Endogenous Tradability and Macroeconomic Implications

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Abstract:

This paper argues that a natural explanation for puzzling behavior in international relative prices can be found by viewing the tradedness of a good as an endogenous decision of the seller. International macroeconomic models long have had difficulty explaining the surprisingly low volatility of the relative price between traded and nontraded goods compared to real exchange rates. This apparent puzzle may reflect a restrictive way of thinking about the nature of nontraded goods. Rather than imposing an artificial dichotomy between traded and nontraded, we regard all goods as parts of a single continuum, where the margin between traded and nontraded is endogenous. This implies that their prices are linked together via a marginal good and a new equilibrium condition. A simple and transparent model is used to demonstrate this approach, featuring a small open economy where differentiated goods are heterogeneous in terms of their iceberg trade costs. The paper goes on to find implications for other basic macroeconomic issues, such as limiting the potency of real exchange rate movements to correct large current account imbalances.

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1. Introduction

While open economy macroeconomics by definition analyzes trade across national borders, the field has long found it useful to allow for the fact that some goods are not traded internationally. As examples, nontraded goods have been central in prominent theories explaining the determination of real exchange rates (Balassa, 1964; Samuelson,1964) and current accounts (Dornbusch, 1983). However, nontraded goods raise some puzzles for international macroeconomics. Consider the puzzling stylized fact featured in several recent empirical papers that the relative price of nontraded goods to traded goods is not very volatile. Empirical measures in Betts and Kehoe (2001a, 2005) indicate that movements in the relative price of nontraded goods are only about 37% as large as movements in the real exchange rate. Empirical work by Engel (1993, 1999) indicates this ratio may be a good deal smaller yet. This fact stands in contrast to standard theoretical models such as that used by Balassa and Samuelson above; these presume that traded goods are constrained by the law of one price, and they explain movements in the real exchange rate primarily in terms of movements in the relative price of nontraded goods. Theoretical models with nontraded goods have had difficulty replicating the relative price volatilities found in empirical work.

The purpose of this paper is to argue that this difficulty may stem from the habit of open economy macroeconomics to think about nontraded goods in a restrictive way. It may be misleading to impose a rigid and artificial dichotomy, where some goods exogenously are labeled as tradable and other as nontradable. Instead, we find insight in recent advances in trade theory regarding the endogenous nature of tradedness, modeling the firm decision of whether to enter the global marketplace. We regard all goods as parts of a single continuum, where the margin between traded and nontraded is endogenous. Whether a good is tradable or not depends on whether the costs of trading that good make trade profitable or not. This fundamental idea of endogenous tradability offers an elegant and simple explanation for the price puzzle above. On the margin there is a seller who is indifferent between selling his good domestically only, or branching out into the international market. As a result, this marginal nontraded good forms a link between the prices of goods that are traded and other similar goods that are nontraded.

We use a simple and transparent model to explain this point and illustrate its usefulness. We build on the small open economy model with trade costs used in Obstfeld and Rogoff (2000), where monopolistically competitive firms can sell differentiated goods at home and in a world market. Goods differ in terms of the iceberg trade costs they face, and we model these costs as following a particular distribution over the continuum. By integrating over a continuum of goods, our approach allows us to avoid the difficulty implied by the Kuhn-Tucker conditions in Obstfeld and Rogoff (2000), whereby the relevant equilibrium conditions for an individual good change discontinuously as it switches between being traded and nontraded.

The model shows that when the small open economy is subjected to demand shocks, the degree to which the relative price of nontraded goods moves depends on the degree of heterogeneity of the trade cost distribution. Depending on the value chosen for the curvature parameter in this distribution, the model can easily generate the range of relative price volatilities found in the empirical literature.

This modeling approach responds to recent findings in empirical research on trade costs. Empirical work by Hummels (1999, 2001) has emphasized that trade costs -- including tariff and nontariff barriers, shipping costs, and other associated costs of marketing and distribution -- vary greatly across classes of goods and play an important role in trade decisions. Collecting detailed trade data for individual goods, he finds that freight costs alone can range from more than 30 percent of value for raw materials and mineral fuels down to 4 percent for some manufactures. Depending on factors such as weight, distance, and the time sensitivity of demand, trade costs can be high and variable for many manufactured goods as well. In a broad survey of trade cost evidence, Anderson and van Wincoop (2004) likewise reach the conclusion that trade costs are very large and very heterogeneous among goods. Empirical work has also found support for the idea that there is switching over time between status as traded and nontraded. Using a panel of U.S. manufacturing plants from 1987 to 1997, Bernard and Jensen (2001) find that year to year transition rates are noteworthy: on average 13.9% of non-exporters begin to export in any given year during the sample, and 12.6% of exporters stop.[1]

This research also takes inspiration from trade theory, which long has worked to understand firm decisions to engage in trade. However, a prominent distinction of the model here is that it focuses on heterogeneity in trade costs rather than productivity. For example, Dornbusch, Fisher and Samuelson (1977) ranks goods by their productivities, while the size of trade costs are assumed to be uniform. Those goods with the greatest comparative advantage in one country or the other are traded, while those goods with small gains from trading relative to the uniform trade costs remain nontraded. In contrast to this convention, we think that when the issue of primary interest is tradedness, it makes more sense to focus on the variation of trade costs among goods. For example, the reason that many types of services are nontraded is not because

[1] The results to follow in no way rely upon implausibly large numbers of firms switching between traded and nontraded status, but rather upon the simple fact that firms have the ability to choose. In fact, the results are strongest in those cases where only a small number of firms actually do switch in equilibrium.

countries are so similar in their productivities in these sectors; rather, they remain nontraded primarily because such services are particularly costly to trade across borders. Further, the usual convention has some strongly counterfactual implications regarding nontraded goods. For example, it does not account for the empirical observations that there is a great deal of heterogeneity among goods in terms of their deviations from the law of one price across countries, nor that these deviations systematically tend to be greater for nontraded goods than for traded goods (Crucini, Telmer, and Zachariadis, 2005). Dornbusch, Fisher, and Samuelson (1977) imply the opposite of this last observation. While such models are appropriate for explaining the distinction between exported versus imported goods, it arguably is not the most appropriate model for understanding the distinction between traded versus nontraded goods.[2]

This paper is also related to recent research in Ghironi and Melitz (2005), as well as Naknoi (2005) and Bergin, Glick and Taylor (2005), which incorporate trade features in a macro model with a continuum of heterogeneous firms. One significant difference is that these papers follow the trade literature in specifying heterogeneity in terms of firm productivity.[3] Betts and Kehoe (2001b) allow heterogeneous trade costs, but we differ in that we have a share of goods that are fully nontraded, which is what allows us to examine shifts of the nontraded extensive margin.

We also find that endogenous tradedness has implications for other basic macroeconomic issues, such as intertemporal trade. Previous work assuming exogenous tradedness (Dornbusch, 1983) found that the presence of nontraded goods strongly discourages intertemporal trade, where countries with large current account imbalances are discouraged from further borrowing by high domestic interest rates and depreciating real exchange rates. We find that if tradedness is endogenous, the share of nontraded goods will tend to adjust so as to minimize this friction. Our result also differs from that of Obstfeld and Rogoff (2000), which likewise models endogenous tradedness to examine current account dynamics, but under a simpler goods market structure of two goods. In contrast to the preceding papers, our model indicates that exchange rate movements are most potent in helping correct current account imbalances when these are small, and that this mechanism begins to break down when a country's current account deficit is large.

[2] The macro model here will differ also in several other respects from the related trade literature. The model describes a small open endowment economy where world price levels are exogenously given. We abstract from production and entry decisions. We also abstract from monopolistic competition and markup pricing by firms. In this context, we do not need fixed costs of trade to induce some firms to forgo international trade; iceberg costs alone are sufficient.

[3] The model of Ghironi and Melitz (2005) also differs in many other details, as it is geared mainly to analyze business cycle regularities. It utilizes a two-county framework, with monopolistic competition and entry decisions into domestic production as well as international trade, which in turn requires fixed entry costs as well as per-unit costs of exporting.

2. Model

To focus on the issue of tradedness, we follow Obstfeld and Rogoff (2000) in considering a very simple small open endowment economy. This choice is a useful starting point for our analysis because it permits some analytical results and makes very transparent the new insights on which we wish to focus. An endowment economy is clearly a special case, but an addendum available from the authors demonstrates that the results are robust to including production in the model. The small open economy assumption is highly relevant for most countries in the world.

The country is endowed with a continuum of goods indexed by i on the unit interval, where y_i represents the level of endowment, c_i is the level of consumption, and p_i is the domestic price level of this good. All of these home goods have the potential of being exported, but some endogenously determined fraction of the goods, n, will be nontraded in equilibrium. For each traded home good there is a prevailing world price p_i^* that may differ from the home price because of trade costs. The small open economy may also import foreign goods for consumption purposes, with consumption level c_F and price level p_F. We initially omit time subscripts in the notation, but introduce them when extending the framework to two periods. For simplicity, we assume that the endowments and world price levels of all home goods are uniform, implying $y_i = y,\ p_i^* = p^*$ for all i.

The aggregate consumption index is specified as:[4]

$$c = \frac{c_H{}^{q} c_F{}^{1-q}}{q^q (1-q)^{1-q}} . \tag{1}$$

Here c_H is an index of home goods consumption:

$$\begin{aligned} c_H{}^{(f-1)/f} &= \int_0^n (c_i)^{(f-1)/f} di + \int_n^1 (c_i)^{(f-1)/f} di \\ &= n\left(\frac{c_N}{n}\right)^{(f-1)/f} + (1-n)\left(\frac{c_T}{1-n}\right)^{(f-1)/f} \end{aligned} \tag{2}$$

where

$$c_N \equiv n\left(\frac{1}{n}\int_0^n c_i{}^{(f-1)/f} di\right)^{f/(f-1)} .$$

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[4] For simplicity we limit ourselves here to a Cobb-Douglas specification, implying a unitary elasticity of substation between home and foreign goods. Empirical work on this elasticity suggests a value between 0.5 and 1.5, with our value of 1 in the middle; e.g., see Pesenti (2002). In the present case, the Cobb-Douglas specification has the added benefit of making the algebraic results more easily interpretable. See the appendix of the working paper version (Bergin and Glick, 2003) for the derivations of the CES case.

$$c_T \equiv (1-n)\left(\frac{1}{1-n}\int_n^1 c_i^{(f-1)/f}\,di\right)^{f/(f-1)}$$

are consumption indexes of nontraded and traded goods, respectively, and n is the share of goods on the continuum $\{0,1\}$ that are nontraded. Price indexes are defined as usual for each category of goods, in correspondence to the consumption indexes above:

$$p = p_H{}^{q} p_F{}^{1-q} \tag{3}$$

$$\begin{aligned} p_H{}^{1-f} &= \int_0^n \left(p_i\right)^{1-f} di + \int_n^1 \left(p_i\right)^{1-f} di \\ &= n p_N{}^{1-f} + (1-n)\, p_T{}^{1-f} \end{aligned} \tag{4}$$

where p is the aggregate price level, p_H is the price index of all home goods, and the price index of home nontraded goods p_N and the price index of home traded goods p_T are defined as

$$p_N \equiv \left(\frac{1}{n}\int_0^n p_i^{1-f}\,di\right)^{1/(1-f)}$$

$$p_T \equiv \left(\frac{1}{1-n}\int_n^1 p_i^{1-f)/f}\,di\right)^{1//(1-f)}$$

Note that if world prices are normalized to unity, i.e. $p^* = 1$, $p_F = 1$, p may be interpreted as the reciprocal of the real exchange rate for this small open economy.

