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## Endogenous Tradability and Some Macroeconomic Implications

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While nontraded goods play an important role in many open economy macroeconomic models, these models have difficulty explaining the low volatility in the relative price of nontraded goods. In contrast to macroeconomic convention, this paper argues that the share of nontraded goods is endogenous, a time-varying product of macroeconomic shocks and trade costs that are heterogeneous across goods. A simple open economy model demonstrates that trade cost heterogeneity and a time-varying margin of tradedness dramatically reduces the volatility of nontraded prices. This also reduces the ability of real exchange rate adjustments to dampen current account imbalances.

Keywords: nontraded goods, trade cost, heterogeneity, relative prices

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## 1 **1. Introduction**

2           Open economy macroeconomics has long found it useful to assume that some goods are  
3 not tradable. Positing the existence of a nontraded goods sector lies behind some classic results  
4 in the literature, such as the behavior of real exchange rates in Balassa (1964) and Samuelson  
5 (1964), and current account adjustment in Dornbusch (1983). But this assumption implies stark  
6 and counterfactual behavior for the relative price of nontraded to traded goods. The domestic  
7 prices of goods that are freely traded internationally are subject to international arbitrage and  
8 pinned to world prices, while the prices of nontradeds are free to move independently of world  
9 prices. These models imply that real exchange rate movements are attributable primarily to  
10 movements in the relative price of nontraded goods.

11           Recent empirical work has demonstrated this to be far from the truth: nontraded prices  
12 tend to move with traded prices, and only a small fraction of real exchange rate movements are  
13 due to movement in the relative price of nontraded to traded goods. This paper argues that this  
14 counterfactual implication in the macro literature is the result of viewing tradedness as an  
15 exogenous characteristic of a good, and it can be resolved by recognizing that the status of a  
16 good as nontraded is an endogenous response to explicit trade costs that vary heterogeneously  
17 across goods.

18           Betts and Kehoe (2006) document the empirical puzzle, studying the properties of the  
19 real exchange rate and the relative price of nontraded goods between the U.S. and five trading  
20 partners with annual data for the period 1980-2000. Their preferred measure computes the  
21 relative price of nontraded goods as the ratio of the gross output deflator for manufacturing,  
22 agriculture, and mining to the gross output deflator for all sectors. For the case of Canada, the  
23 standard deviation of nontraded relative prices to that of the overall real exchange rate varies  
24 from 0.40 to 0.47, depending on the method of detrending. For other countries the ratio is even

1 lower: varying from 0.13 to 0.20 for Germany, all at 0.12 for Japan, 0.18 to 0.23 for Korea, and  
2 0.25 to 0.38 for Mexico. Earlier work by Engel (1999) suggests the relative volatility of  
3 nontraded prices can be yet lower, below 0.10. As an additional stylized fact, Betts and Kehoe  
4 (2006) note that the volatility ratio is systematically related to the strength of the trading  
5 relationship: those countries that trade more with the U.S. have more volatility in nontraded  
6 prices.

7 This paper argues that the difficulty in explaining the low volatility in the relative price of  
8 nontraded goods may stem from a rigid and artificial dichotomy, where some goods exogenously  
9 are labeled as tradable and others as nontradable. Recent advances in trade theory provide insight  
10 regarding the endogenous nature of tradedness. Consider all goods as parts of a single  
11 continuum, where the iceberg costs of trade vary by good. Whether a good is tradable or not  
12 depends on whether the costs of trading that good make trade profitable or not. On the margin  
13 there is a good whose seller is indifferent between selling his good domestically only, or  
14 branching out into the international market. As a result, this marginal nontraded good forms a  
15 link between the prices of goods that are traded and other similar goods that are nontraded.

16 To explain this point and illustrate its usefulness in a transparent manner, this paper  
17 builds on the two-period small open economy model with trade costs used in Obstfeld and  
18 Rogoff (2000). When the small open economy is subjected to demand shocks, the degree to  
19 which the relative price of nontraded goods moves depends on the degree of heterogeneity of the  
20 trade cost distribution. A calibration exercise shows that this model generates the low volatility  
21 in nontraded prices found in the empirical literature.

22 While our model endogenizes the tradedness of goods by building upon recent  
23 developments in the trade literature, it differs in focusing upon heterogeneity in trade costs  
24 among goods rather than upon heterogeneity in productivity among firms. In contrast to the

1 latter convention, we think that when the issue of primary interest is tradedness, it makes more  
2 sense to focus on the variation of trade costs among goods. For example, the reason that services  
3 comprise a disproportionate share of nontradeds is because most services are particularly costly  
4 to trade across borders. Further, the usual convention does not account for the empirical  
5 observations that there is a great deal of heterogeneity among goods in terms of their deviations  
6 from the law of one price across countries, nor that these deviations systematically tend to be  
7 greater for nontraded goods than for traded goods (Crucini, Telmer, and Zachariadis, 2005).

8 This paper is also related to recent research in Ghironi and Melitz (2005), Bergin, Glick  
9 and Taylor (2006), and Naknoi (2008) which incorporate trade features in a macro model with a  
10 continuum of heterogeneous firms. One significant difference is that these papers follow the trade  
11 literature in specifying heterogeneity in terms of firm productivity. Betts and Kehoe (2001) allow  
12 heterogeneous trade costs, but we differ in that we have a share of goods that are fully nontraded,  
13 which is what allows us to examine shifts of the nontraded extensive margin.

14

## 15 **2. Empirical Motivation**

16 Because the argument in this paper relies upon two new ideas, a time-varying share of  
17 nontraded goods and heterogeneity in trade costs over goods, this section provides some  
18 empirical support for these features. Canada's trade with the United States is chosen as a case  
19 that generally suits the small open economy assumption. First, consider movements in the share  
20 of nontraded goods. The NBER-UN trade data base reports the value of U.S. imports from  
21 Canada in the harmonized system 10 digit industries (HS-10).<sup>1</sup> These data reveal that there are  
22 many goods categories with zero trade in any given year, and that there is significant variation  
23 over time in the total number of goods traded. Fig. 1 plots the number of HS-10 goods

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<sup>1</sup> This analysis parallels that in Bergin et al. (2008) for Mexican trade with the U.S. associated with offshoring.

1 classifications traded at some point during a given year, starting from 1989 when the CUSFTA  
2 trade agreement was implemented between Canada and the United States. Note that there is  
3 significant variation in the number of traded goods both upward and downward over time, with a  
4 standard deviation of 3.4% in the HP-filtered annual data. Further, it appears that this variation is  
5 related to business cycle movements, as low points coincide with economic downturns in Canada  
6 or the U.S. in the early 1990s and 2000s. This provides supportive evidence that the shares of  
7 traded and nontraded goods are time-varying and should be treated as endogenous.

8         Next, consider heterogeneity in trade costs among goods. There is some appreciation of  
9 this point in the existing trade literature. Empirical work by Hummels (2001a, 2001b) describes  
10 how freight costs alone can range from more than 30 percent of value for raw materials and  
11 mineral fuels down to 4 percent for some manufactures. Depending on factors such as weight,  
12 distance, and the time sensitivity of demand, trade costs can be high and variable for many  
13 manufactured goods as well. Bernard et al. (2006), reports a detailed measure of trade costs  
14 including tariff rates for 20 U.S. industries at the 2-digit level. They observe that there is  
15 significant variation in trade costs, ranging from 4.9% to 23.2%.

