Topic 8: Exchange Rate Risk and Welfare

Part 1: Empirical evidence on Risk Premia and UIP

a. Background theory: Recall from earlier lectures:

Uncovered interest rate parity:

$$1+i_t = \frac{1}{e_t}\left(1+i_t^*\right)E_t\left[e_{t+1}\right]$$

or:

$$e_t\left(\frac{1+i_t}{1+i_t^*}\right) = E_t\left[e_{t+1}\right]$$

Or if take logs and use approximations:

$$i_t = i_t^* + E_t\left[\tilde{e}_{t+1}\right] - \tilde{e}_t = i_t^* + \%\,\text{e xpected depretiation}$$

Forward exchange rate efficiency:

$$f_t = E_t \left[e_{t+1} \right]$$

Covered interest rate parity: putting the two together:

$$e_t \left(\frac{1 + i_t}{1 + i_t^*} \right) = f_t$$

Consider a more formal derivation

Assets:

M_t home money

B_t home nominal interest bearing assets, at rate i

B_t* foreign nominal interest bearing asset, at rate i*

F_t forward contract: purchase one unit of foreign currency next period in exchange for f_t units of home currency (f_t known in period t)

Household problem

$$Max \;\; E_t \sum_{t=0}^{\infty} \beta^t U\left(C_t, \frac{M_t}{P_t} \right)$$

$$s.t. \;\; P_t Y_t + \left(1+i_{t-1}\right) B_{t-1} + e_t \left(1+i^*_{t-1}\right) B^*_{t-1} + \left(e_t - f_{t-1}\right) F_{t-1} + M_{t-1}$$

$$= P_t C_t + B_t + e_t B_t^* + M_t$$

FOCS:

(lambda for lagrange multiplier on budget constraint.)

(1) Home bonds: $\beta E_t[\lambda_{t+1}](1+i_t)=\lambda_t$

(2) Foreign bonds: $\beta E_t[e_{t+1}\lambda_{t+1}](1+i^*_t)=e_t\lambda_t$

(3) Forward exchange: $E_t[(e_{t+1}-f_t)\lambda_{t+1}]=0$

Or $E_t[e_{t+1}\lambda_{t+1}]=fE_t[\lambda_{t+1}]$

Consumption (lambda): $\lambda_t=\dfrac{U'_{c,t}}{P_t}$

Combine equations:

(4) Covered interest rate parity: combine (1), (2) and (3):

$$e_t\left(\frac{1+i_t}{1+i_t^*}\right)=f_t$$

This is same as previously.

(5) Uncovered interest rate parity: use (2) and (1):

$$e_t\left(\frac{1+i_t}{1+i_t^*}\right)=E_t\left[e_{t+1}\frac{U'_{c,t+1}}{P_{t+1}}\right]\Big/E_t\left[\frac{U'_{c,t+1}}{P_{t+1}}\right]$$

Is more complicated than simple version given before

(6) Market efficiency condition: rewrite (3):

$$f_t = E_t\left[e_{t+1}\frac{U'_{c,t+1}}{P_{t+1}}\right] \Big/ E_t\left[\frac{U'_{c,t+1}}{P_{t+1}}\right]$$

Also is different from previously; reflects a desire to smooth consumption.

Rewrite to compare to earlier equations:

Risk premium: rewrite (6) to split up expectations:

$$f_t = E_t\left[e_{t+1}\right] + \mathrm{cov}_t\left[e_{t+1}, \frac{U'_{c,t+1}}{P_{t+1}}\right] \Big/ E_t\left[\frac{U'_{c,t+1}}{P_{t+1}}\right]$$

$$= E_t\left[e_{t+1}\right] + RP_t$$

and Uncovered interest rate parity: $e_t\left(\frac{1+i_t}{1+i_t^*}\right) = E_t\left[e_{t+1}\right] + RP_t$

Interpretation:

Both involve a risk premium; may require a higher return on an asset that is riskier.

Risk is a matter of how the value of the asset fluctuates in relationship to things that the households value, like consumption.

Consider a positive risk premium:

Implies the covariance of the exchange rate and marginal utility is positive. Means that foreign assets pay off well during bad times.

Therefore foreign assets are a good hedge against consumption risk.