The home goods are distinguished from each other by the presence of good-specific iceberg costs, (t_i) where a certain fraction of the good disappears in transport. We assume that the home country pays for this cost so that the domestic price will be $p_i = p^*/(1+t_i)$ if the country exports good i.[5] These trade costs are specified to follow the distribution:

$$1 + t_i = a\, i^{-b}\,;\; a \geq 1,\; b \geq 0$$

which implies the following distribution of export prices

$$p_i = \frac{p^*}{1+t_i} = \frac{p^* i^b}{a}. \tag{5}$$

The parameter b controls the curvature of the distribution, while a controls the level.[6] Figure 1 illustrates how the distribution of export prices varies with b (assuming $p^*/a = 1$). The goods at the left end of the continuum (i near 0) tend to have lower prices when exported because the trade cost is large; these goods are less tradable. Goods toward the right end of the continuum (i near 1) have

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[5] The presence of trade costs (obviously) implies segmentation between domestic and foreign markets.
[6] This cost distribution is related to the Pareto function, where a is the "scale" parameter and b is the "shape" parameter.

higher prices because the trade cost is low; they are more tradable. b characterizes how quickly the price of an individual good rises with the goods index -- in fact, it can be viewed as an elasticity. For example, for a high b, the percent change in costs is high for a given percent change in the index.[7]

In positing a distribution of transport costs over a continuum of firms, we do not take a stand on how much of this heterogeneity is due to differences across industries versus differences across plants within an industry, as there is empirical evidence indicating heterogeneity on both levels. Our continuum simply ranks all firms according the trading costs they face, without regard for whether this coincides with any notion of industrial grouping.

In the endowment economy in our model the decision of whether to export a good is determined solely on the basis of whether the export price (i.e. the world price) less iceberg costs, exceeds the domestic price. If the export price is higher, then the good is exported, if it is lower, then it is not traded.

Given the cutoff between traded and nontraded goods at index n, it is straightforward to compute the price index for traded goods from the price distribution of exported varieties:

$$p_T = \left(\left(\frac{1}{1-n} \right) \left[\int_n^1 \left(\frac{p^* i^b}{a} \right)^{1-f} di \right] \right)^{1/(1-f)} = \left(\frac{p^*}{a} \right) \left(\left(\frac{1}{1-n} \right) \left(\frac{1}{w} \right) \left\{ \left(\frac{1}{n} \right)^w - 1 \right\} \right)^{1/(1-f)} \qquad (6)$$

where we define $w \equiv b(f-1)-1$, $w \geq -1$ (since $b \geq 0$ and $f > 1$).[8] Keep in mind that this n is itself an endogenous variable that will be solved as part of the general equilibrium system. Equation (6) expresses the price of traded goods as a function of the share of traded goods n, the elasticity of substitution across domestic goods f, and the trade cost parameters, b and a. It is straightforward to establish that $\partial p_T / \partial n > 0$; i.e. the price of traded goods increases with the share of nontraded goods. The reason is that, as the proportion of home goods that are nontraded rises, it is no longer profitable to export goods with marginally higher trade costs; as these goods are withdrawn from export markets, the average price of the remaining export goods rises.[9]

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[7] That the domestic price of more tradable goods is greater than that of less tradable goods can be attributed to our normalization that the world price of all goods is constant. Had we assumed that the foreign price p_i^* rises with i at a rate faster than trade costs fall, the domestic price of exported goods could be higher than that of nontraded goods. In Section 4 in an extension to the model we show how export prices may be relatively higher as a result of heterogeneous productivity in domestic production. Note also that we abstract from possible heterogeneity in import goods; see footnote 17.

[8] Note $p_T \geq 0$ with our specification of trade costs, since for $0 > w \geq -1$ and for $w > 0$, it follows that $(1/w)\{n^{-w} - 1\} > 0$ for $1 \geq n \geq 0$; for $w = 0$, $p_T = \left(\frac{-p^* \log(n)}{a(1-n)} \right) \geq 0$ as well.

[9] This conclusion is robust to the particular definition of the price index. If a naïve statistician did not know the set of traded goods had changed, but collected price data on all goods that previously had been traded,

The price index of nontraded goods is even easier to determine. As usual, intratemporal optimization implies relative demands for each pair of home goods i and j:

$$\frac{c_i}{c_j} = \left(\frac{p_i}{p_j}\right)^{-f} .$$

Since consumption must equal the endowment of nontraded goods, and endowments are uniform for all goods here (i.e. $y_i = y$ for all i), we can conclude that for any pair of nontraded goods it will be true that $c_i / c_j = y_i / y_j = 1$, and so $p_i / p_j = 1$.[10] In other words, the price of each nontraded good will be identical, because they each are by definition not affected by the trade costs which vary by good. This logic applies equally well to the home good that is just on the margin between being traded and nontraded (i=n). The marginal trader decides to export solely on the basis of whether the world price less iceberg costs exceeds the domestic price. But because this good is on the margin of being traded, the domestic price must be the same as that as if it were sold in the world market: $p_n = \left(p^*/a\right)n^b$. As a result, the price index of nontraded goods is pinned down as the price of the marginal traded good:

$$p_N = \left(\left(\frac{1}{n}\right)\left[\int_0^n (p_i)^{1-f} \, di\right]\right)^{1/(1-f)} = \left(\left(\frac{1}{n}\right)\left[\int_0^n (p_n)^{1-f} \, di\right]\right)^{1/(1-f)} = p_n = \left(\frac{p^*}{a}\right)n^b . \qquad (7)$$

This equilibrium condition will be important in the analysis to follow, and it will be referred to as the "marginal nontraded condition." It implies that the price of nontraded goods rises with the share of nontraded goods with elasticity b . Figure 2 below illustrates how this equilibrium price level varies with the share of nontraded goods (still assuming $p^*/a = 1$).

It is easily verified that there can be no discontinuous jump in price either up or down between the last nontraded good and the first traded good. Note that the iceberg trading costs for adjacent goods are essentially identical and that there is no fixed cost to trade. Suppose that the price of the first traded good jumped discontinuously above the price of the last nontraded good; then it would be profitable for the last nontraded good to become traded instead. Similarly, suppose that the

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this average price level would still rise. However, the reason would be that the average includes newly nontraded goods, whose individual prices have risen, rather than the fact that an average is being taken over a subset of goods where the lower price items have been removed.

[10] This assumption can be relaxed without undermining our ability to compute a price index for nontraded goods; the only difference is that the distribution of productivities and endowments would have to be included in the integral, making the resulting price index more complicated. Because our focus here is on the role of heterogeneous trade costs, we utilize the assumption of uniform endowments to make the results more transparent.

price of the first traded good jumped discontinuously below the price of the last nontraded good; then it would be profitable for the first traded good to become nontraded instead.[11]

The price indices of traded and nontraded goods are related to each other. Figure 3 shows their relationship as the share of nontraded goods varies. Observe that (i) p_T is everywhere higher than p_N, since traded goods are less costly to transport,[12] and (ii) both p_N and p_T rise with n.[13]

Equations (6) and (7) can be combined to obtain a characterization of how the relative price structure is pinned down by the share of nontraded goods n, the elasticity of transportation costs b, and the elasticity of substitution of home goods f:[14]

$$\frac{p_N}{p_T} = \left[\left(\frac{n}{1-n}\right)\left(\frac{1}{w}\right)\left(1-n^w\right)\right]^{\frac{1}{f-1}}$$

where once again $w \equiv b(f-1)-1$.

As additional equilibrium conditions, intratemporal optimization implies the demand functions:

$$c_N = n\left(\frac{p_N}{p_H}\right)^{-f} c_H \tag{8}$$

$$c_T = \left(1-n\right)\left(\frac{p_T}{p_H}\right)^{-f} c_H \tag{9}$$

$$c_H = q\left(\frac{p_H}{p}\right)^{-1} c \tag{10}$$

$$c_F = \left(1-q\right)\left(\frac{p_F}{p}\right)^{-1} c \tag{11}$$

It is assumed that residents of the small open economy must pay the cost of transport for imports of foreign goods. The price of imported foreign goods is normalized to unity in the world market, so its domestic price is set exogenously as

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[11] If we included a constant fixed cost of exporting per firm f_X, the price setting condition for the marginal exporter changes from $p_n = p^*/(1+t_n)$ to $p_n = p^*/(1+t_n) - f_X$. This implies a vertical jump down in our figure 2. Note that these price setting conditions are equivalent to zero profit conditions in our endowment economy framework with quantities normalized to unity.

[12] It is not clear how one should compare this prediction to data, given that in the model quantity units are normalized to be constant for all goods, while in actual data they obviously vary across goods. The main testable implication is that there is a greater price wedge on average between the level of prices of individual nontraded goods across countries than there is for the prices of traded goods across countries. This implication is easily verified in data (e.g., see Crucini, Telmer, and Zachariadis, 2005).

[13] These results should hold for any cost distribution that is monotonically increasing in i. We can verify this at least for the class of power functions $\left(l_0 + l_1 i\right)^b$, which are easily integrable.

[14] Note that the absence of trade cost heterogeneity ($b = 0$) implies $w = -1$ and $p_N = p_T$.

$$p_F = 1 + t_F \equiv a_F$$

for some given t_F representing iceberg trade costs for imported goods.[15] [16]

Market clearing for nontraded goods requires

$$c_N = \int_0^n y_i \frac{p_i}{p_N} di = ny \tag{12}$$

given our assumption $y_i = y$ for all i and that $p_i = p_n = p_N$ for all $i \in \{0, n\}$.

The goods market described above will be analyzed in the context of a two-period model with a representative consumer. The consumer maximizes two-period utility

$$dU(c_1) + U(c_2)$$

subject to the intertemporal budget constraint.

$$\left(\frac{p_{H2}}{p_2} y_{H2} - c_2\right) = -\left(\frac{p_1}{p_2}\right)(1+r)\left(\frac{p_{H1}}{p_1} y_{H1} - c_1\right). \tag{13}$$

Here r is the world interest rate. The term d is an exogenous discount factor that can change, thereby allowing us to consider shifts in demand from one period to the next. Intertemporal optimization implies the usual intertemporal Euler equation:

$$U^{'}_{c1} = \frac{1}{d}\left(\frac{p_1}{p_2}(1+r)\right)U^{'}_{c2}. \tag{14}$$

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[15] We assume transport costs on imports may differ from those on exports (i.e., $a \neq a_F$) because of, for example, differential tariff costs. We do not consider heterogeneity in the trade costs of foreign goods nor the endogenous determination of which goods produced abroad are traded (and imported by the domestic economy), and which are nontraded, as the small open economy framework is better suited for considering the endogeneity of home-country variables. Extending the model to endogenize heterogeneous imports from abroad would not harm our main results, since our price indices are defined in terms of exported, not imported, goods. Further, including the latter under heterogeneity would likely only enhance our result that positive demand shocks raise the price of traded goods, since a rise in demand would induce the importing of foreign goods with higher trade costs and lead to higher import prices and hence higher traded good prices in the domestic market.