16         In their review of the literature on trade costs, Anderson and van Wincoop (2004) note  
17 that the most common method of measuring trade costs in macroeconomics is in terms of their  
18 implications for prices, that is, deviations from the law of one price across countries. Here we  
19 can provide some original evidence well suited for the theoretical analysis to follow. Denote by  
20  $\tau_i$  the iceberg trade cost for a good  $i$ , which is the fraction of a good that disappears in  
21 transshipment. If this good is exported by the home country to a foreign market, then arbitrage  
22 implies that the wedge between home and foreign price (the latter indicated by a \*) will be  
23  $p_i/p_i^* = 1 - \tau_i$ . Price wedges are an appropriate measure of trade costs here, first, because it is  
24 iceberg trade costs that are the type of trade cost that the theoretical model will deal with, and

1 second, because it is ultimately puzzles involving such international relative prices that the  
2 model is trying to explain.

3         The Economist Intelligence Unit collects retail price data on a range of consumer goods  
4 in cities around the world, such as 100-count aspirin bottles and a pair of 60 watt light bulbs. The  
5 analysis will focus on U.S. and Canada in 2007, and on the 101 goods classified as traded by  
6 Engel and Rogers (2004) when they used this data set. Average relative prices wedges will be  
7 computed for each good by averaging over the 64 Canadian-U.S. city pairs with available data.  
8 One immediate conclusion is that there is a great deal of heterogeneity in price dispersion:  
9 relative prices range from 62.4% cheaper in Canada to 62.2% more expensive, and the cross-  
10 sectional standard deviation of the price-wedge observations is 39%. Interestingly, the average  
11 level of price dispersion is 0.1%, with about as many goods overpriced as underpriced. Fig. 2  
12 plots the values of price wedges for the half of the sample with values less than unity, where  
13 goods are cheaper in Canada, ranked in order of decreasing price wedge. (A symmetric set of  
14 goods are greater than unity). Price wedges in principle can also reflect other differences  
15 between countries such as exchange rate fluctuations and Balassa-Samuelson effects on the real  
16 exchange rate. But the fact that these factors should affect all goods suggests that they can in part  
17 be removed by demeaning the price wedges by their cross-good average. The fact that this  
18 average value is so near zero in the Canada-U.S. 2007 sample suggests it is not a large issue in  
19 this particular sample.

20         Fig. 2 suggests that the distribution of trade costs across goods is fairly uniform, with a  
21 steady progression of rising trade costs as one moves from right to left. This is very different  
22 from the distribution of productivity heterogeneity across firms, which has been the focus in  
23 trade literature. The standard assumption in recent trade models is that productivity heterogeneity  
24 follows a Pareto distribution, arguing that this fits the highly skewed distribution of firm size,

1 where there is a large number of small firms and a very small number of very large firms. The  
2 model developed below will propose a distribution better suited to capturing the particular  
3 features of trade cost heterogeneity. For corroboration, Fig. 3 plots the trade costs reported in  
4 Bernard et al. (2006) for their sample of 20 2-digit industries. Their numbers for  $\tau_i$  are plotted in  
5 the form of  $1 - \tau_i$  to be comparable to Fig. 2. Again the distribution of trade costs appears quite  
6 linear.

7

### 8 **3. Model and Analytical Results**

9 To facilitate tractability and transparency, the paper follows Obstfeld and Rogoff (2000) in  
10 studying a simple small open endowment economy.<sup>2</sup> The discussion begins with a static version,  
11 helpful in understanding the determination of tradedness. It then extends the model to a second  
12 period, to study dynamics in relative prices.

13

#### 14 **3.1 Static Model**

15 The country is endowed with a continuum of goods indexed by  $i$  on the unit interval, where  $y_i$   
16 represents the level of endowment,  $c_i$  is the level of consumption, and  $p_i$  is the domestic price  
17 level of this good. All of these home goods have the potential of being exported, but some  
18 endogenously determined fraction of the goods,  $n$ , will be nontraded in equilibrium. For each  
19 traded home good there is a prevailing world price  $p_i^*$  that may differ from the home price  
20 because of trade costs. The small open economy may also import foreign goods for consumption  
21 purposes, with consumption level  $c_F$  and price level  $p_F$ . For simplicity, assume that the

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<sup>2</sup> Bergin and Glick (2003, revised 2007) demonstrate that the results are robust to including production in the model.

1 endowments and world price levels of all home goods are uniform, implying  $y_i = y$ ,  $p_i^* = p^*$   
 2 for all  $i$ .

3 The aggregate consumption index is specified as:

$$4 \quad c = \frac{c_H^\theta c_F^{1-\theta}}{\theta^\theta (1-\theta)^{1-\theta}}. \quad (1)$$

5 Here  $c_H$  is an index of home goods consumption:

$$6 \quad c_H^{(\phi-1)/\phi} = \int_0^n (c_i)^{(\phi-1)/\phi} di + \int_n^1 (c_i)^{(\phi-1)/\phi} di = n \left( \frac{c_N}{n} \right)^{(\phi-1)/\phi} + (1-n) \left( \frac{c_T}{1-n} \right)^{(\phi-1)/\phi} \quad (2)$$

$$7 \quad \text{where } c_N \equiv n \left( \frac{1}{n} \int_0^n c_i^{(\phi-1)/\phi} di \right)^{\phi/(\phi-1)} \quad \text{and } c_T \equiv (1-n) \left( \frac{1}{1-n} \int_n^1 c_i^{(\phi-1)/\phi} di \right)^{\phi/(\phi-1)}$$

8 are consumption indexes of nontraded and traded goods, respectively, and  $n$  is the share of goods  
 9 on the continuum  $\{0,1\}$  that are nontraded. Price indexes are defined as usual for each category  
 10 of goods, in correspondence to the consumption indexes above:

$$11 \quad p = p_H^\theta p_F^{1-\theta} \quad (3)$$

$$12 \quad p_H^{1-\phi} = \int_0^n (p_i)^{1-\phi} di + \int_n^1 (p_i)^{1-\phi} di = n p_N^{1-\phi} + (1-n) p_T^{1-\phi} \quad (4)$$

13 where  $p$  is the aggregate price level,  $p_H$  is the price index of all home goods, and the price  
 14 index of home nontraded goods  $p_N$  and the price index of home traded goods  $p_T$  are defined as

$$15 \quad p_N \equiv \left( \frac{1}{n} \int_0^n p_i^{1-\phi} di \right)^{1/(1-\phi)} \quad \text{and} \quad p_T \equiv \left( \frac{1}{1-n} \int_n^1 p_i^{(1-\phi)/\phi} di \right)^{1/(1-\phi)}.$$

16 Note that if world prices are normalized to unity, i.e.  $p^* = 1$ ,  $p_F = 1$ , then  $p$  may be interpreted  
 17 as the reciprocal of the real exchange rate for this small open economy.

