1/28/08

Lecture 4: Asset Market Integration

Part 1) Working through a complete markets case
(from Obstfeld and Rogoff, 1996 ch. 5)

- In the previous lecture, I claimed that assuming complete asset markets produced a perfect-pooling equilibrium. We begin this lecture on asset markets by demonstrating this claim.

Model setup:

- Two countries, denoted home and foreign
- Only one good in the world.
- Two periods: denoted 1 and 2.
- Output in period 1 is known.
- Output in period 2 varies by state of nature, s=1,2,...,S.
- Assume output is an endowment.
- Worldwide asset market in Arrow-Debreu securities, with period 2 payoffs that vary according to state of nature.

Notation:

Y_1	is home output endowment in period 1
$Y_2(s)$	is home output endowment in period 2 if state s occurs
Y^*_1	is home output endowment in period 1
$Y^*_2(s)$	is home output endowment in period 2 if state s occurs
C_1	is home consumption in period 1
$C_2(s)$	is home consumption in period 2 if state s occurs
C^*_1	is home consumption in period 1
$C^*_2(s)$	is home consumption in period 2 if state s occurs
$B_2(s)$	is home net purchase of state s Arrow-Debreu securities in period 1 (for payoff in period 2 if state s occurs)
$p(s)$	is world price of one of state-s security.
$\pi(s)$	is probability of state s occurring, where $\sum_{s=1}^{S} \pi(s) = 1$.

Home Problem:

Household maximizes the intertemporal (two period) expected utility:

$$U = u(C_1) + \beta \sum_{s=1}^{S} \pi(s) u(C_2(s))$$

subject to the home sequence of S+1 budget constraints:

$$\sum_{s=1}^{S} p(s) B_2(s) = Y_1 - C_1 \quad \text{in period 1}$$

$$C_2(s) = Y_2(s) + B_2(s) \quad \text{in period 2, for each state s=1...S}$$

Or combining these into the intertemporal budget constraint:

$$C_1 + \sum_{s=1}^{S} p(s) C_2(s) = Y_1 + \sum_{s=1}^{S} p(s) Y_2(s)$$

First order conditions:

$$p(s)u'(C_1) = \pi(s)\beta u'(C_2(s)) \quad \text{for each s==1...S}$$

or rewriting this: $$\frac{\pi(s)\beta u'(C_2(s))}{u'(C_1)} = p(s)$$

This is a form of intertemporal consumption smoothing.

It also implies consumption smoothing across states:

$$\frac{\pi(s)u'(C_2(s))}{\pi(s')u'(C_2(s'))} = \frac{p(s)}{p(s')}$$

An analogous problem applies to the foreign household, and will produce analogous first order conditions.

Note that the security prices in these conditions $p(s)$ will be identical, since it is the same securities being traded globally.

Implications:

So we have the following implications:

$$\frac{\pi(s)\beta u'(C_2(s))}{u'(C_1)} = p(s) = \frac{\pi(s)\beta u'(C^*{}_2(s))}{u'(C^*{}_1)}$$

and

$$\frac{\pi(s)u'(C_2(s))}{\pi(s')u'(C_2(s'))} = \frac{p(s)}{p(s')} = \frac{\pi(s)u'(C^*{}_2(s))}{\pi(s')u'(C^*{}_2(s'))}$$

This indicates that that home and foreign marginal rates of substitution in consumption are equal – across time and states.

If we assume a standard CRRA utility function

$$U(C) = \frac{1}{1-\rho}C^{1-\rho}$$

and define world output: $Y^W = Y + Y^*$

The first order conditions imply (across states in period 2):

$$\frac{C_2(s)}{C_2(s')} = \frac{C^*{}_2(s)}{C^*{}_2(s')} = \frac{Y^W{}_2(s)}{Y^W{}_2(s')}$$

and (across periods)

$$\frac{C_2(s)}{C_1} = \frac{C^*{}_2(s)}{C^*{}_1} = \frac{Y^W{}_2(s)}{Y^W{}_1}$$ for all states.

This means

$$\frac{C_2(s)}{Y^W{}_2(s)} = \frac{C_2(s')}{Y^W{}_2(s')}$$

and the same for foreign consumption.

<u>This means home consumption is always a constant fraction of world output, regardless of state</u>. In other words:

$$\frac{C_2(s)}{Y^W{}_2(s)} = \mu = \frac{C_1}{Y_1^W}$$

and similarly for the foreign household:

$$\frac{C^*{}_2(s)}{Y^W{}_2(s)} = 1 - \mu = \frac{C^*{}_1}{Y_1^W}$$

This property of complete assets markets helps explain why models like Backus et al (1992) which assume complete asset markets have such a hard time reproducing the low consumption correlations observed in the data.