[16] The existence of a world price for all varieties of home goods implies that these goods must all be available abroad as well as in the home country. We rule out the possibility that any goods with which the home country is endowed are ever imported because of the transport costs incurred by domestic residents of doing so. Specifically, if the home country started importing what it had previously been exporting, the price of these goods would jump from below the world price to above the world price. It would only be an extreme case where domestic residents would be willing to pay this price and still consume enough of these good to import them. Moreover, as long as some goods are always exported each period (i.e. $n < 1$), such an extreme case will never be reached. Intuitively, since the last goods to stop being exported have the lowest transport costs, they would also have to be the first ever to be imported. Hence, the exporting of *all* home goods would need to cease in a period before importing any of them would begin, implying a huge current account deficit and a zero level of gross (not merely net) exports. This is ruled out as long as n <1.

Equilibrium here determines values each period for the variables c_t, c_{Ht}, c_{Tt}, c_{Nt}, c_{Ft}, p_t, p_{Ht}, p_{Tt}, p_{Nt}, n_t, satisfying equations (3-4, 6-12) for each period as well as the intertemporal budget constraint (13) and the intertemporal consumption Euler equation (14). This system is identical to a standard two-period model, with the addition of one extra endogenous variable, n, which is pinned down by one additional equilibrium condition, the marginal nontraded condition (7).

3. Results

A. Solution for the share of nontraded goods under balanced trade

Viewing nontradedness as endogenous offers some new insights into what drives the degree of international integration and the openness of a country's goods markets. Consider first a static version of the model where d is constant at a value of unity (and accordingly r = 0). We will refer to this version as a steady state of the model, in that consumption and all other variables are constant across the two periods. According to the intertemporal budget constraint, the value of domestic production equals the value of domestic consumption in this case, and the trade balance is zero: $p_{H1}y_{H1} - p_1c_1 = p_{H2}y_{H2} - p_2c_2 = 0$.[17] In the appendix we show that the equilibrium conditions above can be solved together to yield the following expression for the equilibrium trade balance (surplus) Z:

$$Z \equiv \frac{1+n^{b+1}b}{1+b} - \frac{1}{wq}\left[n^{b+1}(w+1) - n^{bf}\right] = 0 \qquad (15)$$

where $w \equiv b(f-1)-1$ (and time subscripts are still omitted). In appendix A we show that the trade balance Z falls as n increases. Intuitively, increasing n implies trade in fewer varieties of goods and lowers the trade surplus. Condition (15) implies that the balanced trade condition determines the steady-state share of nontraded goods, $\bar{n}$. It is easily verified that this solution is the unique solution that lies within the permissible range of zero to one (see the appendix). It is clear that if n were 0 and all goods were traded, then the trade balance is positive here. For some $n > 0$, the trade balance will fall to zero.

Condition (15) provides a number of insights concerning the determinants of the equilibrium share of nontraded goods. One observation is that the curvature parameter in the distribution of trade costs (b) plays an important role in determining $\bar{n}$. Table 1 reports

[17] Note that $p_H y_H$ is the value of the endowment.

numerical simulations for a benchmark calibration of $f = 10$, $p^*/a = 1$, $q = 0.5$, $t_F = 0.1$.[18] Column 2 shows that a rise in b progressively raises the share of home goods that are nontraded. This result is fairly intuitive: if trade costs rise very quickly as one exports more classes of goods, it is optimal to export a smaller number of classes of goods. A country should then concentrate its exports in those commodities for which international trade is so much less costly.

Another important determinant of tradedness is the elasticity of substitution between home goods (f). Table 2 shows in column 2 that as this elasticity rises, $\bar{n}$ rises gradually. The intuition is that if home goods are highly substitutable in consumption, one can conserve on trade costs by concentrating one's exports in the goods that are easiest to trade. This means there will be a smaller quantity of these particular classes of goods to consume, but under a high elasticity, it is easy to compensate for this by consuming a greater quantity of other types of goods. On the other hand, if home goods were less substitutable with each other, one would want to consume a more even distribution of home goods, thereby requiring the country to export a smaller portion of a larger number of goods to pay the bill for imports.

Lastly, observe that the scale parameter in the distribution of trade costs, a, does not appear in equation (15) above. When one considers the effects of trade costs here, it is their relative levels between goods (summarized in b), not their overall level (summarized in a) which determines the varieties of goods that are nontraded. In part, this last implication results from the assumption of Cobb-Douglas preferences over home and foreign goods, which is a common assumption in this literature, known to have certain implications that help simplify analytical solutions.[19] Some intuition can be found in the fact that a unitary elasticity of substitution between home and foreign goods implies that a constant share of consumption expenditure goes toward foreign goods, regardless of the relative price between goods, and hence regardless of the size of transport costs. A sufficient quantity of home goods then must be traded and exported to pay for these imports under balanced trade.[20]

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[18] A calibration of $f = 10$ is based on estimates in Basu (1996) and Basu and Fernald (1995).

[19] See Corsetti and Pesenti (2001) for an example.

[20] Condition (A6) in the appendix shows that under balanced trade and Cobb-Douglas preferences, a constant fraction of home goods will be consumed domestically and a constant fraction will be exported, without any regard for the relative price of home to foreign goods. Because the scale parameter of transport costs enters only through price terms, it does not enter in this condition. As long as the world prices of home goods are uniform, the same result holds for changes in p^*.

However, if we consider a more general CES specification between home and foreign goods, the scale of trade costs does affect the share of nontraded goods. The counterpart to equation (15) for the CES case is:[21]

$$\left[\frac{1+n^{b+1}b}{1+b}\right]\left[n^{-w}\left(\frac{1+w}{w}\right)-\frac{1}{w}\right]^{\frac{f-g}{1-f}} = n^{bf}\left(\frac{1}{q}\right)\left(q\left[n^{-w}\left(\frac{1+w}{w}\right)-\frac{1}{w}\right]^{\frac{1-g}{1-f}}+(1-q)\left(a\,p_F\right)\right)^{1-g}\right) \tag{15'}$$

where g is the elasticity of substitution between home and foreign goods and $1 > q > 0$ reflects the degree of bias for home goods. A rise in a raises trade costs which lowers the price of home goods and raises the price of imported goods in our model, shifting demand towards home goods. For an elasticity of substitution between home and foreign goods greater than unity, it can be confirmed that a rise in the scale of trade costs (a) then raises the share of nontraded goods, $\bar{n}$, as one might expect. In words, expenditures on home goods rise and expenditures on foreign goods falls. Since the domestic country is importing less, it need not export as much to balance trade. Hence it need not export as many varieties of goods, leading to a rise in $\bar{n}$. This result is reversed if the elasticity between home and foreign goods is less than unity; in this case a rise in a, increases expenditures on imports relative to that on home goods, leading to lower exports, and lower $\bar{n}$. For a unitary elasticity, as shown here for the Cobb-Douglas case, a has no effect on $\bar{n}$, since the level of relative expenditures on home and foreign goods is unchanged. Empirical work on this elasticity suggests a value between 0.5 and 1.5, with our value of 1 in the middle (see Pesenti, 2002).

B. Implications for the relative price of nontraded goods

Viewing nontradedness as endogenous also offers some new insights into the behavior of international relative price dynamics. If we wish to solve for the dynamics of the model when trade is not restricted to be balanced, the equilibrium conditions cannot be summarized in a single equation as in (15); instead there is a system of four equations that must be solved numerically for n_1, n_2, c_1 and c_2:

[21] See the appendix in the working paper version of Bergin and Glick (2003) for the derivation of this condition. In the case of Cobb-Douglas preferences for the home and foreign good, $g = 1$, and (15') reduces to (15).

$$y_1 n_1^{bf}\left\{\frac{1}{w}\left[n_1^{-w}(1+w)-1\right]\right\}^{\frac{1-f-q}{1-f}} = q a^{1-q} p_{F1}^{1-q} c_1 \qquad (16)$$

$$y_2 n_2^{bf}\left\{\frac{1}{w}\left[n_2^{-w}(1+w)-1\right]\right\}^{\frac{1-f-q}{1-f}} = q a^{1-q} p_{F2}^{1-q} c_2 \qquad (17)$$

$$c_2 = \left[(1+r)\left(\frac{y_1\left[1+n_1^{b+1}b\right]}{b+1} - \left(\left\{\left[n_1^{-w}\left(\frac{1+w}{w}\right)-\frac{1}{w}\right]^{\frac{1}{1-f}}\right\}^{q}(a\,p_{F1})^{1-q}c_1\right)\right) + \frac{y_2\left[1+n_2^{b+1}b\right]}{b+1}\right] \bullet \left\{\left[n_2^{-w}\left(\frac{1+w}{w}\right)-\frac{1}{w}\right]^{\frac{1}{1-f}}\right\}^{-q}(a\,p_{F2})^{q-1} \qquad (18)$$

$$c_1 = d\left(\frac{p_2}{p_1(1+r)}\right)c_2 \qquad (14')$$

See the appendix A for derivations. Equations (16) and (17) reflect the intratemporal allocation of domestic consumption for home goods in periods 1 and 2 respectively, while (18) reflects the effects of the budget constraint on intertemporal allocation. Together these three equations define the set of combinations of c_1 and c_2 that are permissible for the small open economy, characterizing the intertemporal tradeoffs that are possible. The fourth equation is the intertemporal Euler equation (14) written for the particular case of additively separable log utility. This condition indicates the intertemporal tradeoff between c_1 and c_2 that consumers in the small open economy prefer, and captures how demand shocks to d enter the system.

Columns (3-6) of Table 1 illustrate the ability of the model to generate a range of possible relative price volatilities. This is done for the case of a shock to d that raises period-one consumption by 1.5 percent relative to its steady-state level under balanced trade. (This is the standard deviation of U.S. consumption typically used in calibration studies.)[22] The benchmark calibration will be used again here: $f = 10$, $p^*/a = 1$, $q = 0.5$, $t_F = 0.1$, $r = 0$.

Column (3) shows that the volatility of the relative price of nontraded goods depends a great deal on the curvature parameter b. As there is only one intertemporal shock in this two-period experiment, this column reports the percentage "standard deviation" of p_N / p_T as $\log\left(\frac{p_{N1}/\overline{p_N}}{p_{T1}/\overline{p_T}}\right)$, where overbars indicate levels in the balanced trade steady state. This volatility

[22] The rise in consumption requires a shock to d that varies from 3.14% for the case of b =0.1, to 4.26% for b =1.5, and 4.96% for b =10.

is reported as a ratio to the percentage standard deviation in the real exchange rate for $1/p$ computed in the same manner in absolute value. This relative volatility falls dramatically as the curvature of trade costs rises, and for a value of $b = 1.5$, the model is able to approximately replicate the value of 0.37 found in the empirical study by Betts and Kehoe (2001a, 2005).[23] Empirical work by Engel (1999) finds that the volatility of nontraded prices may yet be lower than this, but the table shows that the model is capable of replicating even very low values of volatility as the curvature parameter b is assumed to be progressively larger.

