite the firm being successful, all along, in modifying the union's initial beliefs). Thus in this situation the firm has to be careful not to become disheartened and give up too soon, as a further tiny step in the same direction may suddenly produce the desired result.

The paper is organized as follows. In Section 2 we set up the background to the negotiation stage, which is then studied in Section 3. Section 4 offers some final remarks and a conclusion.

* A first version of this paper was written when the author was Heyworth Research Fellow at Nuffield College, Oxford (U.K.). The comments of an anonymous referee are gratefully acknowledged.

2. *The Origin of the Pay Dispute*

Much of the background to the negotiation stage will be outlined only briefly and we shall not formalize every aspect of it. The content of this Section is nothing more than a plausible series of events which can lead to the negotiation phase analysed in Section 3.

Assume that at date zero it becomes known (possibly through the media) that a certain firm has received a major order, which will yield a *net surplus* to the firm equal to S. Without loss of generality we can assume that

$$0 < S \leq 1 \tag{1}$$

However, the contract between the firm and the customer will not be signed before date T and the (potential) customer has made it clear that he will finalize the contract at that date only if he does not foresee any problems, in particular, if there are no signs of industrial disputes or other obstacles to normal production [1].

The firm's work force is unionized and we first assume that the union's reaction to the news is to ask for a *pay rise* x, with

$$0 < x < S \tag{2}$$

and threaten to call a strike if the request is not met. This gives rise to the following game, illustrated in Figure 1, where the firm has two strategies — to give the pay rise or refuse to — and the union has also two strategies — to carry out the threat or not to.

In Figure 1 the union's preferences over the possible outcomes are shown in the top right-hand corner of each box. The best outcome for the union is to obtain the pay rise without having to call a strike [2]. The worst outcome, on the other hand, is «defeat»: the union does not get the pay rise and does not carry out its threat either [3]. Finally we assume that the union prefers pay-rise/strike to no-pay-rise/strike, but our results would not be affected if the preferences over these two outcomes were interchanged.

The firm's preferences, on the other hand, are shown in the bottom, left-hand corner of each box. The firm's most preferred outcome is no-pay-rise/no-strike (it gets all the surplus). The worst outcome is pay-rise/strike (it loses the

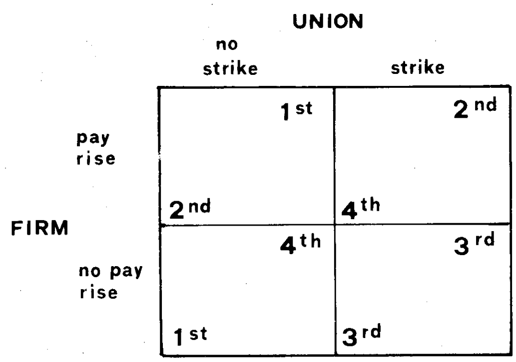
[1] The assumption here is that the customer can easily find another firm, possibly in another country, which is able to provide the same goods or services (maybe at a higher price, but such that the difference in prices is lower than the cost associated with the delay caused by a strike). The assumption that, if a strike is called before the contract is signed, the firm will lose the order, can be interpreted as an extreme case of the "decay" of profitability experienced by a strike-bound firm as modelled by Hart (1989).

[2] Here we are implicitly assuming that strikes are costly to the union (either in monetary or non-monetary terms, as in the case where the union is afraid of losing some of its members), so that the union's most preferred outcome is to obtain a pay rise without having to resort to a strike.

[3] The assumption here is that the union is concerned with its reputation and credibility and the outcome just described would be deleterious from the point of view of future disputes with the same firm (or possibly other firms).

potential surplus S and it still has to give a pay rise to the workers). As long as the requested pay rise x is strictly less than the net surplus S, the firm will prefer pay-rise/no-strike to no-pay-rise/strike (strikes are costly to the firm in terms of lost production; on the other hand we are ignoring issues of reputation for the firm).