Part 2) Portfolio Diversification Puzzle

a) Documenting the Puzzle:

Source IMF

- Shares have increased over time.
- The UK leads the sample with the most diversification,

Why is this a puzzle: One might think that if the a country is 10% of the world equity market, it should hold only 10% of its equity portfolio in home equities; with 90% in foreign assets.

Table 1.1 : International Portfolio Diversification

Country	1986	2001
Australia	6.58%	18.35%
Austria	13.29%	77.28%
Belgium	24.39%	42.32%
Canada	7.16%	16.01%
Finland	0.04%	24.41%
Germany	8.39%	39.65%
Italy	5.36%	32.75%
Netherlands	28.02%	57.43%
Spain	1.06%	14.04%
Switzerland	33.67%	55.36%
United Kingdom	23.02%	29.10%
United States	2.87%	11.30%

This measure of international portfolio diversification is equal to the foreign equity assets held by the country divided by the sum of its stock market capitalization and foreign equity assets adjusted for its foreign equity liabilities. It is discussed in more detail in the text.

Source: dissertation of Amir Amadi, 2004

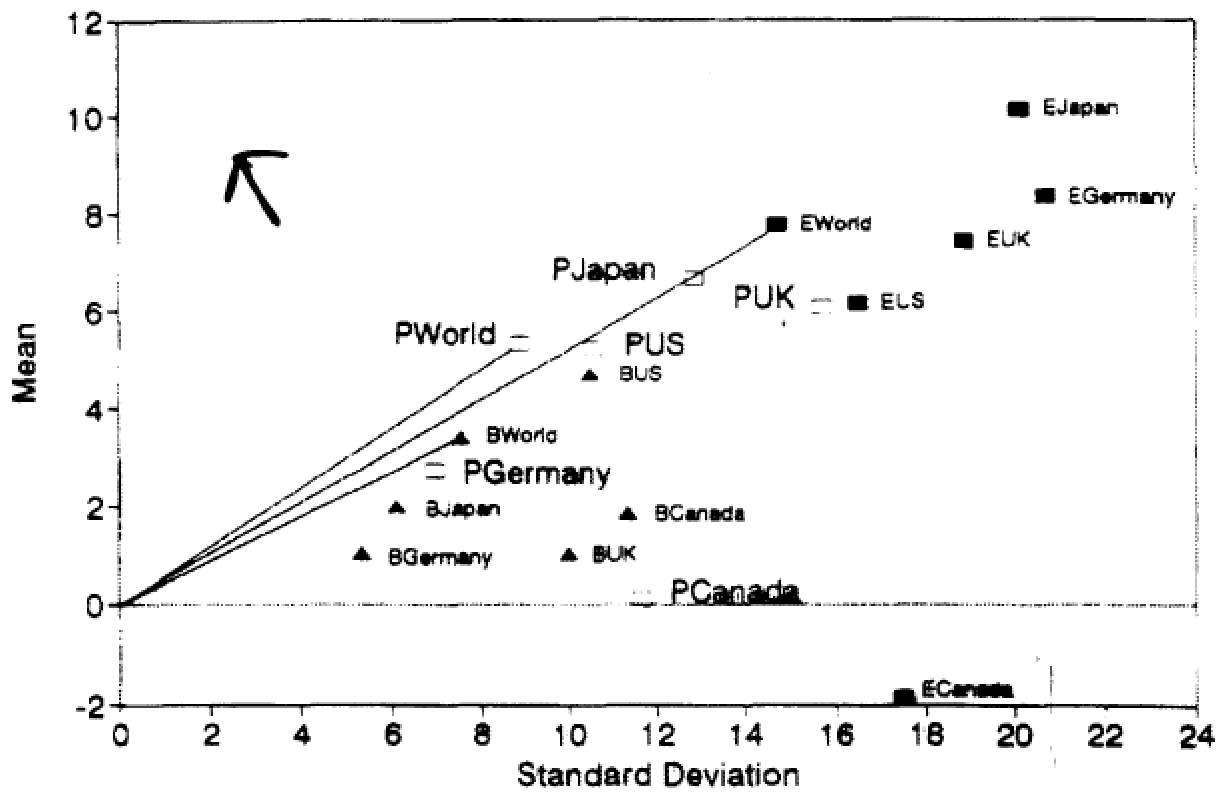

FIGURE 1. Hedged excess returns: monthly data 1980: 12–1990: 12.

b) Demonstrate that the "world portfolio" is optimal:
(from Obstfeld and Rogoff, 1996, chapter 5)

Model setup:

- Like model above: one world good, endowment economy, two periods.
- Many countries: n=1...N
- Asset market allows trade in shares of endowment process in other countries (equities), as well as a noncontingent bond

Note: the equities are like a share in a mutual fund that pays its owner $Y^{n}{}_{2}(s)$ in state *s.*

<u>Notation:</u> same as model above for most variables:

$Y^n{}_1$	income of country *n* in period 1.
$Y^n{}_2(s)$	income of country *n* in period 2 in state s.
$C^n{}_1$	consumption in country n in period 1.
$C^n{}_2(s)$	consumption in country n in period 2.
$\pi(s)$	the probability of state *s* occurring.