Note that even if households are risk neutral (U' is constant), we still have:

$$f_t = E_t[e_{t+1}] + \text{cov}_t\left[e_{t+1}, \frac{1}{P_{t+1}}\right] \Big/ E_t\left[\frac{1}{P_{t+1}}\right]$$

So the forward rate still is not equal to the expected future spot rate. This extra term is often called a "Jensen inequality term"

b. Empirical literature

Early Tests of Interest parity and market efficiency

Framework:

The earliest tests of forward efficiency regressed the future spot rate on the forward rate in logs:

$$e_{t+1} = a_0 + a_1 f_t + \varepsilon_t$$

Tested if $a_0=0$ and $a_1=1$. Null hypothesis is that the forward rate provides an unbiased forecast of the future spot exchange rate.

Note the equation tested here replaces the expected value of e_{t+1} in the original condition with the actual future value, due to lack of data on expectations.

Another problem is nonstationarity of exchange rate. Most researchers subsequently have tested the equation with e_t subtracted on each side:

$$(e_{t+1} - e_t) = a_0 + a_1 (f_t - e_t) + \varepsilon_t$$

Related test: Since covered interest parity holds well, replace the right term with the interest rate differential. We then have a test of uncovered interest parity:

$$(e_{t+1} - e_t) = a_0 + a_1 (i_t - i^*_t) + \varepsilon_t$$

If the risk premium assumed to be constant, it would appear in the a_0 term, making it deviate from the hypothesized value of zero. But it should not affect the a_1 term. So the researchers tended to focus on the hypothesis that a_1=1.

Results:

Froot (1990 JEPerspectives:) Summarize the literature of 75 papers on the subject. The average estimate of a1 over 75 papers is -0.88, only a few find $a_1>0$, and none find $a_1>1$.

Conclusions:

In general, papers reject the hypothesis that $a_1=1$, and most find that it is actually negative.

This means that a country's currency is expected to appreciate in future periods when its interest rate is high.

Rather than offsetting a high interest rate, future appreciation makes it even more profitable to buy a currency.

How can this be consistent with market equilibrium? How can we explain this finding?

An explanation : Time-varying risk premia

If the risk premium varies over time, we could write the equation from before:

$$\left(e_{t+1} - e_t\right) = a_0 + a_1\left(f_t - e_t\right) - RP_t + \varepsilon_t$$

Where RP_t and e_t together are the error term in the regression. So the error term includes a component that may be correlated with regressors, and this would bias the estimate of a_1.

To solve this problem, some researchers have tried to use ARCH models to model the risk premium. (Domowitz and Hakkio, 1995 JIE).

They propose a separate regression for the risk premium itself as a function of the interest rate differential.

But results were not encouraging. It appears that the risk premium may well be time-varying, but it is not a simple function of the interest rate differential.

For example, recall the results in Eichenbaum and Evans (QJE). The forward premium there appeared to move conditionally on monetary policy shocks.

GMM tests:

Several tests in late 1980s and 1990s have estimated the nonlinear Euler equations above, by GMM methods.

The idea is to test the conditions in a form where the risk premium is still built in.

General Method: rearranging condition (6):

$$E_t\left[\left(e_{t+1} - f_t\right) \frac{U'_{c,t+1}}{P_{t+1}} \right] = 0$$

The term inside brackets may be regarded as an excess return on forward exchange, and the condition states that this excess return should be zero.

This implies that any excess return that exists should not be predictable using any information know in period *t*.

Can test this by taking a vector of various instruments z_t know at t and requiring:

$$E_t\left[(e_{t+1} - f_t)\frac{U'_{c,t+1}}{P_{t+1}} z_t\right] = 0$$

This forms a set of restrictions that can be used to estimate parameters in the utility function (risk aversion), and test the equation.

<u>The GMM estimator</u> is the value which minimizes the quadratic form

$$J = g'Wg$$

Where g is the sample counterpart of the set of orthogonality conditions above, and W is a weighting matrix of the orthogonality conditions.

Can use *J* to evaluate the validity of the overidentifying restrictions. When multiplied by the number of regression observations, J is asymptotically chi-square.

Mark (1985 JME): This method first done by Mark.

Data: Work with four currencies simultaneously: Can dollar, DM, Neth guilder, UK pound, all relative to the U.S. dollar. Monthly data 1973-83.

Instruments:

For instruments want variables which might reasonably be of importance to agents in solving their forecasting problem: current consumption and current and past movements in the exchange rates.

Use two versions of each instrument:

Consumption: nondurables with services and without.

Exchange rates:

1) Current and past value for realized profits from foreign exchange speculation on currencies (gap between forward rate and actual spot rate one month later).

2) Current and past forward premia on the currencies (gap between forward rate and actual current spot rate).