Fig. 1 — In each box the union's preference ordering is marked in the top right-hand corner and the firm's in the bottom left-hand corner



It can be seen from Figure 1 that no-pay-rise is a strictly dominant strategy for the firm and, therefore, there is a unique Nash equilibrium of this game given by no-pay-rise/strike. The union, however, by taking the initiative and moving first, can achieve a better outcome. Its most preferred outcome is pay-rise/no-strike and the union can obtain this by setting an *ultimatum*: if by date T-1 the firm has not agreed to give the requested pay rise, workers will go on strike at date T (just before the contract is signed). Now the firm knows that if by time T-1 it has not agreed to the pay rise, the game will be played simultaneously at date T and the outcome will be no-pay-rise/strike (the unique Nash equilibrium of the game illustrated in Figure 1). If, on the other hand, the firm agrees to the requested pay rise by time T-1, then the union will definitely *not* call a strike ("no strike" is the union's best response to "payrise"). The firm will therefore quickly announce that it has agreed to the union's request and the «dispute» is over.

We have therefore obtained the «reasonable» solution of this game (pay-rise/no-strike), but so far there is no room for negotiations. The union has nothing to gain from negotiating and the firm has no way of influencing the union's position.

There *will* be room for negotiation if we introduce uncertainty. In particular, we shall assume that the union does not know the exact value of the net surplus S. The union, however, has some beliefs about it, which can be expressed by a density function h(s) whose support is a subset of [0,1]. Let H(x) be the correspondig cumulative distribution function, that is

(3) $$H(x) = \int_0^x h(s)ds$$

and let

(4) $$P(x) = 1 - H(x).$$

Thus P(x) is the probability, according to the union's beliefs, that *S* is greater than *x*, that is, that — having requested a pay rise of *x* and having set an ultimatum — the firm will grant *x*. Let U(x) be the union's utility-of-money function, normalised so that U(0) = 0. Then the *expected utility* of asking for a pay rise of *x* is given by

(5) $$f(x) = U(x)\ P(x)$$

If the union asks for a small pay rise, it has a high probability of getting it, but its utility would be small. If, instead, it asks for a high pay rise, its utility would be high, but the probability of getting it is small. We shall assume that the union asks for that pay rise x^* which maximizes f(x) [4].

If the union's utility-of-money function U(x) is increasing and U(0) is normalized to be zero, we have that $f(x) \geq 0$ for all *x*, with strict inequality for some *x*. Thus — assuming that U(x) is continuous and noting that f(0) = f(1) = 0 — it follows that x^* has the property that $0 < H(x^*) < 1$. This means that there is a positive probability (according to the union's beliefs) that $S < x^*$ and that, therefore, the firm — if faced with a request of x^* — will have no choice but refuse. Therefore there is now room for negotiation. The union has nothing to lose from agreeing to negotiate and indeed it may gain: if during the negotiation the union can acquire information about the true value of *S*, it will be able to ask for the «right» pay rise. The firm, on the other hand, has everything to gain from negotiating with the union: if $S > x^*$ the firm would agree to the pay rise anyway, but it now has a chance to try and convince the union that x^* is an unreasonable request; if, on the other hand, $S < x^*$, the firm would refuse to give the pay rise, but if it can persuade the union to reduce its request to a value $x_0 < S$, it will avoid a costly strike and obtain a positive surplus equal to $S - x_0$.

We would therefore expect the union to set an ultimatum (as explained above) without however committing itself to x^*: the union can say that — on the basis of the information it has — a reasonable pay rise seems to be x^*, but it is willing to examine any evidence which the firm might care to produce until date T-2. At time T-2, in the light of the information obtained during the

[4] For example, if U(x) = x and the union's beliefs are expressed by the uniform distribution (whose density is given by h(s) = 1 if $0 \leq s \leq 1$ and zero otherwise), then $x^* = 1/2$. It will become clear later that our results are independent of the union's initial beliefs. Also, we are not assuming that the union's beliefs are common knowledge.

negotiation, the union will make its final request (and reiterate its ultimatum; the firm will then have a chance at time T-1 to agree to the final request, as explained above).

In the next section we shall look at the negotiation phase from the point of view of the firm, in particular at the way in which it should not handle the negotiation.