<u>New notation</u> for new variables:

$x^n{}_m$	fractional shares of country *m*'s future output bought by residents in country *n*.
$V^n{}_1$	the market value in period 1 of country *n*'s uncertain output in period 2. The price of the shares in $\boldsymbol{x^n{}_m}$.
$B^n{}_2$	non-contingent bonds purchased in period 1, which pay off at rate (1+r) in period 2

Household Problem

The resident maximizes the two-period utility:

$$U = u(C_1^n) + \beta E_1\left[u(C_2^n)\right] = u(C_1^n) + \beta \sum_{s=1}^{S} \pi(s) u(C_2^n(s))$$

subject to the budget constraint for period 1:

$$Y_1^n + V_1^n = C_1^n + B_2^n + \sum_{m=1}^{N} x_m^n V_1^m$$

and subject to the budget constraint for period 2:

$$C_2^n(s) = (1+r)B_2^n + \sum_{m=1}^{N} x_m^n Y_2^m(s)$$

First order conditions

- With respect to bond holdings ($B^n{}_2$):

$$u'(C_1^n) = (1+r)\beta\sum_{s=1}^{S}\pi(s)u'(C_2^n(s))$$

Which is the usual consumption-smoothing Euler equation.

- With respect to portfolio shares ($x^n{}_m$):

$$V_1^m u'(C_1^n) = \beta\sum_{s=1}^{S}\pi(s)u'(C_2^n(s))Y_2^m(s) \quad m = 1...N$$

The left-hand side is the marginal utility cost of buying country *m*'s risky future output on date 1, and the right-hand side is the expected marginal utility gain from this.

Solution:

We will assume a CRRA utility function: $u(C) = \frac{1}{1-\rho} C^{1-\rho}$

We conjecture that the solution is for all agents to hold shares in the same fully-diversified global portfolio, where the share of each country in this global portfolio is the country's share in world wealth.

We conjecture also consumption levels that divide up total world production (Y^w) between the countries according to their shares in world wealth.

Define country *n*'s share of initial world wealth as:

$$\mu^n = \frac{Y_1^n + V_1^n}{\sum_{m=1}^{n} (Y_1^m + V_1^m)}$$

So our guess for the portfolio shares is μ^n:

$$x^n{}_m = \mu^n \quad for \quad m = 1 \ldots N$$

and our guess is that the country share of world consumption in each period and state is also μ^n:

$$C_1^n = \mu^n \sum_{m=1}^{N} Y_1^m = \mu^n Y_1^w$$

$$C_2^n(s) = \mu^n \sum_{m=1}^{N} Y_{21}^m(s) = \mu^n Y_2^w(s) \quad for \quad s = 1 \ldots S$$

Also conjecture that $B^n{}_2 = 0$.

Now we need to verify that this solution does actually satisfy the budget constraints and first order conditions.

1) Budget constraint for second period:

Plugging the conjectured asset allocation $x^n{}_m = \mu^n$ and $B^n{}_2 = 0$ into period 2 budget constraint, we find the consumption equation for period 2 above.

So we know this consumption allocation is consistent with the period2 budget constraint for each country.

2) <u>Bond Euler equation</u>:

For the CRRA utility, the bond FOC becomes

$$1+r=\frac{(C_1^n)^{-\rho}}{\beta\sum_{s=1}^{S}\pi(s)u'(C_2^n(s))^{-\rho}}$$

The consumption plans above will satisfy this Euler equation for each country if the equilibrium real interest rate is:

$$1+r=\frac{(Y_1^w)^{-\rho}}{\beta\sum_{s=1}^{S}\pi(s)u'(Y_2^w(s))^{-\rho}}$$

So the real interest rate is lower if the level of world output is high today relative to what it is expected to be next period.