18 The home goods are distinguished from each other by heterogeneous iceberg costs ( $\tau_i$ ),  
 19 where a certain fraction of the good disappears in transport. As discussed above, arbitrage



1 requires that the domestic price will be  $p_i = p^*(1 - \tau_i)$  if the country exports good  $i$ . These trade  
 2 costs are defined to follow the specification:  $\tau_i = 1 - (i^\beta / \alpha)$ ;  $\alpha \geq 1, \beta \geq 0, i \in [0, 1]$ , which  
 3 implies the following distribution of export prices

$$4 \quad \frac{p_i}{p_i^*} = \frac{i^\beta}{\alpha}. \quad (5)$$

5 This implies that the trade cost and price ratio vary between 0 and 1 (for  $\alpha = 1$ ) as the goods  
 6 index varies over the unit interval. The parameter  $\beta$  controls the curvature of the distribution,  
 7 while  $\alpha$  scales the level. This specification is easy to integrate over, as is the Pareto distribution  
 8 commonly used in the trade literature to characterize productivity heterogeneity. But the present  
 9 specification is better suited to the case of trade costs in two respects. First, the support for  
 10 iceberg trade costs needs to be the interval from 0 to 1, whereas that for a Pareto distribution is  
 11 from some positive lower bound (usually taken to be unity) to infinity. While such an assumption  
 12 is well suited for a firm's productivity level, it is not well suited for a fractional trade cost.  
 13 Second, a Pareto distribution famously implies a high degree of concentration of firms near the  
 14 lower bound, whereas Figs. 2 and 3 suggested a more uniform distribution of trade costs over  
 15 goods.<sup>3</sup> This need not be the case for our specification, depending on the choice of curvature  
 16 parameter  $\beta$ .

17 In the endowment economy the decision of whether to export a good is determined solely  
 18 on the basis of whether the export price (i.e. the world price) less iceberg costs, exceeds the  
 19 domestic price. If the export price is higher, then the good is exported, if it is lower, then it is not  
 20 traded.

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<sup>3</sup> As discussed in Ghironi and Melitz (2005), the curvature parameter in the Pareto distribution needs to be higher than the elasticity of substitution; this restriction does not apply here.

1           Given the cutoff between traded and nontraded goods at index  $n$ , it is straightforward to  
 2 compute the price index for traded goods from the price distribution of exported varieties:

$$3 \quad p_T = \left( \left( \frac{1}{1-n} \right) \left[ \int_n^1 \left( \frac{p^* i^\beta}{\alpha} \right)^{1-\phi} di \right] \right)^{1/(1-\phi)} = \left( \frac{p^*}{\alpha} \right) \left( \left( \frac{1}{1-n} \right) \left( \frac{1}{\omega} \right) \left\{ \left( \frac{1}{n} \right)^\omega - 1 \right\} \right)^{1/(1-\phi)} \quad (6)$$

4 where  $\omega \equiv \beta(\phi - 1) - 1$ . Equation (6) expresses the price of traded goods as a function of the  
 5 share of nontraded goods  $n$ , the elasticity of substitution across domestic goods  $\phi$ , and the trade  
 6 cost parameters,  $\beta$  and  $\alpha$ . It is straightforward to establish that  $\partial p_T / \partial n > 0$ ; i.e. the price of  
 7 traded goods increases with the share of nontraded goods. The reason is that, as the proportion  
 8 of home goods that are nontraded rises, it is no longer profitable to export goods with marginally  
 9 higher trade costs; as these goods are withdrawn from export markets, the average price of the  
 10 remaining export goods rises.<sup>4</sup>

11           The price index of nontraded goods is even easier to determine. As usual, intratemporal  
 12 optimization implies relative demands for each pair of home goods  $i$  and  $j$ :  $c_i / c_j = (p_i / p_j)^{-\phi}$ .  
 13 Since consumption must equal the endowment of nontraded goods, and endowments are uniform  
 14 for all goods here (i.e.  $y_i = y$  for all  $i$ ), then for any pair of nontraded goods it will be true that  
 15  $c_i / c_j = y_i / y_j = 1$ , and so  $p_i / p_j = 1$ . In other words, the price of each nontraded good will be  
 16 identical, because they each are by definition not affected by the trade costs which vary by good.  
 17 This logic applies equally well to the home good that is just on the margin between being traded  
 18 and nontraded ( $i=n$ ). The marginal trader decides to export solely on the basis of whether the  
 19 world price less iceberg costs exceeds the domestic price. But because this good is on the margin

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<sup>4</sup> This conclusion is robust to the particular definition of the price index. If a naïve statistician did not know the set of traded goods had changed, but collected price data on all goods that previously had been traded, this average price level would still rise. However, the reason would be that the average includes newly nontraded goods, whose individual prices have risen, rather than the fact that an average is being taken over a subset of goods where the lower price items have been removed.

1 of being traded, the domestic price must be the same as that as if it were sold in the world  
 2 market:  $p_n = (p^*/\alpha)n^\beta$ . As a result, the price index of nontraded goods is pinned down as the  
 3 price of the marginal traded good by the following marginal tradability condition:

$$4 \quad p_N \equiv \left( \left( \frac{1}{n} \right) \left[ \int_0^n (p_i)^{1-\phi} di \right] \right)^{1/(1-\phi)} = \left( \left( \frac{1}{n} \right) \left[ \int_0^n (p_n)^{1-\phi} di \right] \right)^{1/(1-\phi)} = p_n = \left( \frac{p^*}{\alpha} \right) n^\beta. \quad (7)$$

5 This implies that the price of nontraded goods rises with the share of nontraded goods with  
 6 elasticity  $\beta$ .

7 The tradability condition (7) provides intuition for why the model will imply a low  
 8 degree of volatility in the relative price of nontraded to traded goods. The price indexes of  
 9 nontraded and traded goods are linked together through this condition, and therefore tend to  
 10 move together. In particular, the condition states that the nontraded price index equals the price  
 11 of the marginal traded good, and in turn, the price of this marginal traded good is linked to all  
 12 other traded goods in the traded price index by the distribution of trade costs that determine all  
 13 traded prices. In a standard small open economy model a shock raises the price of nontraded  
 14 goods dramatically without any change in the price of tradeds which are pinned at world prices.  
 15 In our model the movement in nontraded prices is dampened by the linkage to traded prices. And  
 16 the prices of traded goods differ from the world pricy by varying amounts depending on the size  
 17 of heterogeneous trade costs. As the set of traded goods changes, the set of trade costs that enter  
 18 the price index changes.

19 As additional equilibrium conditions, intratemporal optimization implies the demand  
 20 functions:

$$21 \quad c_N = n \left( \frac{p_N}{p_H} \right)^{-\phi} c_H, \quad c_T = (1-n) \left( \frac{p_T}{p_H} \right)^{-\phi} c_H, \quad c_H = \theta \left( \frac{p_H}{p} \right)^{-1} c, \quad \text{and } c_F = (1-\theta) \left( \frac{p_F}{p} \right)^{-1} c. \quad (8-11)$$

1 It is assumed that residents of the small open economy must pay the cost of transport for imports  
 2 of foreign goods. The price of imported foreign goods is normalized to unity in the world market,  
 3 so its domestic price is set exogenously as  $p_F = 1 / (1 - \tau_F) \equiv \alpha_F$  for some given  $\tau_F$  representing  
 4 iceberg trade costs for imported goods.