3. *The Negotiation Phase*

We assume that the firm cannot «prove» to the union what the true value of *S* is [5]. The firm, however, has available (or will have available) some informative signals (or data) which are not known to the union. The firm's problem is to decide whether or not to reveal this information to the union during the negotiation phase and whether it should disclose all of it or only part of it [6]. Let *D* be the set of data which, potentially, convey information about the value of the surplus and let

$$g: D \times [0,1] \rightarrow R \tag{6}$$

be a function which represents the «objective» relationship between data and surplus, that is, for each *s*, *g*(*d*,*s*) is the probability of observing data *d* given that the state of the environment is such that the surplus will be *s*. If h_t represents the union's beliefs at time *t* and the firm reveals data *d*, the union's beliefs at time $t+1$, updated according to Bayes' rule, will be given by

$$h_{t+1}(s) = \frac{g(d,s)h_t(s)}{\int_0^1 g(d,s)h_t(s)ds} \tag{7}$$

Intuitively, it seems that a good policy for the firm would be to reveal any piece of information which makes the union more pessimistic. We say that between time t and $t+1$ the union has become *more pessimistic* if

$$P_t(x) \geq P_{t+1}(x) \qquad \text{for all } x \in [0,1] \tag{8}$$

that is, if — in the union's mind — the probability of obtaining any given pay rise x has become smaller (recall that $P_t(x) = 1 - H_t(x)$, where H_t is the c.d.f. corresponding to h_t). Thus if the union becomes more pessimistic, its maximum expected utility decreases. Our definition is equivalent to saying that P_t

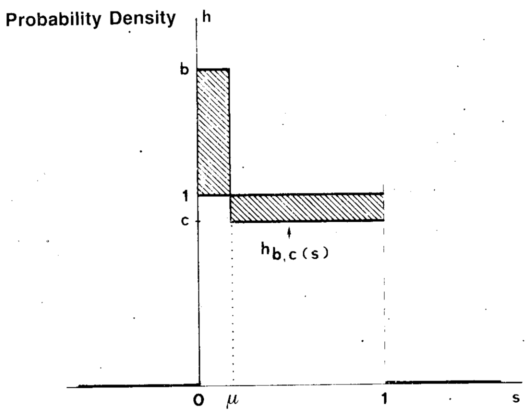
[5] The reason for this could be that the firm itself can only make a forecast about the value of *S* and/or that the union — knowing that the firm has an incentive to make things look worse than they actually are — would always be suspicious of any «evidence» produced.

[6] The data could refer to input prices, demand, competitors' behaviour, technological innovation, likely outcomes of R&D, etc.

dominates P_{t+1} in the sense of first-order stochastic dominance [7]. Using a terminology introduced by Milgrom (1981) we can express the same concept by saying that if at time *t* information *d* is *bad news* for the union, then it is a good idea for the firm to reveal it.

We now give an analytic example which shows that the above intuition is wrong. In the Appendix it is shown that the evolution of beliefs illustrated in the example is consistent with Bayesian updating [8].

Fig. 2 — The density function $h_{b,c}(s)$. Given b and c, μ is determined and equal to (1-c)/(b-c), so that the two shaded areas are equal



Let the union's utility function be given by

$$U(x) = x \tag{9}$$

and its initial beliefs by the uniform distribution ($h(s) = 1$ for $s \epsilon [0,1]$ and zero otherwise). We will show later that our results do *not* depend on the assumption of risk-neutrality (nor do they depend on the above specification of initial beliefs).

Consider now the following two-parameter family of beliefs, illustrated in Figure 2.