Choose to use range of lags for the instruments.

Note: assume a CRRA utility, so need to estimate intertemporal elasticity (1/gamma)

Results:

Estimates of risk aversion are large (intertemporal elasticities small).

Instrument list #1 (using speculative profits): Cannot reject model for any of the lags considered, for either definition of consumption. (*Tmin* is the statistic distributed chi-square, the minimized value of distance function *J*, times the sample size).

Instrument list #2 (using forward premium): Can reject for case of zero lags (only current values of the instruments), but not for cases including lags.

Conclude: Mixed result: Reject model in a few cases (when forward premium is used as an instrument).

Speculative profits used as instrument:

Consumption	lag	gamma	J-stat	Confidence
NDS	0	1.4	19.2	0.56
	1	40.0	41.3	0.79
	2	50.4	59.8	0.81
ND	0	0.0	19.0	0.54
	1	2.8	41.2	0.78
	2	7.3	60.2	0.82

Forward premium used as instrument:

Consumption	lag	gamma	J-stat	Confidence
NDS	0	43.5	32.3	0.97
	1	45.0	43.2	0.84
	2	41.5	56.5	0.72
ND	0	17.5	33.2	0.98
	1	13.8	44.4	0.87
	2	12.7	57.1	0.74

Other papers:

Hodrick (1989 JME): more recent data and more countries (seven). Uses quarterly data from 73:III-87:IV. Also estimates large values of gamma in most cases. Not reject.

Modjtahedi (1991 JIE) Again similar to Mark (1985), but look at other maturities (1,3, and 6 months). Reject the model, a stronger rejection than Mark (1985).

Backus, Gregory and Telmer (1993 JOF): Try with other utility functions than CRRA – habit persistence. Reject the model.

Overall conclusion from the GMM literature: Very mixed results.

Generally consumption is not variable enough to explain the high risk premium, unless it uses a suspiciously high value of risk aversion for utility.

Questions for Discussion:

1) Compare and contrast these tests of UIP with the VAR studies we read earlier.

2) Could you make money off of these findings?

3) Does your study of related financial market topics in macro, such as the equity premium puzzle or term premium puzzle, give you any insight into UIP failures?

Part 2: A theoretical model of UIP deviations
a) Obstfeld and Rogoff (2001)

Motivation

- Include risk in a sticky price model. Try to solve model without linearization, which implies risk neutrality.

- After all, the main motivation for having fixed exchange rates is the fact that people don't like exchange rate uncertainty.

  Interested in questions:
  1) How does a model that includes sticky prices help us explain the forward premium puzzle?

  2) What are the welfare implications of exchange rate variability, and does this provide a justification for having fixed exchange rate regimes?

Model

Very similar to Obstfeld and Rogoff (1995 JPE).

- Two countries. Not necessarily same size. Home is size n, and foreign is (1-n).

- Home preferences separable in consumption (C), real money balances (M/P), and output (y) representing lost leisure:

$$U_t = \frac{1}{1-\rho} C_t^{1-\rho} + \frac{\chi}{1-\varepsilon} \frac{M^{1-\varepsilon}{}_t}{P_t} - \frac{1}{2} y_t^2$$

More general than log utility of their 1995 paper:
Interest elasticity of money is $1/\varepsilon$,
Interest elasticity of consumption $1/\rho$

Sub-utility over consumption also a bit different.
Rather than CES over all goods, consumption is Cobb-Douglas over home and foreign sub-aggregates:

$$C = C_H^n C_F^{1-n} \quad \text{where}$$

$$C_H = \left[\int_0^n c(z)^{\frac{\theta-1}{\theta}} dz \right]^{\frac{\theta}{\theta-1}} \qquad C_F = \left[\int_n^1 c(z)^{\frac{\theta-1}{\theta}} dz \right]^{\frac{\theta}{\theta-1}}$$

Budget constraint is standard from 1995 paper:

$$P_t B_{t+1} + M_t = P_t (1 + r_t) B_t + M_{t-1} + p_t y_t - P_t C_t - P_t \tau_t$$

A useful simplifying assumption:

The Cobb-Douglas form implies a constant split of total consumption expenditure between home and foreign goods:

$$C_H = n \frac{P}{P_H} C \quad \text{and} \quad C_F = (1-n) \frac{P}{P_F} C$$

Price index is Cobb-Douglas as well. And assume LOP holds, so here PPP will hold as well: $P_H = eP_H^*, \quad P = eP^*$