This result stands in sharp contrast to the standard result of open economy models in the literature, where the share of nontraded goods is taken to be exogenous. For example the classic Balassa-Samuelson model explains real exchange rate levels exclusively in terms of shifts in the relative price of nontraded goods. The same is true for the well-known two-period model of Dornbusch (1983), which is very similar to the model considered here, except for the assumption that the share of nontraded goods is fixed. Under such an assumption, a rise in consumption demand will tend to push up the price of consumption goods, but this will be expressed only for nontradeds, because the price of traded goods is pinned down to the world price level by arbitrage. A rise in the relative price of nontraded goods is necessary for equilibrium, to convince households to take their extra consumption in the form of additional imports of tradable goods, given that the consumption of nontraded goods is limited by definition to the domestic supply of such goods.

This conclusion is illustrated in column (7) of Table 1, where the movement in the relative price of nontraded goods is solved for a version of the model here where n is taken to be exogenous. The model is identical to the one reported in the earlier columns, except that the "marginal nontraded condition" (equation 7) is dropped. To maintain comparability with the earlier columns of the table the exogenous value of the nontraded share, n, is set at the level of $\bar{n}$ found for the corresponding endogenous nontraded model reported in the preceding columns. Note that it is true for all the cases in the table, that the relative price of nontraded goods moves

[23] The traded goods included in the aggregate price index include only home traded goods and exclude imported foreign goods. This is in part a matter of technical necessity: the model is designed to avoid an a priori demarcation between different types of home goods, so there is no clear way to define a price index combining imported foreign goods together with a subset of goods in the home goods CES index, while excluding other goods in this CES index. Very fortunately, the stylized fact which the model is trying to replicate is defined in precisely the same manner. When Betts and Kehoe (2001a) compute the relative price of nontraded to traded goods, they likewise define p_T in terms of the prices of goods in traded sectors that are produced at home (using either gross output deflators by sector or a domestic producer price index). In addition, the statistic we report for our model likewise reflects Betts and Kehoe by using the full consumer price index for the domestic price level, p.

much less under the assumption of endogenous tradedness than for the standard assumption of exogenous tradedness. In fact, it is easy to demonstrate that the ratio of volatilities reported in column (7) must always be greater than unity when n is exogenous. Since the aggregate price level p is a weighted average of nontraded prices (p_N), traded home goods prices (p_T), and import prices (p_F), where the latter two are fixed by world levels, the movement in the first component must always be larger than the movement in the overall average that it induces. This explains why a small open economy model with exogenously determined nontraded goods has such difficulty explaining a low volatility in the price of nontraded goods relative to the overall real exchange rate.

A comparison of columns (3) and (7) makes clear that the one change of making n endogenous has a very dramatic effect on the ability of the model to explain this empirical regularity. The chain of events characteristic of standard models, explained above, no longer applies. Now, as a rise in demand starts to push up the relative price of nontraded goods, some traded goods sellers on the margin will find it profitable to sell more in the home market, to the point of abandoning attempts to market their good abroad where they need to deal with costs of trade. This endogenous rise in the share of nontraded goods allows the supply of nontraded goods to rise, despite the fact that the endowment of each individual good is fixed. This rise in supply reduces the pressure for the relative price of nontradeds to rise in the face of the higher demand.

The main insight here is that, when one begins to view nontraded goods as being endogenously determined, one can see there is a potentially strong force limiting the movement in the relative price of these nontraded goods. The marginal trading condition from the model (eqn. 7) is useful in seeing how this result arises. Recall that this equation states that the price index of nontraded goods will equal the price of the marginal traded good. This linkage between nontraded and traded prices prevents one price index from straying too far from the other, and thus helps dampen the volatility in their ratio.

It is interesting to note that this dampened volatility in the relative price of nontradeds does not rule out volatility in the overall price index or real exchange rate here. Columns (5) and (6) in Table 1 show that for high levels of b, the price of nontraded and traded goods tend to move more volatility and in a synchronized fashion. Given that these two prices are important components in the overall CPI, this overall price index moves a good deal. But because the two components are moving in synchronization, the relative price of one in terms of the other is not moving significantly. This explains why the ratio reported in column (3) is able to take on such small values under endogenous tradedness, whereas it can never take a value less than unity under the assumption of exogenous tradedness.

Why does this mechanism work best for high values of b ? Looking at the marginal condition (equation 7), it becomes clear that b is the elasticity of the nontraded price index with respect to changes in n. It is at high values of b where the demand shock induces a small change in n and a large change in the price of nontraded goods. But this also requires a larger change in the price index of traded goods, so the overall price index changes more. One interesting implication of this logic, is that the mechanism outlined here to explain the stylized fact does not require an implausible degree of movement in the share of nontraded goods. In fact, inspection of column (4) of Table 1 confirms that the mechanism is at its most potent when n moves the least between the two periods. For the benchmark case of b =1.5, the nontraded share moves 1.67% between the periods, from a share of about 0.585 to 0.575, and this shift is yet smaller for cases with higher b in the table.

The curvature parameter is not the only parameter to play an important role in this mechanism. Table 2 shows that a higher elasticity of substitution between home goods (f) also plays an important role. Column (3) shows that as f rises, the volatility in relative nontraded prices as a ratio to that of the real exchange rate falls. Intuitively, if the last nontraded good and the marginal traded good are highly substitutable, this makes the link between their two prices stronger. This in turn strengthens the linkages between the price indexes of traded and nontraded goods.

C. Implications for the intertemporal price and intertemporal trade

Through its effects on relative price movements, endogenous tradability has important implications for other macroeconomic issues. One such issue is intertemporal trade, the ability of a country to borrow in world financial markets to finance a current account deficit in a given period. Recently, large current account deficits in the U.S. have renewed interest in this issue. Dornbusch (1983) demonstrated that when nontraded goods are present, a change in their relative price can discourage intertemporal trade.[24] Looking at the intertemporal budget constraint (equation 13), one sees that the cost of borrowing in foreign markets includes not only the world rate of interest, r, but also the change in the price level or real exchange rate over time. Since borrowing takes place in units of the world consumption index, a change in the relative price of

[24] This version of the intertemporal theory of the current account receives empirical support in Bergin and Sheffrin (2000).

home to foreign goods affects the cost of repaying the loan. In particular, if a temporary rise in consumption induces a temporary rise in the domestic price level, the expected fall in price for the next period implies that repayment of the loan will be larger in units of the home consumption index than implied by the interest rate alone. This rise in the "intertemporal price" can discourage such intertemporal trade.

This theory was extended in a limited but important way to endogenously nontraded goods by Obstfeld and Rogoff (2000). In a model with one home good that can switch into and out of being traded, it was shown that changes in the intertemporal price may be highly nonlinear, and may come into effect only for large current account imbalances. A disadvantage of this approach is the difficulty of dealing with Kuhn-Tucker conditions which imply discrete changes in equilibrium conditions for various ranges of variable realizations. The model in the present paper reformulates the equilibrium conditions for the case of a continuum of goods. Rather than making the solution yet more complex, this permits us to eliminate the discrete changes and discontinuities in the prices of individual goods, and instead focus on smoothly changing levels of various integrals over regions of the continuum. As shown above, this method of dealing with endogenous tradedness is much easier to work with, and has the promise of being incorporated into a wide range of international macro models.

To gauge the effect of endogenous tradedness on intertemporal trade, we use our model to compute the intertemporal price (p_1/p_2) for various levels of intertemporal borrowing. Figure 4 plots this intertemporal price against various levels of intertemporal reallocation of consumption (c_1/c_2). The solid line represents the benchmark model and the dashed line the exogenous nontraded case defined above. The exogenous share of nontraded goods for this case is calibrated to equal the share of the endogenous model in its balanced-trade steady state.

Several conclusions emerge. First, the intertemporal price rises smoothly in the endogenously nontraded model, in contrast to the earlier papers with only one or two home goods. The absence of price changes for small shocks to the current account and the dramatic kinks and sudden price rises for large imbalances characteristic of the earlier models disappear here in the more realistic case of many goods.[25] This smooth rise in intertemporal price indicates that there is no special cost that kicks in to discourage only large current account deficits**.**

A second conclusion is that the intertemporal price rises less steeply when tradedness is endogenous, compared to the standard model with exogenous tradedness. The general insight of

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[25] In these models the kinks in the price response occur because there are a finite number of domestic goods with discontinuously differing trade costs. Hence, as goods shift from being traded or nontraded, export prices jump suddenly.

Dornbusch (1983) is still correct, that the rise in nontraded prices implied by the presence of nontraded goods drives up the intertemporal price. However, when goods can switch in and out of being nontraded, they will tend to do so in a way to minimize this cost. When consumption rises in period 1 and falls in period 2, the share of nontraded goods rises in period 1 to free up more domestic goods for home consumption, and the share of nontraded goods falls in period 2 as the country needs to export more goods to repay its debt. In each case, the endogenous movement in the quantity of nontraded goods partly insulates the price of nontraded goods and thereby the intertemporal price from the shock. The difference between the two models is small for small current account imbalances, where the share of nontraded goods is about the same for both models. But the difference grows for larger current account imbalances, as the share of nontraded goods in the endogenous model deviates more from the steady-state level.

We conclude that the model with a continuum of endogenously traded goods differs from past studies, both those assuming exogenous tradedness and those assuming simple forms of endogeneity with a small number of goods. Not only does the more general model warn that there is no special intertemporal price effect that might kick in to help restore balance when a country reaches an especially large current account imbalance, but to the contrary, the automatic mechanism working through intertemporal prices begins to break down as current account imbalances become progressively larger.

5. Conclusions

This paper has proposed a new way of thinking about nontraded goods in a macro model, focusing on tradedness as an endogenous decision in the face of good-specific trading costs. The paper develops a very tractable way of dealing with this endogeneity, and explores its implications in the context of a simple small open economy macro model. This way of thinking offers an appealing explanation for a long-standing puzzle : the relative price of nontraded goods tends to move with much less volatility than the real exchange rate. This fact stands in contrast to standard theoretical models such as Balassa-Samuelson, which rely almost entirely on such relative price movements.

The paper then shows that the endogeneity of tradedness can have implications for other macroeconomic issues. In particular, the ability of nontraded goods to discourage intertemporal trade will be less severe than in past models, which assumed goods were exogenously nontraded.

We do not view endogenous tradability as the sole explanation for the many puzzles in international macroeconomics. Rather we view our mechanism as complementary to other explanations that suggest roles for sticky prices, nontraded distributive services, vertical production arrangements, etc. In fact, the incorporation of this approach into models with these other features is a fruitful line of research. The mechanism developed here is sufficiently simple that it has the potential for being applied to a wide variety of macro models to analyze a range of macroeconomic issues.