3) <u>Equities Euler equation</u>:

Similarly satisfied for equilibrium equity share prices:

$$V_1^m = \sum_{s=1}^{S} \pi(s)\beta\left[\frac{Y_2^w(s)}{Y_1^w}\right]^{-\rho} Y_2^m(s) \quad m = 1 \ldots N$$

4) <u>Budget constraint for period 1</u>:

$$Y_1^n + V_1^n = C_1^n + B_2^n + \sum_{m=1}^{N} x_m^n V_1^m$$

Sub in for $\boldsymbol{C^n}_{\mathbf{1}}$, $B^n{}_2$ and $x^n{}_m$:

$$Y_1^n + V_1^n = \mu^n Y_1^w + \sum_{m=1}^{N} \mu^n V_1^m$$

Simplify and sub in the definition of μ^n:

$$Y_1^n + V_1^n = \frac{Y_1^n + V_1^n}{\sum_{m=1}^{n}(Y_1^m + V_1^m)} Y_1^w + \frac{Y_1^n + V_1^n}{\sum_{m=1}^{n}(Y_1^m + V_1^m)} V_1^w$$

multiply by the reciprocal of μ^n:

$$\sum_{m=1}^{n}(Y_1^m + V_1^m) = Y_1^w + V_1^w$$

Which is true by definition.

This verifies that in a simple model it is optimal for people to hold equity portfolios that are fully diversified internationally.

<u>In conclusion</u>, this underscores that it is a puzzle that in practice people have a bias toward domestic equities.

Part2: Theoretical Explanations

a): Asset Transactions costs

Tesar and Werner (1995) dismiss this because volume of international asset trade is large, even though the net positions are small. (Table 5B (12))

1. <u>Domestic Turnover Rates:</u>

	Total Trans. (A)	Market Cap. (B)	Total Turnover (A/B)
Canada	177.8	290.1	0.61
Germany	628.2	361.5	1.74
Japan	5.218.5	4,102.1	1.27
U.K.	635.0	823.2	0.77
U.S.	3,223.9	3,027.1	1.07
World	11,716.9	10,140	1.16

2. Turnover Rates in Foreign Equity held by Domestic Residents:

	Trans.in Foreign Equity (C)	Inv.Pos. in For. Equity (D)	Turnover Rate (C/D)	Pct.For. Equity in Total Trans. (C/A)
Canada	43.1	5.6	7.7	24.2%
Germany	73.1	n.a.	n.a.	11.6%
Japan	166.1	n.a.	n.a.	3.2%
United Kingdom	n.a.	226.2	n.a.	n.a.
United States	232.8	91.7	2.54	7.2%

b) Nontraded goods (Obstfeld and Rogoff text section 5.5)

Setup:

- Extend model above to have nontraded goods (N) and equities in nontraded goods endowment.

- Note: the payoffs of equities in the nontraded good process must be paid abroad in the form of traded goods (T).

- Model generates analogous equations to above, but specific to T.

- Implications depend on how T and N goods interact in the utility function.

Consider case where T and N goods are additively separable, so N no effect on U'_T

Solution for traded equities: follows share of home T wealth in total world T wealth.

$$x^n{}_{T,m} = \mu_T^n$$

Solution for nontraded equities: not buy foreign shares

$$x^n{}_{N,m} = \begin{cases} 1 & (m = n) \\ 0 & (m \neq n) \end{cases}$$

Intuition: foreign shares not help in case of shock to home nontraded endowment, because only pay of in units of traded goods.

Implication: even if portfolios are diversified over equities in T, since the portfolio also includes equities on N, the OVERALL portfolio will be biased toward home equities.

Is this enough to explain bias in data?

Tesar (JIE 1993) case

Suppose T and N interact in utility, and intratemporal elasticity greater than intratemporal.

Now shock that lowers the endowment of N also lowers desired consumption of T. So it is optimal to have short position in foreign N equities, which biases overall portfolio more.

d) Kollmann (2006):

motivation: Use an RBC model to explain portfolio diversification and CA behavior.

To be studied in homework assignment.

Notes on Devereux and Sutherland (2006)

Motivation: Methodological paper. Provides a methodology for solving for portfolio allocation without imposing complete risk sharing.

Idea:

- solve for asset FOC as a second order approximation, to include risk premium terms. Rest of model can be solved as first order approximation. Solve linear system, then plug into nonlinear asset equation. This paper computes a closed for solution (simple case). More complicated models solved numerically would require iteration on the system of equations for a solution.

- Works because portfolio allocation only enters the rest of the linear model (BConst in particular) in steady state form. So if only care about linear approximation to model dynamics (ie, not care about how portfolio allocation responds dynamically to shocks), then only need SS for portfolio, and this can be solved on the basis of a second order approx to the portfolio optimization equation.

Application: shows that if government consumption is biased toward home goods, than government spending shocks can explain large bias in home equities.

We will be able to talk more about second moments and risk premia more later. A later lecture will discuss how to handle these issues.