5 Market clearing for nontraded goods requires

$$6 \quad p_N c_N = \int_0^n p_i y_i di \text{ or } c_N = ny \quad (12)$$

7 since (7) implies  $p_N = p_i$  for all  $i \leq n$  with uniform endowments  $y_i = y$  for all  $i$ . The static  
 8 model is closed by assuming balanced trade:

$$9 \quad p_H y_H - pc = 0. \quad (13)$$

10

### 11 **3.2 Implications for the Share of Nontraded Goods**

12 Viewing tradedness as endogenous offers some new insights into what drives the degree  
 13 of openness of a country's goods markets. The equilibrium conditions above can be solved  
 14 together to yield the following expression for the equilibrium trade balance (surplus)  $Z$ :

$$15 \quad Z \equiv \frac{1 + n^{\beta+1} \beta}{1 + \beta} - \frac{1}{\omega \theta} [n^{\beta+1} (\omega + 1) - n^{\beta \phi}] = 0. \quad (14)$$

16 See the supplementary material in the appendix for derivation of this condition and the proofs of  
 17 the conclusions that follow in this section. The trade balance  $Z$  falls as  $n$  increases. Intuitively,  
 18 increasing  $n$  implies trade in fewer varieties of goods and lowers the trade surplus. Condition  
 19 (14) implies that the balanced trade condition determines the steady-state share of nontraded  
 20 goods,  $\bar{n}$ . It is easily verified that this solution is the unique solution that lies within the  
 21 permissible range of zero to one. It is clear that if  $n$  were 0 and all goods were traded, then the  
 22 trade balance is positive. For some  $n > 0$ , the trade balance will fall to zero.

1           Condition (14) provides a number of new insights into factors that determine the  
2 endogenous share of nontraded goods. One such factor is the curvature in the distribution of  
3 trade costs,  $\beta$ . Implicit differentiation of (14) indicates that  $\partial \bar{n} / \partial \beta > 0$ , as shown in the  
4 appendix. The nontraded share rises as the curvature of the trade cost distribution rises.  
5 Intuitively, if trade costs rise very quickly as more classes of goods are exported, it is optimal to  
6 export a smaller number of classes of goods. A country should then concentrate its exports in  
7 those commodities for which international trade is so much less costly.<sup>5</sup>

8           Another determinant of tradedness is the elasticity of substitution between home goods,  
9  $\phi$ . Implicit differentiation of (14) indicates that  $\partial \bar{n} / \partial \phi > 0$ , as shown in the appendix. The  
10 intuition is that if home goods are highly substitutable in consumption, one can conserve on trade  
11 costs by concentrating one's exports in the goods that are easiest to trade. This means there will  
12 be a smaller quantity of these particular classes of goods to consume, but under a high elasticity,  
13 it is easy to compensate by consuming a greater quantity of other types of goods.

14

### 15 **3.3. Two-Period Model**

16           To study the dynamics of relative prices, the goods market described above will be  
17 analyzed in the context of a two-period model with a representative consumer. Variable time  
18 periods will be indicated by subscript. The equilibrium conditions developed above apply in both  
19 periods 1 and 2. In addition, the consumer maximizes two-period utility

$$20 \quad \delta U(c_1) + U(c_2),$$

21 subject to the intertemporal budget constraint

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<sup>5</sup> The level parameter of trade costs ( $\alpha$ ) does not appear under the Cobb-Douglas preferences assumed. It does appear if preferences are generalized to a CES case, as shown in Bergin and Glick (2003, revised 2007).

$$\left( \frac{p_{H2}}{p_2} y_{H2} - c_2 \right) = - \left( \frac{p_1}{p_2} \right) (1+r) \left( \frac{p_{H1}}{p_1} y_{H1} - c_1 \right). \quad (15)$$

Here  $r$  is the world interest rate on debt in world currency units. The term  $\delta$  is an exogenous discount factor that can change, thereby allowing us to consider shifts in demand from one period to the next. Intertemporal optimization implies the usual intertemporal Euler equation:

$$U'_{c1} = \frac{1}{\delta} \left( \frac{p_1}{p_2} \right) (1+r) U'_{c2}. \quad (16)$$

Equilibrium here determines values each period for the variables  $c_t, c_{Ht}, c_{Tt}, c_{Nt}, c_{Ft}, p_t, p_{Ht}, p_{Tt}, p_{Nt}, n_t$ , satisfying equations (3-4, 6-12) for each period as well as the intertemporal budget constraint (15) and the intertemporal consumption Euler equation (16); see the appendix. This is the model simulated in the following section. Note that the static model specified in section 3.1 represents the steady state of the dynamic model, defined when the disturbance  $\delta$  is set to zero, so that consumption and all other variables are constant across the two periods. According to the intertemporal budget constraint, the value of domestic production equals the value of domestic consumption in this steady state, and the trade balance is zero:  $p_{H1}y_{H1} - p_1c_1 = p_{H2}y_{H2} - p_2c_2 = 0$ , as was assumed in the specification of the static model.

#### 4. Calibration Exercise

Because the two-period model cannot be solved analytically, a calibration experiment is used to study its implications for relative price movements.

##### 4.1 Calibration

The model is calibrated to Canadian data, as representative of the small open economy case. First, to calibrate the distribution of trade costs, distribution (5) is fit to the US-Canadian price

1 data used to create Fig. 2. In particular, the price wedges are ordered for the goods between a  
2 Canadian and U.S. city pair in 2007, and the log of this is regressed on the log of the good index.  
3 Because the goods are all traded goods, the index is rescaled to run from  $\bar{n}$  to 1 rather than from  
4 0 to 1. A value for  $\bar{n}$  is computed by collecting data on Canadian GDP in the categories of  
5 manufacturing, mining, and agriculture; as a share of overall GDP this averages 0.24 in annual  
6 data over 1981-2000, the period for which data was available. This implies a share for  
7 nontradeds of 0.76. Regressing the log of the price gap on the log of the index for each of the  
8 goods adjusted by  $\bar{n}$ , the average estimate of  $\beta$  over the goods is 3.1. Foreign trade costs are  
9 calibrated at  $\tau_F = 0.1$ , following Obstfeld and Rogoff (2000). We retain the normalization  
10 that  $p^*/\alpha = 1$ .

11 The home bias preference parameter,  $\theta$ , is calibrated at 0.73 as the share of domestic  
12 goods in the consumption bundle of Canada in 2007. The standard calibration in  
13 macroeconomic models for our parameter  $\phi$ , the elasticity of substitution between home goods is  
14 6 (see Rotemberg and Woodford 1992, as well as Ghironi and Melitz, 2005).

15 We employ the usual assumption that the steady state value of the exogenous discounting  
16 factor  $\delta$  equals the reciprocal of the gross world interest rate  $(1+r)$ , which then cancel out each  
17 other. The calibration experiment will take 1000 independent random draws for  $\delta$  and feed them  
18 into the two-period model. Standard deviations are computed over the logged values of variables  
19 in period 1 of the model. Shocks to  $\delta$  are log normally distributed, and calibrated to imply the  
20 consumption level has a standard deviations of 0.95%, which is the standard deviation in annual  
21 Canadian real consumption data during 1980-2000.