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Table 1: Demand shock, role of b

		Endogenous n				Exogenous n*
(1)	(2)	(3)	(4)	(5)	(6)	(7)
b	$\bar{n}$	$\frac{sdev(p_N/p_T)}{sdev(1/p)}$	$\log\left(\frac{n_1}{n_2}\right)$	$sdev(p_N)$	$sdev(p_T)$	$\frac{sdev(p_N/p_T)}{sdev(1/p)}$
0.1	0.1966	3.5350	0.0458	0.0023	0.0008	5.5679
0.5	0.4507	0.8988	0.0239	0.0059	0.0038	2.5509
1.5	0.5802	0.3810	0.0167	0.0124	0.0103	2.1689
5	0.7184	0.1623	0.0085	0.0209	0.0193	2.0500
10	0.7963	0.1109	0.0025	0.0246	0.0232	2.0251

Benchmark parameter values: $f = 10,\ p^*/a = 1, q = 0.5, t_F = 0.1,\ r = 0$.
Computed for a taste shock that leads to a 1.5% rise in period one consumption.
The volatility of variables, reported as 'sdev,' is computed as the absolute value of the log deviation between the period 1 and steady-state values. For example: $\frac{sdev(p_N/p_T)}{sdev(1/p)} = \left|\log\left(\frac{p_{N1}/p_{T1}}{\bar{p}_N/\bar{p}_T}\right)\Big/\log\left(\frac{\bar{p}}{p_1}\right)\right|$.

*Computed for the corresponding level of $\bar{n}$, to facilitate comparison with the endogenous n case.

Table 2: Demand shock, role of f

		Endogenous n				Exogenous n*
(1)	(2)	(3)	(4)	(5)	(6)	(7)
f	$\bar{n}$	$\frac{sdev(p_N/p_T)}{sdev(1/p)}$	$\log\left(\frac{n_1}{n_2}\right)$	$sdev(p_N)$	$sdev(p_T)$	$\frac{sdev(p_N/p_T)}{sdev(1/p)}$
5	0.5503	0.7787	0.0165	0.0124	0.0083	2.3895
10	0.5802	0.3810	0.0167	0.0124	0.0103	2.1689
20	0.5937	0.1801	0.0169	0.0126	0.0115	2.0815

Benchmark parameter values: $b = 1.5,\ p^*/a = 1,\ q = 0.5,\ t_F = 0.1,\ r = 0$.
Computed for a taste shock that leads to a 1.5% rise in period one consumption.
The volatility of variables, reported as 'sdev,' is computed .as the absolute value of the log deviation between the period 1 and steady-state values.
*Computed for the corresponding level of $\bar{n}$, to facilitate comparison with the endogenous n model.

Fig. 1: Price of good if traded $(p^*/a = 1)$

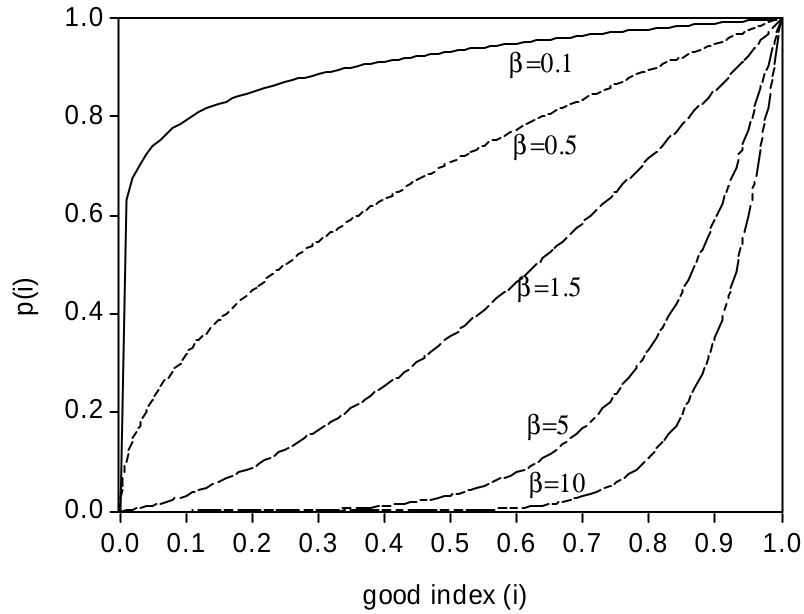


Fig. 2: Aggregate price level of nontraded goods

(shown for $b = 1.5, \quad p^*/a = 1$)

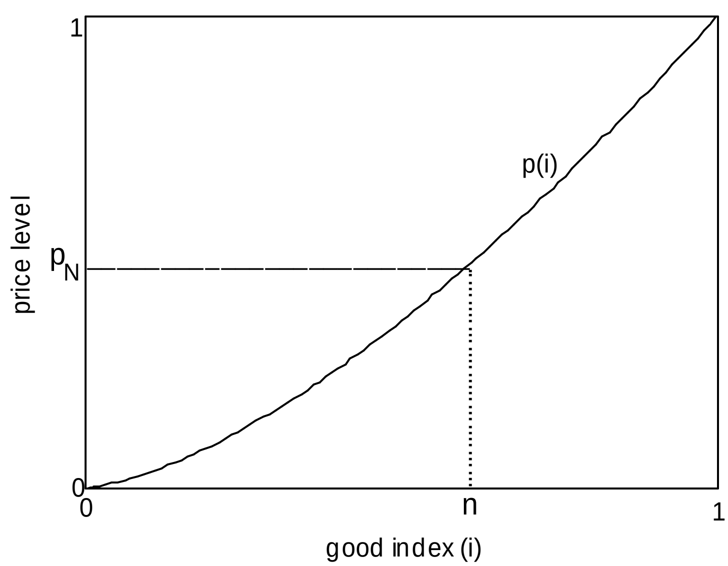


Fig. 3: Price indexes of traded and nontraded goods as a function of n

(shown for $b = 1.5$, $p^*/a = 1$, $f = 10$)

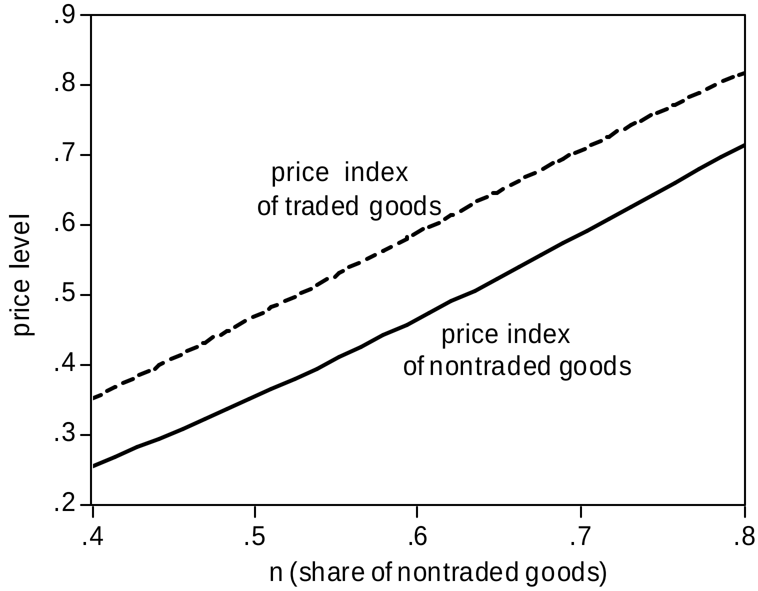
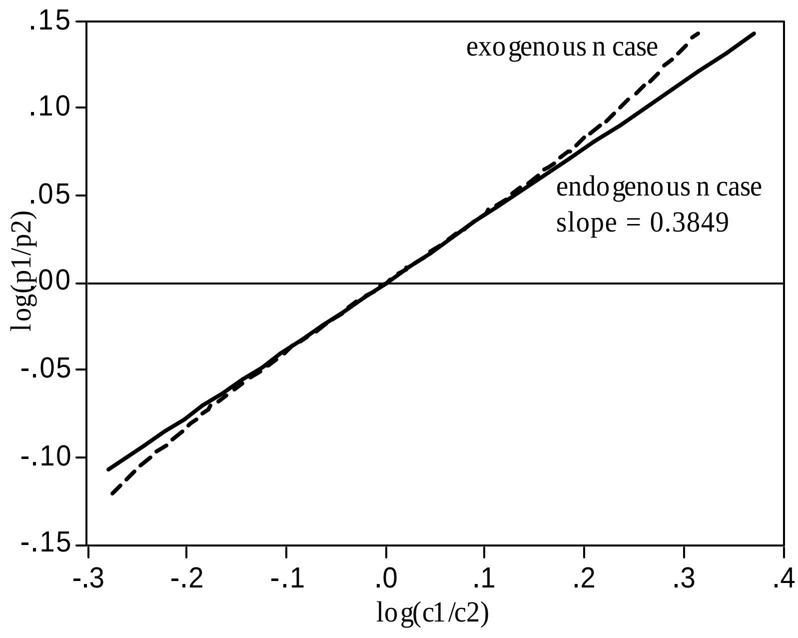


Fig 4: Intertemporal price, log(*p*1/*p*2)



Plotted for $b = 1.5$, $a = 1$, $q = 0.5$, $t_F = 0.1$, Cobb-Douglas preferences, where $\bar{n}$ set to 0.5802 for the exogenous *n* case.

Appendix: Derivation of equilibrium conditions

Combine (8) and (12) to solve out for c_N:

$$c_H = y\left(p_N / p_H \right)^f \tag{A1}$$

Substitute in (A1) for p_N with (7):

$$p_H c_H = \left(p^*/a \right)^f y n^{bf} p_H{}^{1-f} . \tag{A2}$$

Substitute in (4) for p_T with (6) and for p_N with (7):

$$p_H{}^{1-f} = \left(\frac{p^*}{a} \right)^{1-f} \frac{1}{w}\left[n^{-w}(1+w) - 1 \right] \tag{A3}$$

where $w \equiv b(f-1) - 1$. Combine (A3) with (A2) to obtain

$$p_H c_H = \frac{p^* y}{a}\left(\frac{n^{bf}}{w} \right)\left[n^{-w}(1+w) - 1 \right]. \tag{A4}$$

Note next that the domestic value of aggregate home production can be derived as

$$\begin{aligned} p_H y_H &= \int_0^n p_i y_i di + \int_n^1 p_i y_i d i = \int_0^n p_N y di + \int_n^1 p_i y di \\ &= \left(p^*/a \right)\left(n^b \right) ny + y \int_n^1 \left(\frac{p^* i^b}{a} \right) di \\ &= \frac{p^* y}{a} n^{b+1} + \frac{p^* y}{a}\left(\frac{1}{b+1} \right)\left(1 - n^{b+1} \right) \end{aligned}$$

implying

$$p_H y_H = \frac{p^* y}{a}\left[\frac{1 + n^{b+1} b}{1+b} \right]. \tag{A5}$$

With balanced trade, $p_H y_H = pc$. Noting that (10) implies $p_H c_H = q\, pc$ and combining this with the balanced trade condition gives

$$p_H y_H = \left(\frac{1}{q} \right) p_H c_H . \tag{A6}$$

Substituting in (A6) on the lefthand side for $p_H y_H$ with (A5) and on the righthand side for $p_H c_H$ with (A4):

$$\left(\frac{p^* y}{a} \right)\left[\frac{1 + n^{b+1} b}{1+b} \right] = \frac{p^* y}{qa}\left(\frac{n^{bf}}{w} \right)\left[n^{-w}(1+w) - 1 \right].$$

Canceling $p^* y / a$ from both sides, recalling $w \equiv b(f-1) - 1$, and rearranging gives equation (15) in the text, the equilibrium condition for n in the case of a zero trade balance surplus Z:

$$Z \equiv \frac{1 + n^{b+1} b}{1+b} - \frac{1}{wq}\left[n^{b+1}(w+1) - n^{bf} \right] = 0 . \tag{15}$$

To show a unique solution exists for condition (15), it is straightforward to see that for $n=0$, $Z=1/(1+b)>0$, and for $n=1$, $Z=-(1-q)/q<0$. Showing that $\partial Z/\partial n<0$ implies that Z crosses the 0 axis only once and is sufficient to establish the existence of a unique solution for n:

$$\frac{\partial Z}{\partial n}=\frac{(b+1)n^{b}b}{1+b}-\frac{1}{qw}\left[(b+1)n^{b}(w+1)-bfn^{bf-1}\right]$$

$$=\frac{1}{qw}\left[n^{b}b(q-1)w-n^{b}bf(1-n^{w})\right]<0$$

since $q<1$ and $1-n^{w}>0$ for $0<n<1$ and $w>0$.[26]

Given the level of n that implicitly solves condition (15), it is straightforward to solve for the other endogenous variables: first the prices, p_T and p_N through (6) and (7), p_H through (A3), p through (3); and then the quantities, c_N and c_T through (8) and (9), c_H and c_F through (10) and (11), and c through (1).