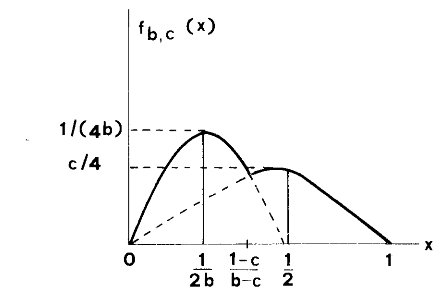
[7] See, for example, Lippman-McCall (1982, pp. 215-6). It can be shown that P_{t+1} is dominated by P_t in the sense of first-order stochastic dominance if and only if

$$\int_{-\infty}^{+\infty} V(s)h_{t+1}(s)ds \leq \int_{-\infty}^{+\infty} V(s)h_t(s)ds$$

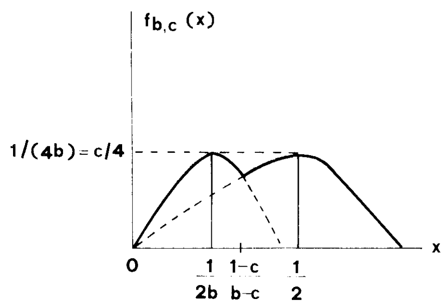
for every non-decreasing function V (in particular, by choosing $V(s) = s$ it follows that if P_t dominates P_{t+1} then the mean at time $t+1$ is less than the mean at time t).

[8] An alternative interpretation of the example of this section is in terms of comparative statics rather than in terms of change over time.

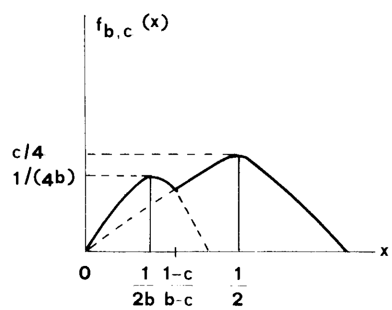
Fig. 3 — The function $f_{b,c}(x)$



(a) Case $\frac{1}{4b} > \frac{c}{4}$



(b) Case $\frac{1}{4b} = \frac{c}{4}$



(c) Case $\frac{1}{4b} < \frac{c}{4}$

(10) $$h_{b,c}(s) = \begin{matrix} b & \text{if } 0 \leq s \leq \mu \\ c & \text{if } \mu < s \leq 1 \end{matrix}$$

where

(11) $$\mu = (1-c)/(b-c)$$

(in fact, given *b* and *c*, the point μ is determined by the condition that the integral of *h* between 0 and 1 be equal to 1: the shaded areas in Figure 2 are equal). When $b = c = 1$ we have the uniform distribution.

Let $H_{b,c}$ be the cumulative distribution function corresponding to $h_{b,c}$ and let $P_{b,c} = 1 - H_{b,c}$. Then

(12) $$P_{b,c}(x) = \begin{matrix} 1-bx & \text{if } 0 \leq x \leq \mu \\ c-cx & \text{if } \mu \leq x \leq 1 \end{matrix}$$

It is easy to check that if

(13) $$b \geq b' \text{ and } c \leq c' \text{ and not both equal}$$

then $h_{b,c}$ represents more pessimistic beliefs than $h_{b',c'}$, according to the definition given above. Thus if, for example, the initial beliefs of the union are given by $b = c = 1$ (that is, by the uniform distribution), then the firm will try and force the union's beliefs to lie in the region defined by the inequality

$$0 \leq c < 1 < b$$

which is the shaded area in Figure 4. At any point in that area, a movement in the East, South or South-East direction represents an increase in pessimism.

Let $f_{b,c}(x)$ be given by (5), that is — using (9) —,

(15) $$f_{b,c}(x) = U(x)P_{b,c}(x) = xP_{b,c}(x)$$

Then $f_{b,c}(x)$ is the «union» of two parabolas, as shown in Figure 3 [9]. Finally, let

(16) $$x^*(b,c) = \underset{x}{\text{argmax}}\ f_{b,c}(x)$$

It is easy to check that

(17) $$x^*(b,c) = \begin{matrix} 1/2 & \text{if } c > 1/b \\ 1/(2b) & \text{if } c < 1/b \end{matrix}$$

[9] Figure 3 does not illustrate all the possible cases. The case $1/(4b) > c/4$ (Figure 3a) includes two more cases, where the maximum of the parabola on the right goes inside the other parabola and therefore the function f becomes unimodal. Similarly for the case $1/(4b) < c/4$ (Figure 3c).

while for $c = 1/b$, $x^*(b,c) = \{1/2,\ 1/(2b)\}$. We call the set $\{(b,c)/c = 1/b\}$ the *Maxwell set* (see Bonanno-Zeeman, 1988).