So can write foreign demand for home good as:

$$C_H^* = n \frac{P^*}{P_H^*} C^* = n \frac{P}{P_H} C$$

Insert these into the market-clearing condition for home goods:

$nC_H + (1-n)C_H^* = nY$ (scaled by population size)

substitute in demands:

$$n\left(n\frac{P}{P_H}C\right) + (1-n)\left(n\frac{P*}{P_H*}C*\right) = nY$$

use LOP and PPP:

$$n\left(n\frac{P}{P_H}C\right) + (1-n)\left(n\frac{P/e}{P_H/e}C*\right) = nY$$

$$n\left[nPC + (1-n)PC^*\right] = nP_HY$$

$$n\left[nPC + (1-n)PC^*\right] = nP_HY$$

Do same for demands for foreign goods and market clearing condition:

$$(1-n)\left[nPC+(1-n)PC^{*}\right]=(1-n)P_{F}Y^{*}$$

Together, these imply that: $P_{H}Y = P_{F}Y^{*}$.

<u>This shows</u>:

- The income levels in both countries will be the same, regardless of the shocks.

- Relative prices adjust to insure against any output shock, so no need for asset trade: a result shown in Cole and Obstfeld (1991).

Implications for borrowing and the current account:

Given consumption smoothing implied by the intertemporal Euler equation, the fact that that the incomes in both countries move exactly together indicates that the consumption levels in both countries will move together.

In particular, the solution implies:

$$C = \frac{P_H Y}{P}, \quad C^* = \frac{P_F Y^*}{P^*}, \text{ and } \quad C = C^*$$

The fact that there is no borrowing and hence no current account or wealth reallocation greatly simplifies the solution.

This is a widely used device, which we will see employed in several subsequent papers in this literature.

Implications for Risk Sharing:

Consider the risk sharing condition characteristic of complete asset markets:

$$\frac{U_C^{*'}}{U_C^{'}} = \frac{eP^*}{P}$$ for all periods and all states of nature.

Under purchasing power parity, this requires:

$$U_C^{'} = U_C^{*'}$$

Will be true here because $C = C^*$

So the allocation here will mimic exactly the complete markets equilibrium.

Note: will not be valid under some alternative specifications:
(We will see both of these cases in subsequent papers)

1) Local currency pricing: If law of one price violated, then cannot derive the real income condition above.

2) Presence of nontraded goods
Suppose we specify consumption as including nontraded home goods:

$$C = C_N^{\gamma} C_T^{1-\gamma}$$

where T goods are a Cobb-Douglas aggregate of home and foreign

The law of one price does not apply to the nontraded part of consumption, so the real income condition above implies

$$C_T = C_T^{*}$$

for traded goods alone, instead of $C = C^{*}$.

Price-setting behavior:

Assume that prices must be set one period in advance, and goods markets are demand determined.

First order condition with respect to price (maximize U, subject to budget constraint and demands functions)

$$E_{t-1}\left[C^{1-\rho}(\theta-1)\right]=E_{t-1}\left[\theta\left(\frac{PC}{P_H}\right)^2\right]$$

Is similar to what found in 1995 paper.

- If the agent lowers price, it knows it will increase demand, and have to increase output.
- The left side represents the benefit of extra consumption made possible.
- The right side represents disutility of lost leisure.

Rewrite this:

Use definition of price index

$$P = P_H^n (eP_F{}^*)^{1-n}$$

to write FOC as function of $P_H, P_F{}^*, C, e$

Instead of linearizing, assume shocks and hence variables are log-normally distributed.

So use:

$$E \log(x) = \log Ex + \frac{1}{2} \operatorname{var} \log(x)$$

and

$$\operatorname{var}(xy) = \operatorname{var} x + \operatorname{var} y + 2\operatorname{cov}(xy)$$

to write (See appendix B for details)

$$p_H - p_F^* - E_{t-1}e = (1-2n)\sigma_e^2 + 2\sigma_{ce}$$

Interpret:

- Suppose covariance term is positive. Says that currency is weak at same time when consumption level high.
- Both events tend to raise demand for home good and make disutility of leisure higher than expected.
- Response of firm: set price higher than otherwise would, because want higher marginal revenue to hedge against the risk of this variance in marginal disutility of effort.
- Main idea: Exchange rate risk can affect price-setting behavior of the firms.