For the multiperiod case, we introduce time subscripts and solve out for c_{Ht} with (A2) and (10) together to get

$$\frac{y_t}{a^{f}}n_t^{bf}p_{Ht}^{1-f}=q\,p_t c_t. \tag{A7}$$

Substitute in (3) for p_{Ht} with (A3) to get

$$p_t=\frac{1}{a^{q}}\left\{\frac{1}{w}\left[n_t^{-w}(1+w)-1\right]\right\}^{q/(1-f)}p_{Ft}^{1-q}. \tag{A8}$$

Substitute in (A7) for p_{Ht} with (A3) and for p_t with (A8):

$$\frac{y_t}{a}n_t^{bf}\frac{1}{w}\left[n_t^{-w}(1+w)-1\right]=q\frac{1}{a^{q}}\left\{\frac{1}{w}\left[n_t^{-w}(1+w)-1\right]\right\}^{q/(1-f)}p_{Ft}^{1-q}c_t. \tag{A9}$$

Rearranging gives the equations (16) and (17) that express the intratempoal consumption allocation relation between c_t and n_t that holds for each period t =1,2:

$$y_t n_t^{bf}\left\{\frac{1}{w}\left[n_t^{-w}(1+w)-1\right]\right\}^{\frac{1-f-q}{1-f}}=q\,a^{1-q}p_{Ft}^{1-q}c_t. \tag{16,17}$$

Lastly, we rearrange the intertemporal budget constraint (13) to get

$$c_2=\left[(1+r)(p_{H1}y_{H1}-p_1c_1)+p_{H2}y_{H2}\right]/p_2. \tag{A10}$$

Substituting in (A10) for $p_{Ht}y_{Ht}$ with (A5) and for p_t with (A8), t =1,2 gives (18):

$$c_2=\left[(1+r)\left(\frac{y_1\left[1+n_1^{b+1}b\right]}{b+1}-\left(\left\{\left[n_1^{-w}\left(\frac{1+w}{w}\right)-\frac{1}{w}\right]^{\frac{1}{1-f}}\right\}^{q}(a\,p_{F1})^{1-q}c_1\right)\right)+\frac{y_2\left[1+n_2^{b+1}b\right]}{b+1}\right]\bullet\left\{\left[n_2^{-w}\left(\frac{1+w}{w}\right)-\frac{1}{w}\right]^{\frac{-q}{1-f}}\right\}^{-q}(a\,p_{F2})^{q-1}. \tag{18}$$

[26] If $w<0$, then $1-n^{w}<0$, but it is straightforward to see that $\partial Z/\partial n<0$ still.

The system of three equations – (16), (17), and (18) -- can be solved numerically for n_1, n_2, and c_2, given a value of c_1. The Euler equation (14) completes the system.

Addendum for the Referees: Generalizing to Production Economies

While the price implications of endogenous tradability were most transparently demonstrated in an endowment economy, we demonstrate here the robustness of the result to a more general environment, including production and productivity shocks. We begin by allowing production of home goods with a homogenous production function, while maintaining the assumption of heterogeneous trade costs. We then go on to consider an alternative case where productivity, rather than transport costs, vary heterogeneously across goods.

A. Homogenous productivity and heterogeneous transport costs

We introduce output through a Ricardo-Viner specific factors production function which implies a decreasing returns to scale in the variable factor:

$$y_i = A\left(l_i\right)^a, \qquad 0 \le a \le 1 \tag{19}$$

where l_i denotes workers employed in production of each individual good i, and A is a productivity level parameter. This approach is necessary here to ensure a non-degenerate set of traded goods -- the "full specialization problem" -- and is a common device in the trade literature (see Jones, 1971; Samuelson, 1971; and Mussa, 1974 for early examples). Under a constant returns alternative, the small open economy would concentrate production for export in only the final variety in the continuum that has the lowest transport cost. We employ the usual assumption that labor is mobile across sectors within each economy, but immobile internationally.

With perfect competition, marginal costs are equalized to price:

$$p_i = \frac{W}{a y_i / l_i} = \frac{W}{a(A)^{1/a}}\left(y_i\right)^{1/e},$$

implying output of good i is [27]

$$y_i = \left(p_i \ \frac{a(A)^{1/a}}{W}\right)^e, \ e \equiv a/(1-a) > 0 \tag{20}$$

where W denotes the domestic wage rate and e is the price elasticity of output. Output rises as productivity increases or wages decline.

As in the endowment case, the small open economy assumption implies traded goods prices are pinned down by the world price (still normalized to a uniform constant for all goods)

[27] Note $\lim_{a \to 0} a^a = 1$.

and transport costs. Thus the export prices of individual goods p_i are still given by (5) and the price index for traded goods p_T is still given by (6).

To determine the price of nontraded goods, note that (20) implies that the relative supplies for each pair of goods *i* and *j* depend positively on their relative prices:

$$\frac{y_i}{y_j} = \left(\frac{p_i}{p_j}\right)^e,$$

while intratemporal optimization implies their relative demands are

$$\frac{c_i}{c_j} = \left(\frac{p_i}{p_j}\right)^{-f}.$$

Since consumption must equal production of nontraded goods, $c_i / c_j = y_i / y_j$, it follows that $p_i / p_j = 1$, $y_i / y_j = 1$ in equilibrium for $i, j \in \{0, n\}$. In other words, if there are no productivity differences among home goods, then their prices and quantities are identical when they are not traded. (When they are traded, their prices differ if trade costs are heterogeneous.) The uniformity of nontraded prices is the same result derived in the uniform endowment case. As before, the price of the marginally traded good n, $p_n = \left(p^*/a\right)n^b$, pins down the price level of all nontraded goods, and the average price of nontradeds p_N is still given by (7). So the inclusion of production has no effect on the equilibrium condition for nontraded prices, and it can only influence the equilibrium value of these prices via its effects on the share of nontradeds, ***n*,** in that condition.

Output levels reflect the pattern of prices. Inserting the expression for traded goods prices (5) into (20) yields

$$y_i = \left(\frac{p^* i^b}{a}\frac{a(A)^{1/a}}{W}\right)^e \quad i \in \{n,1\} \tag{21}$$

while the price of the marginally traded good n, and the property of uniform nontraded prices imply

$$y_i = y_n = \left(\frac{p^* n^b}{a}\frac{a(A)^{1/a}}{W}\right)^e \quad i \in \{0, n\}. \tag{22}$$

Thus, for given levels of wages and the nontraded share, the output of nontraded goods is constant, while the output of tradeds increases as trade costs fall.

In appendix B we show that the labor market equilibrium condition yields an equation linking wages to the share of nontraded goods and other exogenous variables:

$$W = \frac{p^{*} a A}{a L^{1-a}} \left[\frac{1 + n^{b(1+e)+1} b(1+e)}{b(1+e)+1} \right]^{1-a} \qquad (23)$$

where L is the fixed labor supply. With export prices pinned down by the world market, wages rise in response to an increase in productivity or to an increase in the share of nontraded goods n. In the single period setting the model is closed by the trade balance condition (see the appendix):

$$Z \equiv \frac{1 + n^{b(1+e)+1} b(1+e)}{1 + b(1+e)} - \frac{1}{wq}\left[n^{b(1+e)+1}(w+1) - n^{b(f+e)} \right] = 0 \qquad (24)$$

It is readily apparent that (24) is a generalized version of the trade balance condition derived in the endowment case that determines the equilibrium share of nontraded goods $\bar{n}$. (In the special case that the price elasticity of output e is zero eqn. (24) reduces to eqn. (15)). As in the prior case, with Cobb-Douglas preferences, $\bar{n}$ is independent of homogenous shocks to transport costs a; here it is independent of homogenous productivity level shocks in A as well.

In the multiperiod case allowing unbalanced trade, equilibrium involves solving a set of equations analogous to (16)-(18). However, in addition to solving for n_1, n_2, and c_2, given a value of c_1, now we must also determine wages W_1, W_2. We do so by adding to the system the wage equation (23) for periods 1 and 2. We relegate a listing of this system of equations to appendix B and report in Table 3 the results of a demand shock in the production model. Numerical experiments in this expanded model require calibration of some additional parameters. We set the steady-state level of technology A, and the labor supply L to unity. This implies that as the production scale term a goes to zero, the economy converges to the endowment economy shown earlier in the paper. For the purpose of our experiments, we set a at 0.5. All other parameters are the same as in the endowment model, as reported in Table 1. Once again, the shock is a rise in d sufficient to raise consumption in period one by 1.5%.[28]

The results of the experiment are similar to those in Table 1 for the endowment economy. Increases in b and greater heterogeneity in transport costs dampen movements in the relative price of nontradables to tradables, so it is still true that a low magnitude in the movement of this relative price is possible for the appropriate choice of heterogeneity in transport costs.

Several differences with the endowment economy model are worth highlighting. First, the steady-state share of nontraded goods in column 2 is somewhat higher with production. Intuitively, endogenous production allows the small open economy to take advantage of the heterogeneity in transport costs more fully. Even in the endowment economy trade was

[28] This implies a rise in d of 3.9% for the main case of b =1.5, compared to 4.26% in the endowment model.

concentrated in sectors with low trade costs, though this came at the cost of shifting consumption toward the remaining sectors; now production can be concentrated in these sectors to permit greater exports in these tradable sectors with smaller costs for consumption allocations. The dynamic response of the share of nontrades to the shock is also somewhat smaller than that in the endowment economy, on a percentage basis.

Second, the fact that the values of n differ somewhat from the endowment economy case means that the behavior of prices differs somewhat. In general, the movement in the relative price of nontradeds (column 3) is somewhat higher than that in the endowment economy for each magnitude of b listed. We know from the analytical results above that this difference comes about simply because of the different values of n; conditional on n, the equilibrium conditions for prices are identical in the production and endowment economies. Nevertheless, the differences in prices are rather small, and they still follow a steady downward trajectory as the magnitude of b rises.