22

## 23 4.2. Implications for Nontraded Prices

1 Simulation results for the benchmark calibration are reported in the first row of Table 1.  
2 Of key interest is the volatility ratio for relative nontraded prices, reported in column 4. The  
3 benchmark calibration, based on Canadian data, generates a price volatility ratio of 0.38. This  
4 compares well with the range of values estimated by Betts and Kehoe (2006) for Canada, ranging  
5 from 0.40 to 0.48, and shows that our model can succeed in generating low degrees of volatility  
6 in the relative price of nontraded goods.<sup>6</sup>

7 In contrast, column (8) of Table 1 shows that a low price volatility ratio is not possible in  
8 a small open economy model where the share of nontraded goods is given as exogenous. The  
9 model is identical to the one reported in the earlier columns, except that the marginal tradability  
10 condition (eqn. 7) is dropped. To maintain comparability with the earlier columns of the table the  
11 exogenous value of the nontraded share,  $n$ , is set at the level of  $\bar{n}$  found for the corresponding  
12 endogenous nontraded model reported in the preceding columns. The price volatility ratio in this  
13 case rises to 1.46. It is easy to demonstrate that the ratio of volatilities reported in column (8)  
14 must always be greater than unity when  $n$  is exogenous. Since the aggregate price level  $p$  is a  
15 weighted average of nontraded prices ( $p_N$ ), traded home goods prices ( $p_T$ ), and import prices  
16 ( $p_F$ ), where the latter two are fixed by the integrated world market at world levels, the  
17 percentage movement in the first component must always be larger than the movement in the  
18 overall average that it induces.

19 This explains why classic small open economy models of nontraded goods are incapable  
20 of reproducing the low volatility in relative nontraded prices. Intuitively, when nontraded goods  
21 are exogenously determined, a rise in home demand requires a rise in the relative price of

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<sup>6</sup> The traded goods included in the aggregate price index include only home traded goods and exclude imported foreign goods. This is in part a matter of technical necessity: the model is designed to avoid an a priori demarcation between different types of home goods, so there is no clear way to define a price index combining imported foreign goods together with a subset of goods in the home goods CES index, while excluding other goods in this CES index. Fortunately, the stylized fact which the model is trying to replicate is defined in precisely the same manner.



1 nontraded goods, to convince households to take their extra consumption in the form of  
2 additional imports of tradable goods, given that the consumption of nontraded goods is limited to  
3 the domestic supply of such goods. But when nontraded goods are endogenously determined,  
4 some traded goods sellers on the margin will respond to the rising price of nontraded goods and  
5 find it profitable to sell more in the home market, to the point of abandoning attempts to market  
6 their good abroad where they need to deal with costs of trade. This endogenous rise in the share  
7 of nontraded goods allows the supply of nontraded goods to increase, despite the fact that the  
8 endowment of each individual good is fixed. This increase in supply reduces the pressure for the  
9 relative price of nontradeds to rise in the face of the higher demand.

10         The mechanism described above does not require an implausibly high degree of  
11 movement in the endogenous nontraded share. As reported in column (5) of the table, the  
12 percentage change in the share of traded goods is a modest 1.7%. It was noted previously that  
13 the standard deviation in the number of traded goods for Canadian data plotted in Fig. 1 was  
14 3.4%. So the model's volatility in the margin of tradability is easily justifiable, and even modest,  
15 by the standards of Canadian data.

16         The remaining rows of Table 1 report sensitivity to alternative calibrations. First, higher  
17 values for the curvature parameter  $\beta$  reduce the relative volatility ratio. From the marginal  
18 tradability condition (equation 7), it is clear that  $\beta$  reflects the elasticity of the nontraded price  
19 index with respect to changes in  $n$ . It is at high values of  $\beta$  where the demand shock induces a  
20 small change in  $n$  and a large change in the price of nontraded goods. But this also requires a  
21 larger change in the price index of traded goods, so the overall price index changes more.  
22 Second, a higher elasticity of substitution between home goods  $\phi$  also lowers the relative price  
23 volatility ratio. Intuitively, if the last nontraded good and the marginal traded good are highly

1 substitutable, this makes the link between their two prices stronger. This in turn strengthens the  
2 linkages between the price indexes of traded and nontraded goods. The table shows that for  
3 sufficiently high values of  $\beta$  or  $\phi$ , it is possible to explain very low degrees of volatility in the  
4 price of nontraded goods, similar to those described by Betts and Kehoe (2006) for non-  
5 Canadian countries in their sample. Our estimates for  $\beta$  for Japan, Korea, and Germany,  
6 applying the same methodology as for Canada, are higher than those for Canada, ranging from  
7 3.8 to 4.2. Higher estimates of  $\phi$  at 10 are suggested by empirical work in Basu (1996).

8         Lastly, higher values of  $\beta$  and  $\phi$  imply a higher steady state share of nontraded goods,  
9 along with the lower relative price volatility. Taking the nontraded share as a measure of a  
10 country's lack of openness to trade, we can conclude that our model offers an explanation for the  
11 finding in Betts and Kehoe (2006) that volatility in the relative price of nontradeds is positively  
12 related to the degree of trade between countries.

13

#### 14 **4.3. Implications for Real Exchange Rates and Current Account Adjustment**

15         Finally, we follow Obstfeld and Rogoff (2000) in using our two-period model to study  
16 how trade costs affect current account dynamics. The current account in period 1 equals the  
17 constant endowment less consumption, so a rise in the consumption ratio  $c_1/c_2$  indicates a rising  
18 current account deficit. The Euler equation (16) indicates that the consumption based real interest  
19 rate used by agents to decide intertemporal consumption allocations is  $(p_1/p_2)(1+r)$ , as the  
20 world interest rate  $r$  needs to be converted to domestic consumption units. Obstfeld and Rogoff  
21 demonstrated that a progressively greater rise in current consumption induces a temporary rise in  
22 the real exchange rate, which raises the cost of borrowing abroad to finance current consumption.  
23 We can simulate a range of shocks to  $\delta$  in our model, and Fig. 4 plots how the log of the

1 intertemporal price,  $p_1/p_2$ , rises with the log of  $c_1/c_2$ . The solid line represents the benchmark  
2 model with endogenous tradability and the dashed line the exogenous nontraded case defined  
3 above. The exogenous share of nontraded goods for this case is calibrated to equal the share of  
4 the endogenous model in steady state.

5 One conclusion is that the intertemporal price rises smoothly with progressively rising  
6 consumption in both cases. This contrasts with Obstfeld and Rogoff, where there is a discrete  
7 jump in  $p_1/p_2$  when the single home good in their model switches from being exported to  
8 nontraded. Our result indicates that there is no nonlinear cost that switches on to strongly  
9 discourage particularly large current account deficits.

10 A second conclusion is that the intertemporal price rises less steeply when tradability is  
11 endogenous rather than exogenous. When consumption rises in period 1 and falls in period 2, the  
12 share of nontraded goods rises in period 1 to free up more domestic goods for home  
13 consumption, and the share of nontraded goods falls in period 2 as the country needs to export  
14 more goods to repay its debt. In each case, the endogenous movement in the quantity of  
15 nontraded goods insulates the price of nontraded goods and thereby the real exchange rates. This  
16 is a further reason that real exchange rate adjustments are dampened by endogenous tradability,  
17 and respond less to discourage current account imbalances.

18

## 19 **5. Conclusions**

20 This paper models tradability as endogenous response to heterogeneous good-specific  
21 trading costs. The model offers an explanation for a prominent puzzle in the empirical literature:  
22 the relative price of nontraded goods tends to move with much less volatility than the real  
23 exchange rate. This fact stands in contrast to standard theoretical models such as Balassa-

1 Samuelson, which rely almost entirely on such relative price movements. Endogenous tradability  
2 also is found to limit the ability of real exchange rates to dampen current account fluctuations.  
3 The mechanism developed here is sufficiently simple that it has the potential for being applied to  
4 a wide variety of macro models to analyze a range of macroeconomic issues.