FOC for Money demand: Like that in 1995 paper:

$$\frac{M_t}{P_t} = \chi^{1/\varepsilon} \left(\frac{1 + i_{t+1}}{i_{t+1}} \right)^{1/\varepsilon} C_t^{\rho/\varepsilon}$$

So a shock to money supply:

$m_t = m_{t-1} + \mu_t$ (in logs)

will either raise P, raise C, or raise or lower i (depending on elasticity) or a combination of the three.

Explaining the forward premium puzzle:

Main idea:

Want to explain why a fall in i predicts a currency depreciation, rather than appreciation as required by interest rate parity.

Must be a large fall in RP.

$$\underset{\downarrow}{i_{t+1}} - i^*_{t+1} = \underset{\downarrow}{E[e_{t+1}]} - e_t + \underset{\downarrow}{RP_t}$$

A simple way to view this is in terms of theory presented earlier in lecture. Recall a general expression for the risk premium:

$$RP_t = E_t\left[e_{t+1} \frac{U'_{c,t+1}}{P_{t+1}} \right] \Big/ E_t\left[\frac{U'_{c,t+1}}{P_{t+1}} \right]$$

Recall also that under a model with perfect risk sharing, we have seen:

$$\frac{U'^{*}_{c}}{P^{*}} = e \frac{U'_{c}}{P} \; ,$$

So we have:

$$RP_t = E_t\left[\frac{C^{*-\rho}_{t+1}}{P^{*}_{t+1}} \right] \Big/ E_t\left[\frac{C^{-\rho}_{t+1}}{P_{t+1}} \right]$$

Rewrite this using: $\log E[X] = E[\log X] + \frac{1}{2}\mathrm{var}[\log X]$

Assume that the shocks are log normally distributed, then:
(lower case letters represent logs, overbars are means)

$$\log RP_t = (\overline{p} - \overline{p^*}) + \rho(\overline{c} - \overline{c^*})$$
$$+ \frac{1}{2}(\mathrm{var}_t[p^*] + \rho^2 \mathrm{var}_t[c^*] + 2\rho \mathrm{cov}_t[p^*, c^*])$$
$$- \frac{1}{2}(\mathrm{var}_t[p] + \rho^2 \mathrm{var}_t[c] + 2\rho \mathrm{cov}_t[p, c])$$

Under symmetry and perfect risk sharing:

$$RP_t = \frac{1}{2}(\sigma^2_{p^*t} - \sigma^2_{pt}) + \rho(\sigma^2_{cp^*t} - \sigma^2_{cpt})$$

where: $\sigma^2_{pt} = \mathrm{var}_t(p_{t+1})$, and $\sigma^2_{cpt} = \mathrm{cov}_t(c_{t+1}, p_{t+1})$

Interpretation:

- The first bracketed term above is a Jensen inequality term.
- The second bracketed term represents how a home nominal asset acts as a hedge against consumption risk.
- If consumption and the price positively correlated, home nominal assets pay off best in real terms (low price) in bad states of the world (low consumption)
- So home assets are a hedge and risk premium is low.
- What could make this covariance and risk premium fall?

Solve for the risk premium as a function of the variance in home monetary shocks (σ_{μ}^{2}):

$$RP_t = -\sigma_{\mu t}^{2}\left(\frac{1}{2} + (1-\beta)n(\varepsilon - 1)\right)$$

This says a rise in the volatility of money supply can lower the risk premium.

Intuition:

- When money supply rises, this only partly translates into a rise in price.

- To the degree that price doesn't rise fully, there is a rise in demand which raises production and consumption.

- So a rise in monetary volatility raises the correlation of price and consumption.

- This makes home assets more attractive as a hedge, and lowers the risk premium.

Welfare implications:

As 1995 paper, plug solutions into the utility function:

$$E[u] = E\left[\frac{C^{1-\rho}}{1-\rho} - \frac{Y^2}{2}\right] = \Omega \exp(1-\rho)\left[-\frac{2n(1-n)}{1+\rho}\sigma_e^2 - \sigma_c^2\right]$$

$$where \quad \Omega = \left(\frac{2\theta - (1-\rho)(\theta-1)}{2\theta(1-\rho)}\right)\left(\frac{\theta-1}{\theta}\right)^{\frac{1-\rho}{1+\rho}}$$

So high variability in consumption or the exchange rate lowers utility. Both are bad.

Intuition:

- Exchange rate uncertainty raises the risk of unexpected rises in demand for your good.

- As we saw above, monopolistic producers hedge against this by raising their price and restricting output.

- So fixed exchange rates can eliminate this, and permit greater welfare.

b) Bacchetta and van Wincoop (AER 2006)

Student presentation.