The Maxwell set is the subset of the parameter space at which the function $x^*(b,c)$ is discontinuous [10]. In Figure 4 we have shown two possible paths, denoted by (1) and (2), which represent *a priori* good handling of the negotiations by the firm, in that they are associated with increasing pessimism on the part of the union. The arrows denote the evolution of the union's beliefs over time.

Let us first consider path (1): $x^*(b,c)$ is the pay rise which maximizes the union's expected utility when its beliefs are given by $h_{b,c}(s)$ (thus, $x^*(b,c)$ is the pay rise the union will request when date T-2 arrives if $h_{b,c}$ are the beliefs it holds at that time). Figure 5a illustrates the evolution of x^* over time along path (1).

The union's response is at first as expected: as the union becomes more pessimistic, it reduces its request (from 1/2 to 1/(2b): b increases along the path), then at time $\hat{t}$ (when the point $b = 3$, $c = 1/3$ is reached) it suddenly switches (discontinuously) to its original request of 1/2 and it sticks to it despite the fact that the firm is still being successful in making the union more and more pessimistic. Intuitively what is happening here is the following. After having initially persuaded the union that *S* cannot be very large, the firm is now concentrating its effort on convincing the union that *S* is very small (and therefore that the pay rise it can afford to give is close to zero). This policy is successful at first (the union reduces its original request), but it then creates a conflict in the union's mind. In fact, the union now «knows» that if it asks for a very low pay rise it is very likely that it will get it, but its utility would be very small. On the other hand, the union does not rule out a very small chance that the firm can actually afford to give a substantial pay rise (the probability that *S* is large is very small but positive). Although the probability of obtaining a large pay rise is very small, the associated utility would be very high. As the union becomes more and more pessimistic the conflict in its mind grows stronger and stronger, and if the firm goes too far in pursuing its policy the union will suddenly decide to opt for the low probability-high utility bet and stick to it no matter how pessimistic it becomes. Therefore, along path (1) the firm runs the risk of «overdoing it»: when it reaches a point like *W* it ought to stop negotiating and accept the pay rise requested by the union, even if it still has the means to make the union more pessimistic.

The conflict in the union's mind, however, can also work in favour of the firm. This is what happens along path (2) in Figure 4. The corresponding evolution of $x^*(b,c)$ is illustrated in Figure 5b. For a long time there is no response on the part of the union and then, suddenly, the union «capitulates» and asks for a pay rise which is one third the original request. The danger with the policy associated with path (2), however, is that — since for a long time there is no response — the firm may be led to infer that it is not being successful in affec-

[10] The Maxwell line has a vertex at $b = 1$, $c = 1$. Outside the shaded region of Figure 4, the function $x^*(b,c)$ is continuous. Therefore the graph of $x^*(b,c)$ is a cusp [see Bonanno-Zeeman, 1988].

ting the union's beliefs and may give up (if the true S is greater than 1/2), when in fact a further (tiny) step in the same direction would suddenly, and all at once, have produced the desired result. That is, the firm may become disheartened and stop at a point like Z in Figure 4.

Fig. 4 — The Maxwell line and two possible paths crossing it

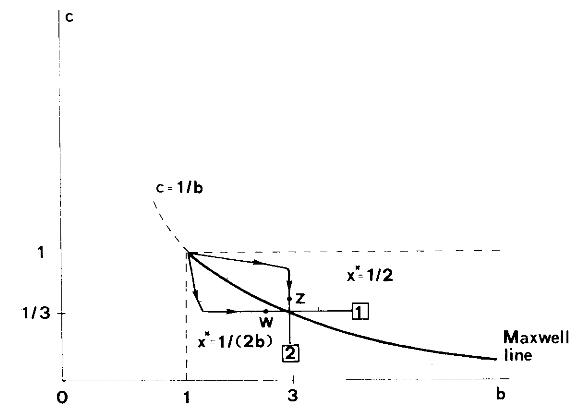


Fig. 5.a — The evolution of the requested pay rise along path (1) in Fig. 4. Time $\hat{t}$ is when the point b = 3, c = 1/3 is reached