5

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- 14

1

**Table 1: Simulation Results**

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	Endogenous $n$						Exogenous $n^1$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
$\beta$	$\phi$	$\bar{n}$	$\frac{sdev(p_N/p_T)}{sdev(1/p)}$	$sdev(1-n)$	$sdev(p_N)$	$sdev(p_T)$	$\frac{sdev(p_N/p_T)}{sdev(1/p)}$
3.1	6	0.772	0.381	0.0169	0.0155	0.0115	1.460
0.1	6	0.275	2.536	0.0090	0.0024	0.0008	3.842
1	6	0.660	0.661	0.0147	0.0076	0.0045	1.645
10	6	0.858	0.197	0.0154	0.0254	0.0218	1.397
3.1	2	0.720	0.797	0.0130	0.0157	0.0083	1.694
3.1	10	0.783	0.233	0.0178	0.0153	0.0128	1.418
10	10	0.860	0.110	0.0150	0.0244	0.0224	1.385

15 Standard deviations are of logged variables.

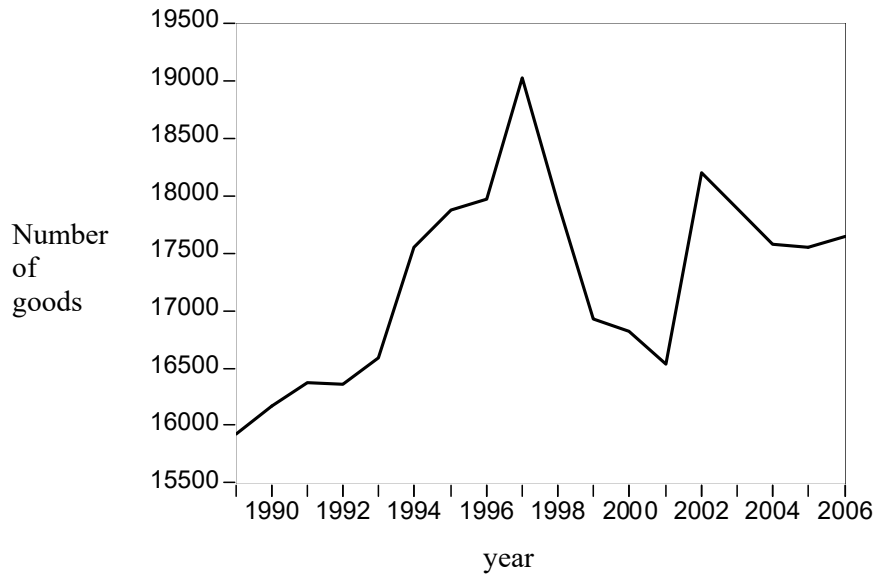
16 Benchmark parameter values:  $p^*/\alpha = 1, \theta = 0.73, \tau_F = 0.1$ .17 <sup>1</sup>Computed for the corresponding level of  $\bar{n}$ , to facilitate comparison with the endogenous  $n$  case.

18

19 Simulations of 1000 draws of the intertemporal preference  $\delta$ , under alternative assumptions for the trade  
 20 cost distribution parameter,  $\beta$ , and the elasticity of substitution between goods,  $\phi$ .  $p_N$  is the price of  
 21 nontraded goods,  $p_T$  is the price of traded goods, and  $p$  is the consumer price index, the reciprocal of  
 22 which here equals the real exchange rate;  $n$  is the share of nontraded goods.

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Fig. 1. Number of Canadian Goods Exported to the United States

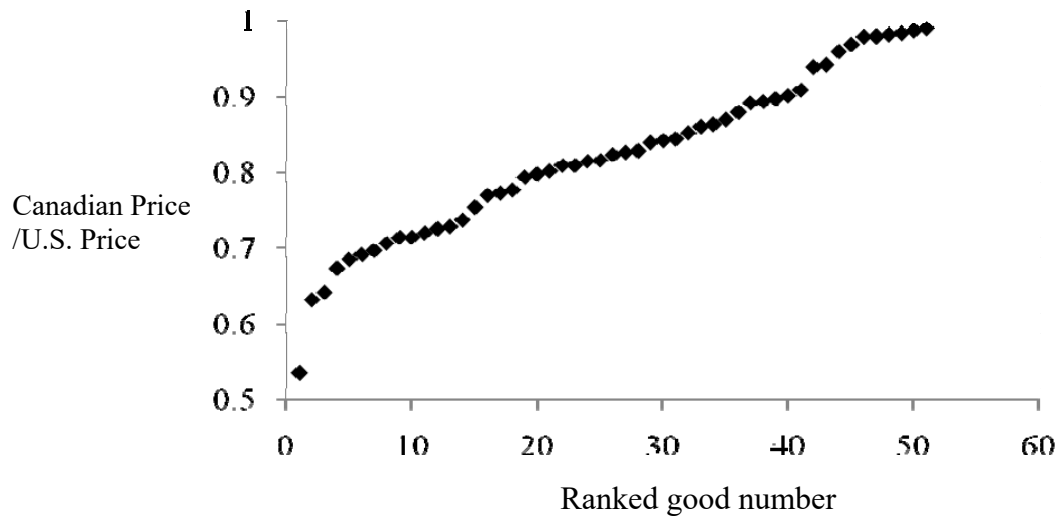


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The figure reports the number of HS 10-digit goods exported from Canada to the United States each year during 1989-2007.  
Source: NBER-UN trade data base

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Fig. 2. Price Wedges for Canada-U.S. goods



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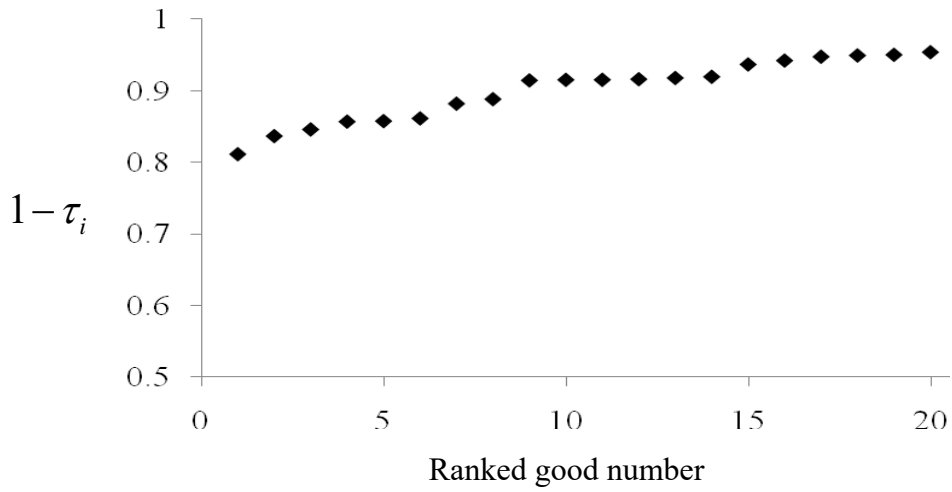
Figure shows the ratio of the Canadian price to the U.S. price for 50 selected traded goods in 2007. They are ordered in increasing order of the price ratio.