Since we now have a model with endogenous production, it is natural to consider another type of experiment, involving a shock to the production function rather than the demand condition. Table 4 presents numerical results for a shock that raises the technology term A in period 2 by 1.5 percent. (This shock raises equilibrium output in period 2 by 1.32 percent for the benchmark case of $b = 1.5$). Note that this is the type of supply shock considered by Dornbusch (1983).

Results for a productivity shock are extremely similar to those for the demand shock. While the movements in the share of nontraded goods are different, they still move in the manner needed to buffer the change in nontraded prices and facilitate movements in the price of traded goods. Once again the role play by nontraded prices in column (3) varies inversely with b, and low magnitudes of relative price movement are possible for the appropriate choice of this heterogeneity parameter. We conclude that our main insight from the simple endowment economy extends to a model including production and even including supply shocks.

B. Heterogeneous productivity and homogeneous transport costs

While the focus of this paper is on the important role of heterogeneity in terms of trade costs, given that the preceding literature has focused on heterogeneity in terms of productivities, we briefly consider this alternative here.

In particular, we assume a production function

$$y_i = A_i \left(l_i \right)^a, \qquad 0 \le a \le 1 \tag{19'}$$

where productivity is an increasing function of i

$$A_i = A\, i^{b_A}$$

with A denoting the homogenous component constant across goods and b_A capturing the degree of heterogeneity across varieties.

To highlight the role of productivity differences we also assume trade costs are homogenous, i.e. $b = 0$ in (5), implying the prices of all traded goods are identical:

$$p_i = \frac{p^*}{1+t_i} = \frac{p^*}{a} \qquad \text{for} \quad i \in \{n,1\}. \tag{5'}$$

Hence

$$p_T = \frac{p^*}{a}. \tag{6'}$$

Thus the average price of traded goods is now independent of n.

The equalization of marginal costs and price implies output of each good depends on its productivity

$$y_i = \left(p_i \, \frac{a(A_i)^{1/a}}{W} \right)^e \tag{20'}$$

implying in turn

$$\frac{y_i}{y_j} = \left(\frac{p_i}{p_j} \right)^e \left(\frac{A_i}{A_j} \right)^{1+e}.$$

It follows from the equilibrium condition for nontradeds $c_i / c_j = y_i / y_j$ that (noting that $e \equiv a/(1-a)$ implies $1 + e = e/a$)

$$\frac{p_i}{p_j} = \left(\frac{A_i}{A_j} \right)^{-\frac{e+1}{e+f}}, \; \frac{y_i}{y_j} = \left(\frac{A_i}{A_j} \right)^{f\left(\frac{e+1}{e+f}\right)} \qquad \text{for} \quad i \in \{0,n\}$$

or, for $j = n$

$$p_i = p_n \left(\frac{A_i}{A_n} \right)^{-\frac{e+1}{e+f}} = p_n \left(\frac{i}{n} \right)^{-b\left(\frac{e+1}{e+f}\right)}, \qquad y_i = y_n \left(\frac{A_i}{A_n} \right)^{f\left(\frac{e+1}{e+f}\right)} = y_n \left(\frac{i}{n} \right)^{b\, f\left(\frac{e+1}{e+f}\right)} \tag{25}$$

Thus, in contrast to the case with homogenous productivity, heterogeneous productivity implies that the prices and output of the intramarginal nontraded goods differ, with their prices falling and output rising as productivity increases across varieties. It follows that the price index of nontraded goods is

$$p_N = \left(\left(\frac{1}{n} \right) \left[\int_0^n (p_i)^{1-f} \, di \right] \right)^{1/(1-f)} = \left(\left(\frac{1}{n} \right) \left[\int_0^n \left(p_n \left(\frac{A_i}{A_n} \right)^{-\frac{1+e}{e+f}} \right)^{1-f} di \right] \right)^{1/(1-f)}$$

$$= p_n \left(\left(\frac{1}{n} \right) \left[\int_0^n \frac{Ai^{b_A \frac{1+e}{e+f}(f-1)}}{An^{b_A \frac{1+e}{e+f}(f-1)}} di \right] \right)^{1/(1-f)} = p_n \left(b_A \left(\frac{1+e}{e+f} \right) (f-1) + 1 \right)^{1/(f-1)} \qquad (7')$$

where the appearance of n in the integral cancels that in the multiplicative weighting term, so that n is eliminated in the final form of the expression (7'). Since p_n, the price of the marginally traded good, is pinned down by its export price, p^*/a, which as noted above is independent of n, the price index of nontradeds is independent of n as well. Because p_T and p_N depend only on constant parameters, their ratio, the relative price of nontradeds p_N / p_T, is clearly invariant to shocks, even if the tradability of goods changes. This does not mean that individual goods prices are invariant; solving for the price of individual nontraded goods prices (see eqn. 25) indicates that these goods prices all rise when a shock raises n. (The mechanism through which this occurs is rising wages, which are passed on to higher marginal costs.) But it is this same shift in n that guarantees that the aggregate price index of nontraded goods does not change. In particular, as the distribution of prices of individual nontraded goods shifts up, the support of this distribution expands to include a new set of cheaper goods, enough to hold constant the average price among nontradeds as a group.

Nor does this result imply that there are no effects on broader price aggregates. Comparison of (6') and (7') indicates $p_N > p_T$. Since nontraded goods by definition display lower average productivity than traded goods, their prices are higher. This contrasts with our earlier analysis where traded prices always exceed nontraded prices (because the latter are pinned down by the marginal traded good which has a *lower* price than that of traded goods on average). Given this fact, inspection of (4) implies that an increase in n raises p_H since it raises the weight of the higher priced nontradeds in the home goods basket. And this in turn raises the overall price level p. Further details of the model in this case are relegated to appendix C.

Since the analytical equations (6') and (7') make clear that shocks have no effect on the relative price of nontraded goods through changes in tradability regardless of the value of the heterogeneity parameter b_A, there is little benefit in presenting numerical simulations in tables analogous to Tables 3 and 4. However, such numerical simulations do confirm that a demand shock raises the share of nontraded goods as the rise in demand in period 1 generates a current account deficit in that period. Similarly, if we consider the same experiment as in Table 4 of a rise in period two technology A_2, this also has very similar effects: an anticipated rise in future output generates a rise in current consumption and hence a current account deficit, which again raises the endogenous share of nontraded goods. The price effects discussed above then naturally ensue.

We conclude that, despite the very different nature of this model with productivity heterogeneity, it generates results broadly consistent with our main insight from the earlier models with endowments or production with homogeneous productivity. Once again it is adjustment in the endogenous share of nontraded goods that buffers the effect of shocks on the relative price of nontraded goods. In fact for the case considered with heterogeneous productivity, the movement in n completely neutralizes the effect that any movement in individual goods prices will have on the nontraded price aggregate. Further, similar to our previous results, this fact does not rule out movements in the overall national price level. The particular mechanism generating this portion of the result is somewhat different in this model specification, in that it no longer is a matter of nontraded and traded prices moving together, but the fact that nontraded prices receive a greater weight in the overall aggregate. But again it is the endogenous movement in the nontraded margin n that facilitates this result.

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Table 3: Demand shock in production economy, role of b

(1)	(2)	(3)	(4)	(5)	(6)
b	$\bar{n}$	$\frac{sdev(p_N/p_T)}{sdev(1/p)}$	$\log\left(\frac{n_1}{n_2}\right)$	$sdev(p_N)$	$sdev(p_T)$
0.1	0.2214	3.3072	0.0417	0.0021	0.0008
0.5	0.4974	0.9302	0.0196	0.0049	0.0031
1.5	0.6490	0.4487	0.0115	0.0086	0.0068
5	0.7948	0.2225	0.0049	0.0122	0.0109
7.5*	0.8363	0.1845	0.0035	0.0130	0.0118

Benchmark parameter values: $f = 10$, $p^*/a = 1, q = 0.5, t_F = 0.1, r = 0$, $A = 1$, $L = 1$, $a = 0.5$.
Computed for a taste shock that leads to a 1.5% rise in period one consumption.
The volatility of variables, reported as 'sdev,' is computed as the absolute value of the log deviation between the period 1 and steady-state values.
* Numerical solutions converge for values of b only up to 7.5.

Table 4: Productivity shock in production economy

(1)	(2)	(3)	(4)	(5)	(6)
b	$\bar{n}$	$\frac{sdev(p_N/p_T)}{sdev(1/p)}$	$\log\left(\frac{n_1}{n_2}\right)$	$sdev(p_N)$	$sdev(p_T)$
0.1	0.2214	3.3173	0.0202	0.0010	0.0004
0.5	0.4974	0.9293	0.0086	0.0021	0.0013
1.5	0.6490	0.4475	0.0045	0.0034	0.0027
5	0.7948	0.2219	0.0017	0.0044	0.0039
9*	0.8531	0.1704	0.0010	0.0046	0.0042

Benchmark parameter values: $f = 10$, $p^*/a = 1, q = 0.5, t_F = 0.1$, $r = 0$, $A = A_1 = 1, L = 1, a = 0.5$.
Computed for a rise in A in period 2 by 1.5%.
The volatility of variables, reported as 'sdev,' is computed as the absolute value of the log deviation between the period 1 and steady-state values.
* Numerical solutions converge for values of b only up to 9.

Technical Appendix to the Addendum

A. Production economy with homogenous productivity

Equations (1) to (11) continue to apply when the model includes production. Given the property that $y_i = y_n$ and $p_i = p_n = p_N$ for $i \in \{0, n\}$ when productivity is homogenous, the market-clearing condition for nontradeds (12) becomes

$$c_N = \int_0^n y_i \frac{p_i}{p_N} di = ny_n \tag{B12}$$

where $y_n \equiv \left(\frac{p^* n^b}{a} \frac{a\,(A)^{1/a}}{W} \right)^e$ is the output of the marginally traded good n, $e \equiv a/(1-a)$, a is the output elasticity of labor in the production function (19). Following the same sequence of substitutions in deriving (A1)-(A4) yields

$$c_H = y_n \,(p_N / p_H)^f \tag{B1}$$

$$p_H c_H = y_n \left(\frac{p^*}{a} \right)^f n^{bf} p_H{}^{1-f} \tag{B2}$$

$$p_H{}^{1-f} = \left(\frac{p^*}{a} \right)^{1-f} \frac{1}{w} \left[n^{-w} (1+w) - 1 \right] \tag{B3}$$

$$p_H c_H = \frac{y_n p^*}{a} \left(\frac{n^{bf}}{w} \right) \left[n^{-w} (1+w) - 1 \right] \tag{B4}$$

where $w \equiv b(f-1) - 1$. (Comparing with (A1)-(A4), note that y_n replaces y in (B1), (B2), and (B4), while (A3) and (B3) are identical.) The value of aggregate home production can be derived as :

$$p_H y_H = \int_0^n p_i y_i di + \int_n^1 p_i y_i d\, i = \int_0^n p_n y_n di + \int_n^1 \left(\frac{i^b p^*}{a} \right) \left(\frac{p^* i^b}{a} \frac{a(A)^{1/a}}{W} \right)^e di$$

$$= np_n y_n + \left(\frac{1}{a} \right)^{1+e} \left(\frac{a(A)^{1/a}}{W} \right)^e \left(p^* \right)^{1+e} \frac{\left(1 - n^{b(1+e)+1} \right)}{b(1+e)+1}$$

by again utilizing the property that $p_i = p_n = p_N$ and $y_i = y_n$ for $i \in \{0, n\}$ and by substituting for p_i, y_i, $i \in \{n, 1\}$ with (5) and (21), respectively. Substituting in for $p_n = \frac{p^* n^b}{a}$,

