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Editors' note

The papers contained in this and the previous issue were presented at the workshop on *Interactive Epistemology in Dynamic Games and Games with Incomplete Information* held in Venice in June 1998, for which we gratefully acknowledge funding from *Research in Economics*, the Department of Economics of the University of Venice and the Italian Centre for Game Theory.

Interactive epistemology is the formal analysis of what different individuals involved in a situation of strategic interaction know and believe about facts concerning the external world as well as facts concerning each other's knowledge and beliefs.† One of the main analytical tools of interactive epistemology, type spaces, has been introduced by the Nobel prize winner John Harsanyi in order to analyse games with incomplete information (Harsanyi, 1967–68). Aumann (1974, 1976) used related tools, information partitions, to illustrate the scope of non-causal correlation in strategic games and to show the impossibility (assuming a common prior) of "agreeing to disagree" about the probability of an event conditional on private information. Since then game theorists have become acquainted with the literature on modal and epistemic logic (Hintikka, 1962; Chellas, 1984) first developed by mathematical philosophers and then also by computer scientists,‡ and interactive epistemology has become a common ground for the three disciplines.

In order to understand the role of interactive epistemology in economics and game theory, it is useful to make a comparison with the situation of oligopoly theory before the publication of *The Theory of Games and Economic Behavior*.§ The primitives of the theory of oligopoly were a market demand function and the technology of the oligopolistic firms. While these primitives could be described in mathematical language, the theory had no formal language to express other assumptions about market interaction such as which variables are actually under the control of each firm, what is the order of moves and the information of each firm about the competitors. Different assumptions about these aspects of the situation were implicitly embedded in competing equilibrium concepts with different behavioural implications (e.g., Cournot vs. Bertrand vs. Stackelberg equilibrium).

† The phrase "interactive epistemology" is due to Aumann (1995).
‡ See Fagin *et al*. (1995) and references therein.
§ Von Neumann and Morgenstern (1944).

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The theory of games has provided the formal language to express these assumptions and a unified equilibrium concept, (Bayesian) perfect equilibrium, which yields (most of) the equilibrium concepts of oligopoly as special cases under appropriate assumptions. As a result, the received theory has been clarified and a wider range of phenomena has been made amenable to formal analysis.

The "standard" solution concepts of game theory also rely on implicit and/or informal assumptions: specifically, assumptions about how the players form and revise their beliefs. On the one hand, it became clear that only in special (though important) cases is equilibrium the result of common knowledge of the strategic situation and common belief in rationality. On the other hand, it has been claimed that some Bayesian perfect equilibria rely on "unreasonable" beliefs. Once again, assumptions that could not be formally and explicitly expressed in a formal language have been embedded in new solution concepts. In particular, a plethora of refinements of the Bayesian perfect equilibrium concept for dynamic games has been put forward by game theorists, much to the confusion of applied economists.

The epistemic analysis of games enriches the formal language of game theory allowing to formulate rigorously and explicitly assumptions about players' knowledge, beliefs and rationality. Old and new solution concepts can be obtained by looking at the behavioural implications of these assumptions, thus clarifying and expanding the existing theory.

Each special issue contains a survey and two original papers. The first issue is relatively general in its scope, while the second is more focused on belief revision, counterfactual reasoning and interactive epistemology in dynamic games.

Michael Bacharach puts forward a new theory of co-operation: interactive team reasoning. Asymmetrically informed agents are assumed to choose sometimes as individuals and sometimes as members of teams. In the latter case they "team reason", that is, they compute and choose their component in a profile evaluated using the team objective function. In doing this they take into account that the other potential members of the team may lapse into selfish behaviour. "Unreliable team interaction" is compared to the Bayesian equilibria of related games of incomplete information.

Pierpaolo Battigalli and **Giacomo Bonanno** provide a brief, self-contained introduction to the analytical tools used in the literature and a selective survey of recent results on interactive epistemology. In particular, they focus on the characterization of the common prior assumption, rationalizability (in static and dynamic games), forward and backward induction.

Dov Samet provides an axiomatic approach to Bayesian belief change. In the standard view a Bayesian agent is one who revises her prior beliefs by conditioning on new information, that is, on new facts she has become certain of. Samet shows that Bayesianism can be characterized without resorting to the notion that the agent acquires new information: an agent is Bayesian if her prior beliefs, when conditioned on her posterior beliefs, agree with the latter.

Counterfactual reasoning is a crucial aspect of strategic decision making in dynamic games. The papers contained in the next issue deal with the formal representation of subjunctive and counterfactual conditionals (including "epistemic" conditionals of the form "If node x were reached, player i would think that …") and their use in game theoretic analysis.

The paper by **Pierpaolo Battigalli** and **Marciano Siniscalchi** studies the interplay between epistemic independence and rationalizability in extensive games. The notion of epistemic independence formalizes the idea that when a player receives information about a particular opponent she should not revise her beliefs concerning other opponents. The authors provide an epistemic characterization of a weak and a strong notion of rationalizability with independent beliefs and relate the two to the notion of backward induction in perfect information games.

The survey by **Brian Skyrms, Gary Bell** and **Peter Woodruff** summarizes and unifies different theories of counterfactual and subjunctive conditionals, such as Stalnaker's selection function, Selten and Leopold's parametric theory and Skyrms' Bayesian theory. The authors show that these seemingly different theories can be unified using the notion of a family of partitions.

Robert Stalnaker clarifies, in the context of epistemic models of games, the relationship between the strategic-form and the extensive form representation of a game. He shows that epistemic models defined for the strategic form provide all the material necessary to build a complete model of the extensive form. He also shows the equivalence, for games with perfect recall, of two definitions of rationality, one for strategy choices in the strategic-form representation and the other for the individual choices that the player is disposed to make in the course of playing the extensive game.

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