Source: Economist Intelligence Unit



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Fig. 3. Distribution of Trade Costs for 2-digit U.S. Industries

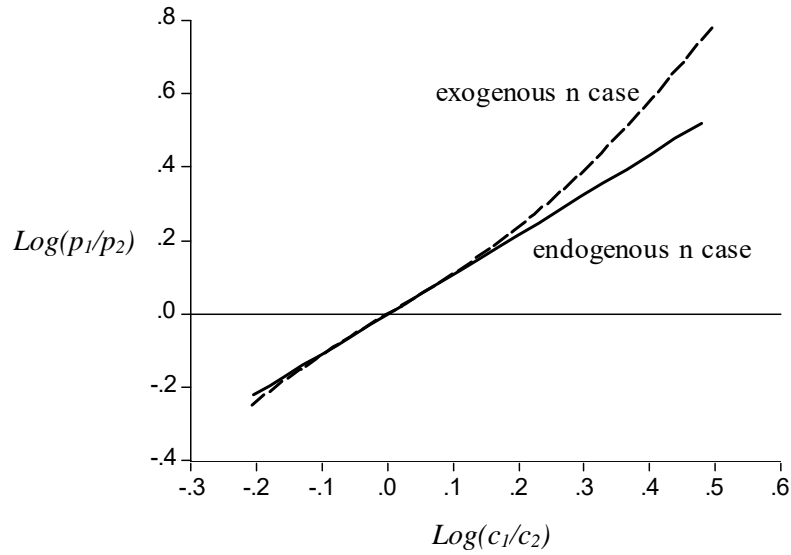


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The figure plots the range of trade costs for 20 2-digit industries reported in Bernard et al. (2006), covering tariffs plus transportation costs, 1988-1992. The trade cost is reported here as  $1 - \tau_i$ , where  $\tau_i$  is the fractional trade cost taken from Table 1 of Bernard et al. (2006).

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Fig 4. Intertemporal Price



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Plotted for  $\beta = 3.1, \alpha = 1, \phi = 6, \theta = 0.73, \tau_F = 0.1$

The figure plots the intertemporal price, log of  $p_1/p_2$ , associated with a range of consumption and current account imbalances, measured as the log of  $c_1/c_2$ . A relative increase in current consumption generates a rise in the current price index relative to the future, implying a rise in the cost of borrowing abroad to finance a current account deficit.

1

2 **Appendix, to be made available as ‘Supplementary Material’**

3

4 1. Derivation of trade balance condition (14) and static model equilibrium

5

6 Combine (8) and (12) to solve out for  $c_N$ :

7

$$c_H = y(p_N / p_H)^\phi. \quad (\text{A1})$$

8 Substitute in (A1) for  $p_N$  with (7):

9

$$p_H c_H = (p^* / \alpha)^\phi y n^{\beta\phi} p_H^{1-\phi}. \quad (\text{A2})$$

10 Substitute in (4) for  $p_T$  with (6) and for  $p_N$  with (7):

11

$$p_H^{1-\phi} = \left(\frac{p^*}{\alpha}\right)^{1-\phi} \frac{1}{\omega} [n^{-\omega}(1+\omega) - 1] \quad (\text{A3})$$

12 where  $\omega \equiv \beta(\phi - 1) - 1$ . Combine (A3) with (A2) to obtain

13

$$p_H c_H = \frac{p^* y}{\alpha} \left(\frac{n^{\beta\phi}}{\omega}\right) [n^{-\omega}(1+\omega) - 1]. \quad (\text{A4})$$

14 Note next that the domestic value of aggregate home production can be derived as

15

$$\begin{aligned} p_H y_H &= \int_0^n p_i y_i di + \int_n^1 p_i y_i di = \int_0^n p_N y di + \int_n^1 p_i y di \\ &= (p^* / \alpha)(n^\beta) n y + y \int_n^1 \left(\frac{p^* i^\beta}{\alpha}\right) di \\ &= \frac{p^* y}{\alpha} n^{\beta+1} + \frac{p^* y}{\alpha} \left(\frac{1}{\beta+1}\right) (1 - n^{\beta+1}) \end{aligned}$$

16 implying

17

$$p_H y_H = \frac{p^* y}{\alpha} \left[ \frac{1 + n^{\beta+1} \beta}{1 + \beta} \right]. \quad (\text{A5})$$

18 With balanced trade,  $p_H y_H = pc$ . Noting that (10) implies  $p_H c_H = \theta pc$  and combining this with the  
19 balanced trade condition gives

20

$$p_H y_H = \left(\frac{1}{\theta}\right) p_H c_H. \quad (\text{A6})$$

21 Substituting in (A6) on the lefthand side for  $p_H y_H$  with (A5) and on the righthand side for  $p_H c_H$  with  
22 (A4):

23

$$\left(\frac{p^* y}{\alpha}\right) \left[ \frac{1 + n^{\beta+1} \beta}{1 + \beta} \right] = \frac{p^* y}{\theta \alpha} \left(\frac{n^{\beta\phi}}{\omega}\right) [n^{-\omega}(1+\omega) - 1].$$

24 Canceling  $p^* y / \alpha$  from both sides, recalling  $\omega \equiv \beta(\phi - 1) - 1$ , and rearranging gives equation (14) in the  
25 text, the equilibrium condition for  $n$  in the case of a zero trade balance surplus  $Z$ :

26

$$Z \equiv \frac{1 + n^{\beta+1} \beta}{1 + \beta} - \frac{1}{\omega \theta} [n^{\beta+1} (\omega + 1) - n^{\beta\phi}] = 0. \quad (14)$$

1            Given the level of  $n$  that implicitly solves condition (14), it is straightforward to solve for the  
 2 other endogenous variables: first the prices,  $p_T$  and  $p_N$  through (6) and (7),  $p_H$  through (A3),  $p$  through  
 3 (3); and then the quantities,  $c_N$  and  $c_T$  through (8) and (9),  $c_H$  and  $c_F$  through (10) and (11), and  $c$   
 4 through (1).

## 5 6 7 2. Demonstrating unique solution for condition (14)

8  
9 It is straightforward to see that for  $n=0$ ,  $Z=1/(1+\beta)>0$ , and for  $n=1$ ,  $Z=-(1-\theta)/\theta<0$ . Showing  
 10 that  $\partial Z/\partial n < 0$  implies that  $Z$  crosses the 0 axis only once and is sufficient to establish the existence of a  
 11 unique solution for  $n$ . Accordingly, it can be proven that

$$12 \quad \frac{\partial Z}{\partial n} = \frac{(\beta+1)n^\beta \beta}{1+\beta} - \frac{1}{\omega\theta} [(\beta+1)n^\beta(\omega+1) - \beta\phi n^{\beta\phi-1}]$$

$$13 \quad = \frac{\theta-1}{\theta} n^\beta \beta - \left(\frac{1-n^\omega}{\omega}\right) \frac{1}{\theta} n^\beta \beta \phi < 0$$

14 since  $\theta < 1$  and  $\frac{1-n^\omega}{\omega} > 0$  for  $0 < n < 1$ .