$y_n \equiv \left(\frac{p^* n^b}{a} \frac{aA^{1/a}}{W} \right)^e$ then gives

$$p_H y_H = \left(\frac{a(A)^{1/a}}{W} \right)^e \left(\frac{p^*}{a} \right)^{1+e} \left[\frac{1 + b(1+e) n^{b(1+e)+1}}{1 + b(1+e)} \right]. \tag{B5}$$

The trade balance condition can be derived by first substituting for y_n in (B4) and then substituting the resulting expression for $p_H c_H$ on the righthand side of (A6) and substituting (B5) for $p_H y_H$ on the lefthand side of (A6) to obtain:

$$\left(\frac{a(A)^{1/a}}{W}\right)^{e}\left(\frac{p^*}{a}\right)^{1+e}\left[\frac{\left(b(1+e)\right)n^{b(1+e)+1}+1}{b(1+e)+1}\right]=\frac{1}{qa}p^*\left(\frac{p^*}{a}\frac{a(A_n)^{1/a}}{W}\right)^{e}\left(\frac{n^{b(e+f)}}{w}\right)\left[n^{-w}(1+w)-1\right].$$

Cancelling the term $(p^*/a)^{1+e}$ from both sides and rearranging gives the trade balance condition (24) in the text:

To derive the wage equation, substitute for l_i with the production function (19) into the labor market equilibrium condition $\int_0^n l_i di + \int_n^1 l_i di = L$, implying

$$\int_0^n \left(\frac{y_i}{A}\right)^{1/a} di + \int_n^1 \left(\frac{y_i}{A}\right)^{1/a} di = L.$$

Further substitution for y_i with (21) and (22) gives

$$\int_0^n \left(\frac{\left(\left(\frac{p^*}{a}\right)\frac{n^b a(A)^{1/a}}{W}\right)^{e}}{A}\right)^{1/a} di + \int_n^1 \left(\frac{\left(\frac{p^* i^b}{a}\frac{a(A)^{1/a}}{W}\right)^{e}}{A}\right)^{1/a} di = L$$

which, upon noting $e/a = 1+e$ and integrating, results in

$$\left(\frac{p^* a A}{aW}\right)^{1+e} n^{b(1+e)} n + \left(\frac{p^* aA}{aW}\right)^{1+e}\frac{1}{b(1+e)+1}\left(1-n^{b(1+e)+1}\right)=L.$$

Solving for W and noting $1/(1+e)=1-a$ gives expression (23) in the text.

To derive the analogues to (16), (17), and (18) in the multiperiod case, we reintroduce time subscripts and solve out for c_{Ht} with (B2) and (10) together to get

$$\left(\frac{p^*}{a}\right)^{f} y_{nt} n_t^{bf} p_{Ht}^{1-f} = q p_t c_t. \tag{B7}$$

Next substitute in (3) for p_{Ht} with (B3) to get

$$p_t = \left(\frac{p_t^*}{a}\right)^{q}\left\{\frac{1}{w}\left[n_t^{-w}(1+w)-1\right]\right\}^{q/(1-f)} p_{Ft}^{1-q} \tag{B8}$$

which is identical to (A8). Substituting in (B7) for p_{Ht} with (B3) and for p_t with (B8) gives an gives expression equivalent to (A9) with y replaced by y_n. Rearranging gives the equations (B16) and (B17) that express the intratemporal consumption allocation relation between c_t and n_t that holds for each period t = 1, 2:

$$y_{nt} n_t^{bf}\left\{\frac{1}{w}\left[n_t^{-w}(1+w)-1\right]\right\}^{\frac{1-f-q}{1-f}} = q\left(\frac{p_{Ft}}{p_t^*/a}\right)^{1-q} c_1. \tag{B16, B17}$$

where (recall) $y_{nt} \equiv \left(\left(\frac{p_t^*}{a}\right)\left(n_t^{\,b}\right)\frac{a\left(A_t\right)^{1/a}}{W_t}\right)^e$. Lastly, substitute for $p_{Ht}y_{Ht}$ with (B5) and for p_t with (B8) in the rearranged intertemporal budget constraint (A9) to get (B18):

$$\begin{aligned} c_2 = \Bigg\{ & (1+r)\left(\left(\frac{p_1^*}{a}\right)^{1+e}\left(\frac{a(A_1)^{1/a}}{W_1}\right)^{e}\frac{1+b(1+e)n_1^{\,b(1+e)+1}}{1+b(1+e)} - \left[(n_1)^{-w}\left(\frac{1+w}{w}\right)-\frac{1}{w}\right]^{\frac{q}{1-f}}\left(\frac{p_1^*}{a}\right)^{q}(p_{F1})^{1-q}c_1\right) \\ & + \left(\frac{p_2^*}{a}\right)^{1+e}\left(\frac{a(A_2)^{1/a}}{W_2}\right)^{e}\frac{1+b(1+e)n_2^{\,b(1+e)+1}}{1+b(1+e)} \Bigg\} \\ & \bullet \left\{\left[(n_2)^{-w}\left(\frac{1+w}{w}\right)-\frac{1}{w}\right]^{\frac{-q}{1-f}}\left(\frac{p_2^*}{a}\right)^{-q}(p_{F2})^{q-1}\right\} \end{aligned}$$

The system of five equations – (B16), (B17), (B18) and the wage equation (23) for periods 1 and 2 – can be solved numerically for n_1, n_2, c_2, W_1, W_2, given a value of c_1.

B. Production economy with heterogeneous productivity

Equations (1) – (11) continue to apply as in the homogenous productivity case. Using (25) to substitute for p_i and y_i in (12), it follows that

$$\begin{aligned} c_N &= \int_0^n y_i \frac{p_i}{p_N}\,di = \frac{1}{p_N}\int_0^n y_n\left(\frac{i}{n}\right)^{b_A f\left(\frac{1+e}{e+f}\right)} p_n\left(\frac{i}{n}\right)^{-b_A\left(\frac{1+e}{e+f}\right)}di \\ &= \frac{y_n p_n}{p_N}\int_0^n\left(\frac{i}{n}\right)^{b_A\left(\frac{(f-1)(1+e)}{e+f}\right)}di = \frac{y_n p_n}{p_N}\left(\frac{1}{b_A\left(\frac{(1+e)(f-1)}{e+f}\right)+1}\right)n. \end{aligned}$$

which, after substituting for p_N with (7'), yields

$$\begin{aligned} c_N &= \frac{y_n p_n}{p_n\left(b_A\frac{(1+e)(f-1)}{e+f}+1\right)^{1/(f-1)}}\left(\frac{1}{b_A\left(\frac{(1+e)(f-1)}{e+f}\right)+1}\right)n \\ &= \left(\frac{1}{1+w_A}\right)^{f/(f-1)} n y_n \end{aligned} \qquad \text{(C12)}$$

where $w_A = b_A\frac{(1+e)(f-1)}{e+f} > 0$, $p_n = \frac{p^*}{a}$, $y_n \equiv \left(\frac{p^*}{a}\frac{a(An^{b_A})^{1/a}}{W}\right)^e$.

The analogue expressions obtained for (A1)-(A4) are given by[29]

$$c_H = y_n (1+w_A)^{\frac{-f}{f-1}} (p_N / p_H)^f \tag{C1}$$

$$p_H c_H = \left(\frac{p^*}{a}\right)^f (p_H)^{1-f} y_n \tag{C2}$$

$$p_H{}^{1-f} = \left(\frac{p^*}{a}\right)^{1-f} \left[1-n\left(1-\frac{1}{1+w_A}\right)\right] \tag{C3}$$

$$p_H c_H = \left(\frac{p^* y_n}{a}\right)\left[1-n\left(1-\frac{1}{1+w_A}\right)\right]. \tag{C4}$$

Following the same sequence of substitutions as in prior variants of the model, we can then derive expressions for the value of aggregate home output:

$$p_H y_H = \left(\frac{a(A)^{1/a}}{W}\right)^e \left(\frac{(p^*)}{a}\right)^{1+e} \left(\frac{1-\frac{w_f}{w_e}+\frac{w_f}{w_e} n^{b_A(1+e)+1}}{1+w_A}\right) \tag{C5}$$

the trade balance:

$$Z = \frac{1}{w_e} + \frac{(w_f / w_e)}{1+w_A} n^{b_A(1+e)+1} - \left(\frac{1}{q}\right)\left[n^{b_A(1+e)} - n^{b_A(1+e)+1}\left(1-\frac{1}{1+w_A}\right)\right] \tag{C15}$$

intratemporal consumption allocation:

$$y_{nt}\left[1-n_t\left(1-\frac{1}{1+w_A}\right)\right]^{\frac{1-f-q}{1-f}} = q\left(\frac{p^*}{a}\right)^{q-1} p_{Ft}{}^{1-q} c_t \tag{C16, C17}$$

budget constraint (C18):

$$c_2 = \left[(1+r)\left(\left(\frac{p_1^*}{a}\right)^{1+e}\left(\frac{a(A_1)^{1/a}}{W_1}\right)^e\left[\frac{1-(w_f/w_e)+(w_f/w_e)(n_1)^{w_e}}{1+w_A}\right] - \left\{1-n_1\left(1-\frac{1}{1+w_A}\right)\right\}^{q/(1-f)}\left(\frac{p_1^*}{a}\right)^q p_{F1}{}^{1-q} c_1\right)\right.$$

$$\left. + \left(\frac{p_2^*}{a}\right)^{1+e}\left(\frac{a(A_2)^{1/a}}{W_2}\right)^e\left[\frac{1-(w_f/w_e)+(w_f/w_e)(n_2)^{w_e}}{1+w_A}\right]\right] \bullet \left\{1-n_2\left(1-\frac{1}{1+w_A}\right)\right\}^{-q/(1-f)}\left(\frac{p_2^*}{a}\right)^{-q} p_{F2}{}^{q-1}$$

and the wage equation:

[29] Solving out for c_N with (8) and (C12) gives (C1). Substituting in (C1) in turn for p_N with (7') gives (C2). Substituting into (4) with expressions for p_T (6') and p_N (7') gives (C3). Combining (C2) and (C3) gives (C4).

$$W_t = \left(\frac{p_t^* a A_t}{a L^{1-a}} \right) \left(\frac{1 - \left(w_f / w_e \right) + \left(w_f / w_e \right) \left(n_t \right)^{b_A (1+e)+1}}{1 + w_A} \right)^{\frac{1}{1+e}} \tag{C23}$$

where $w_e \equiv b_A(1+e)+1>1$, $w_f \equiv b_A\left(\frac{1+e}{e+f}\right)(1+e)>0$, $w_A = b_A \frac{(1+e)(f-1)}{e+f} = w_e - w_f - 1 > 0$,

$e \equiv \frac{a}{1-a} > 0$, and $y_n \equiv \left(\frac{p^*}{a} \frac{a(An^{b_A})^{1/a}}{W} \right)^e$.