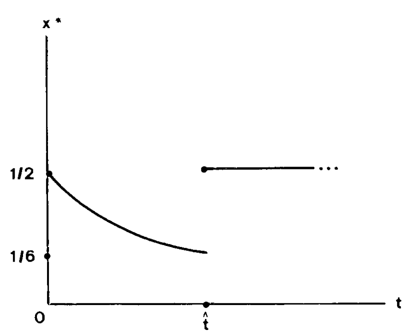
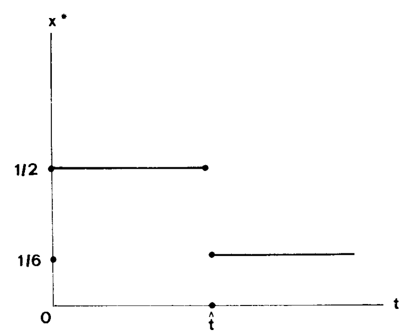


Fig. 5.b — The evolution of the requested pay rise along path (2) in Fig. 4



An interesting question is whether there are conditions which ensure that greater pessimism will lead to lower values of the requested pay rise. One such condition is the monotone likelihood ratio property (MLRP) often used in the literature (cf. Milgrom, 1981). Such property would require, in our example, that *b* and *c* change in a way that keeps μ constant. When μ is constant, beliefs follow a straight-line path through the point ($b=1$, $c=1$) in Figure 4. Therefore in this case the «switch to intransigence» no longer occurs, while the phenomenon of delay and sudden «capitulation» (which occurred along path 2) still remains. In the «revelation of information» interpretation which we followed above, however, the firm has no direct control over the union's beliefs: the firm can only choose whether or not to reveal a given piece of information. In this case an interesting question is whether there are properties of the function *g* (which gives the probabilistic relationship between data and surplus: cf. (6) and (7)) which rule out the possibility of a switch to intransigence. Applying a result given in Bonanno (1988), it can be shown that if *d* is a piece of information such that $g(d,\cdot)$ is a non-constant, non-increasing function of *s*, then revelation of *d* cannot lead to an increase in the requested pay rise. Note, however, that this property of *g* does *not* imply that μ remains constant (that is, it does not imply the MLRP). Furthermore, this property of $g(d,\cdot)$ does not rule out the phenomenon of delay and sudden «capitulation» either.

4. *Final Remarks and Conclusion*

The purpose of this paper was to provide an example (rather than a general model) of what can happen during negotiations and to highlight the delicacy of the choice of policy. However, there is a precise sense in which the results we obtained are robust and go beyond the simple example considered.

The family of beliefs considered here — given by (10) — is just one of an infinite number of possibilities and the choice was motivated by the great analytical simplicity obtained [11]. However, catastrophe theory tells us that given any one-parameter family of beliefs — representing the evolution of beliefs over time (as in paths (1) and (2) in Figure 4) — the qualitative behaviour displayed in our example is «not unlikely» and arises in a structurally stable way [12]. Therefore a different family of beliefs and/or a different choice of parameters (for example, one could take the mean and variance as parameters) can, generically, yield the same qualitative results. Structural stability implies, in particular, that the linearity of the utility function (risk neutrality) was not a necessary condition for our results: a strictly concave per-

[11] The function $P_{b,c}(x)$ given by (12) is continuous but not smooth and, as a consequence, the function $h_{b,c}(x) = -dP_{b,c}/dx$ — given by (10) — is discontinuous. However, since smooth functions are dense in the space of continuous functions (see Hirsh, 1976, theorem 2.4, p. 47), we can choose a smooth approximation of (12) — with corresponding smooth density replacing (10) — and by catastrophe theory all sufficiently close smooth approximations of (12) would yield the same qualitative results (in particular, a Maxwell line which is close to the one we obtained).