## 15 16 17 3. Response of equilibrium $\bar{n}$ to $\beta$ and $\phi$ :

18  
19 First consider  $\frac{\partial n}{\partial \beta} = -\frac{\partial Z}{\partial \beta} / \frac{\partial Z}{\partial n}$

20  
21 From subsection 2, we know that  $\partial Z/\partial n < 0$ .

22  
23 Differentiation of (14) with respect to  $\beta$ :

$$24 \quad \frac{\partial Z}{\partial \beta} = -\frac{1}{(1+\beta)^2} + \frac{\beta}{1+\beta} \exp((\beta+1)\ln(n)) \ln(n) + \exp((\beta+1)\ln(n)) \frac{(1+\beta)-\beta}{(1+\beta)^2}$$

$$- \frac{\beta(\phi-1)}{(\beta(\phi-1)-1)\theta} \exp((\beta+1)\ln(n)) \ln(n) - \exp((\beta+1)\ln(n)) \left( \frac{(\beta(\phi-1)-1)\theta(\phi-1) - \beta(\phi-1)(\phi-1)\theta}{(\beta(\phi-1)-1)^2 \theta^2} \right)$$

$$+ \frac{(\beta(\phi-1)-1)\theta \exp(\beta\phi \ln(n)) \phi \ln(n) - \exp(\beta\phi \ln(n)) (\phi-1)\theta}{((\beta(\phi-1)-1)\theta)^2}$$

25  
26 Rearranging and using the equilibrium condition (14)

$$27 \quad \frac{\partial Z}{\partial \beta} = -\frac{1}{(1+\beta)^2} + \frac{\beta}{1+\beta} n^{\beta+1} \ln(n) + \frac{1}{(1+\beta)^2} n^{\beta+1} - \frac{\beta(\phi-1)}{\omega\theta} n^{\beta+1} \ln(n) + \left(\frac{\phi-1}{\omega^2\theta}\right) n^{\beta+1} + \frac{\omega\phi \ln(n)}{\omega^2\theta} n^{\beta\phi} - \frac{(\phi-1)}{\omega^2\theta} n^{\beta\phi}$$

28 Rearranging further:

$$\frac{\partial Z}{\partial \beta} = \frac{1}{(1+\beta)^2} (n^{\beta+1} - 1 - \ln(n^{1+\beta})) + \frac{\phi-1}{\omega^2 \theta} n^{\beta\phi} (n^{-\omega} - 1 - \ln(n^{-\omega})) + \frac{1}{(1+\beta)^2} n^{\beta+1}.$$

All three terms in the expression above are positive for any  $0 < n < 1$ . The first two terms can be signed because for any value of  $x \in (0,1)$  it is true that  $x > 1 + \ln(x)$ , where  $x$  is taken here to be

$$n^{\beta+1} \text{ and } n^{-\omega} \text{ in turn. So we conclude } \frac{\partial n}{\partial \beta} = -\frac{\partial Z}{\partial \beta} / \frac{\partial Z}{\partial n} > 0.$$

5

$$\text{Next consider } \frac{\partial n}{\partial \phi} = -\frac{\partial Z}{\partial \phi} / \frac{\partial Z}{\partial n}.$$

$$\frac{\partial Z}{\partial \phi} = -n^{\beta+1} \frac{(\beta(\phi-1)-1)\theta\beta - \beta(\phi-1)\beta\theta}{(\beta(\phi-1)-1)^2 \theta^2} + \frac{(\beta(\phi-1)-1)\theta n^{\beta\phi} \beta \ln(n) - n^{\beta\phi} \beta\theta}{(\beta(\phi-1)-1)^2 \theta^2}$$

$$= \left\{ \frac{\theta\beta}{\omega^2 \theta^2} \right\} n^{\beta\phi} \left\{ n^{-\omega} - 1 - \ln(n^{-\omega}) \right\}.$$

This is positive for  $0 < n < 1$ , where the last term in braces is signed the same way as the second

term in the expression for  $\partial Z / \partial \beta$  above. So we conclude  $\frac{\partial n}{\partial \phi} = -\frac{\partial Z}{\partial \phi} / \frac{\partial Z}{\partial n} > 0$ .

11

12

#### 4. Derivation of two-period equilibrium:

14

For the two-period case, we introduce time subscripts and solve out for  $c_{Ht}$  with (A2) and (10) together to get

$$\frac{y_t}{\alpha^\phi} n_t^{\beta\phi} p_{Ht}^{1-\phi} = \theta p_t c_t. \quad (\text{A7})$$

Substitute in (3) for  $p_{Ht}$  with (A3) to get

$$p_t = \frac{1}{\alpha^\theta} \left\{ \frac{1}{\omega} \left[ n_t^{-\omega} (1+\omega) - 1 \right] \right\}^{\theta/(1-\phi)} p_{Ft}^{1-\theta}. \quad (\text{A8})$$

Substitute in (A7) for  $p_{Ht}$  with (A3) and for  $p_t$  with (A8):

$$\frac{y_t}{\alpha} n_t^{\beta\phi} \frac{1}{\omega} \left[ n_t^{-\omega} (1+\omega) - 1 \right] = \theta \frac{1}{\alpha^\theta} \left\{ \frac{1}{\omega} \left[ n_t^{-\omega} (1+\omega) - 1 \right] \right\}^{\theta/(1-\phi)} p_{Ft}^{1-\theta} c_t. \quad (\text{A9})$$

Rearranging gives equations (A10) that express the intratemporal consumption allocation relation between  $c_t$  and  $n_t$  that holds for each period  $t=1,2$ :

$$y_t n_t^{\beta\phi} \left\{ \frac{1}{\omega} \left[ n_t^{-\omega} (1+\omega) - 1 \right] \right\}^{\frac{1-\phi-\theta}{1-\phi}} = \theta \alpha^{1-\theta} p_{Ft}^{1-\theta} c_t. \quad (\text{A10})$$

Lastly, we rearrange the intertemporal budget constraint (15) to get

$$c_2 = \left[ (1+r)(p_{H1} y_{H1} - p_1 c_1) + p_{H2} y_{H2} \right] / p_2. \quad (\text{A11})$$

Substituting in (A11) for  $p_{Ht} y_{Ht}$  with (A5) and for  $p_t$  with (A8),  $t=1,2$  gives (A12):

$$\begin{aligned}
1 \quad c_2 = & \left[ (1+r) \left( \frac{y_1 [1+n_1^{\beta+1} \beta]}{\beta+1} - \left( \left[ n_1^{-\omega} \left( \frac{1+\omega}{\omega} \right) - \frac{1}{\omega} \right]^{\frac{1}{1-\phi}} \right)^\theta (\alpha p_{F1})^{1-\theta} c_1 \right) \right] \\
& + \frac{y_2 [1+n_2^{\beta+1} \beta]}{\beta+1} \left] \bullet \left\{ \left[ n_2^{-\omega} \left( \frac{1+\omega}{\omega} \right) - \frac{1}{\omega} \right]^{\frac{-\theta}{1-\phi}} \right\}^{-\theta} (\alpha p_{F2})^{\theta-1}.
\end{aligned} \tag{A12}$$

- 2 The system of three equations – (A10) for  $t=1,2$  and (A12) -- can be solved numerically for  $n_1$ ,  $n_2$ , and
- 3  $c_2$ , given a value of  $c_1$ . The Euler equation (16) completes the system.