[12] For more details on the claims made here about catastrophe theory see Bonanno-Zeeman (1988).

turbation of the utility function would yield the same qualitative results [13].

One could also generalize our example by allowing the union to become «convinced» that S lies in some intermediate range of values (e.g. between 0.5 and 0.6). That is, instead of the two-step function illustrated in Figure 2, we could have a three-step function. This would imply extending the dimension of the parameter space from two to four. Catastrophe theory then tells us that the cusp catastrophe we obtained would be «globalized» into a butterfly catastrophe, with the added possibility of discontinuous jumps to and from intermediate values of the requested pay rise (while the discontinuities analyzed above would still remain).

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[13] Not necessarily a «small» perturbation, however. For example, if in the model considered in this paper we replace (9) by $U(x)=x^{1/2}$ (so that risk-neutrality is replaced by risk-aversion), the Maxwell line becomes the line of equation $c=b^{-1/2}$ (with a vertex at $b=1$, $c=1$) and

$$x^*(b,c)=\begin{matrix} 1/(3b) & \text{if } c<b^{-1/2} \\ 1/3 & \text{if } c>b^{-1/2} \end{matrix}$$

APPENDIX

We want to show that given any two points (b,c) and (b′,c′) in the shaded area of Figure 4 such that $b \geq b'$ and $c \leq c'$ and not both equal — cf. (13) —, there exists a non-negative function $g(d,\cdot)$ such that — cf. (7) —:

$$h_{b,c}(s) = \frac{g(d,s)h_{b,c'}(s)}{\int_0^1 g(d,s)h_{b',c'}(s)ds} \tag{A.1}$$

(Furthermore, $g(d,\cdot)$ can be chosen in such a way that its image is a subset of [0,1]). Let $\mu' = (1-c')/(b'-c')$ (cf. (11)). Let

$$\phi(s) = \begin{cases} (c'\theta)/b' & 0 \leq s \leq \mu' \\ \theta & \mu' < s \leq 1 \end{cases} \tag{A.2}$$

where θ is an arbitrary positive constant. It is easy to check that:

$$\frac{\phi(s)\ h_{b,c'}(s)}{\int_0^1 \phi(s)\ h_{b,c'}(s)ds} = 1 \qquad \text{for all } s \epsilon [0,1] \tag{A.3}$$

Now let

$$g(d,s) = h_{b,c}(s)\phi(s) \tag{A.4}$$

Lemma. If $g(d,\cdot)$ is given by (A.4), the (A.1) is satisfied.

Proof.

$$\frac{g(d,s)\ h_{b,c'}(s)}{\int_0^1 g(d,s)h_{b,c'}(s)ds} = \frac{h_{b,c}(s)\dfrac{\phi(s)\ h_{b,c'}(s)}{\int_0^1 \phi(s)\ h_{b,c'}(s)ds}}{\int_0^1 h_{b,c}(s)\dfrac{\phi(s)\ h_{b,c'}(s)}{\int_0^1 \phi(s)\ h_{b,c'}(s)ds}ds} =$$

$$= (\text{by (A.3)}) = \frac{h_{b,c}(s)}{\int_0^1 h_{b,c}(s)ds} = h_{b,c}(s)$$

Therefore, given a set of data D and n+1 points (with $n+1 \leq \#D$, where $\#D$ denotes the cardinality of the set D) situated along one of the paths in the shaded area of Figure 4 (such that point t + 1 follows point t in the direction of increasing pessimism), it is possible to construct a non-negative function $g:Dx[0,1]\rightarrow R$ such that

$$\int_D g(d,s)\mathrm{d}d = 1 \quad \text{for all } s\epsilon[0,1]$$

and there are n points $d_t \epsilon D$ $(t = 1,..,n)$ such that

$$h_{t+1}(s) = \frac{g(d_t,s)\ h_t(s)}{\int_0^1 g(d_t,s)\ h_t(s)ds} \qquad (t = 1,..,n).$$

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