Lecture 3: International Real Business Cycle Models

Part1: Quantity Puzzle

a) Motivation

Issue: Course has already analyzed a couple of moments in data, such as cor(S,I) and cor(I,CA). Now we address a wider set – especially comovements across countries.

Questions:

- How much do national business cycles move together?
- Is this due more to similar shocks, or due to spillovers?
- Through what markets are shocks transmitted?

Methodology:

- We here will use the approach of Real Business Cycle (RBC) models. We start with Backus, et al (1992 JPE) which set the agenda for the resulting literature.

- This is different from the models we've used in previous lectures, in that production will depend on labor as well as capital.

- The vast majority of papers in this literature use two-country models. This is different from the models considered so far in class, which were small-open economy models. Now interest rates and relative prices will no longer be exogenous.

b) **Stylized facts**:

Introduction:

- Study tables 11.1 and 11.2 from book chapter by Backus, et al.

- Data: 10 industrial countries and an aggregate of Europe, quarterly, and Hodrick-Prescott (HP) filtered to focus on business-cycle frequencies in data.

- Collect observations on the following:
    - Volatility: standard deviation
    - Persistence: autocorrelation
    - Comovement: correlations

Table 11.1
Properties of Business Cycles in OECD Economies

Country	*Standard Deviation (%)*		*Ratio of Standard Deviation to That of y*					*Autocorr.*
	y	*nx*	*c*	*I*	*g*	*L*	*A*	*y*
Australia	1.45	1.23	0.66	2.78	1.28	0.34	1.00	.60
Austria	1.28	1.15	1.14	2.92	0.36	1.23	0.84	.57
Canada	1.50	0.78	0.85	2.80	0.77	0.86	0.74	.79
France	0.90	0.82	0.99	2.96	0.71	0.55	0.76	.78
Germany	1.51	0.79	0.90	2.93	0.81	0.61	0.83	.65
italy	1.69	1.33	0.78	1.95	0.42	0.44	0.92	.85
japan	1.35	0.93	1.09	2.41	0.79	0.36	0.88	.80
Switzerland	1.92	1.32	0.74	2.30	0.53	0.71	0.67	.90
United Kingdom	1.61	1.19	1.15	2.29	0.69	0.68	0.88	.63
United States	1.92	0.52	0.75	3.27	0.75	0.61	0.68	.86
Europe	1.01	0.50	0.83	2.09	0.47	0.85	0.98	.75

Table 11.2
International Comovements in OECD Economies

	Correlation of Each Country's Variable with Same U.S. Variable					
Country	y	c	i	g	h	e
Australia	.51	−.19	.16	.23	−.18	.52
Austria	.38	.23	.46	.29	.47	.17
Canada	.76	.49	−.01	−.01	.53	.75
France	.41	.39	.22	−.20	.26	.39
Germany	.69	.49	.55	.28	.52	.65
Italy	.41	.02	.31	.09	−.01	.35
Japan	.60	.44	.56	.11	.32	.58
Switzerland	.42	.40	.38	.01	.36	.43
United Kingdom	.55	.42	.40	−.04	.69	.35
Europe	.66	.51	.53	.18	.33	.56

Table 11-1

Domestic Volatilities:

- Consumption is less volatile than output, reflecting consumption-smoothing.
- Investment is more volatile than output: 2-3 times.
- Countries differ much: output is more volatile in the US (sdev = 1.92); least in France (0.9).

Employment is procyclical

- The Solow residual is strongly procyclical, but less volatile than output.
- Technology shocks help explain fluctuations in output, but they need endogenous fluctuations in labor supply to amplify their effects on output.

Net exports:

- Why do we not talk about current account? (Will explain later: due to complete asset markets in theoretical model).
- The trade balance is countercyclical in all 10 countries
  Recall the question from before: Why does a rise in technology lower the trade balance even though output rises? Because investment rises even more.

Persistence: Output quite persistent, autocorr from 0.5 - 0.9.

Table 11-2: International correlations: (with US)

- **Cor(Y,Y*) > cor (C,C*) for all cases**
- Note that the correlation in national output levels is due mainly to the correlation of the Solow residuals, not due to spillovers that work through increased labor supply. We know this because the cor(L,L*) is low.

c) Model

Introduction:

- We present here a streamlined version of Backus, et al (1992 JPE), eliminate inventory accumulation.
- Start with case of a single consumption good, to focus on quantities, not relative prices between goods.

Setup of model:

Utility of representative household:

$$U_{it} = \left(C_{it}^{\ \mu} (1 - L_{it})^{1-\mu} \right)^{1-\frac{1}{\sigma}} \quad i = h, f$$

Allocates one unit of time between work and leisure.

Production of single good uses labor and capital:

$$Y_{it} = F(K_{it}, L_{it}) = A_{it} K_{it}^{\ \theta} L^{1-\theta}{}_{it} \quad i = h, f$$

Since both countries produce the same good, the resource constraint is:

$$Y_{ht} + Y_{ft} = C_{ht} + C_{ft} + I_{ht} + I_{ft} + G_{ht} + G_{ft}$$

Capital formation uses time-to-build structure.

Additions to the stock of fixed capital require inputs of the produced good for 4 periods:

$$K_{t+1} = (1-\delta)K_t + s^1{}_t$$

(For the home country; analogous for foreign. Skipped i subscripts on everything to avoid confusion)

Where $s^j{}_t$ is the number of investment projects at date t that are j periods from completion.

$$s^j{}_{t+1} = s^{j+1}{}_t$$

It takes 4 periods for a capital good to be built and increase the capital stock. So put in 1/4 of value added each period:

If add up all the investment expenditure made in a period on the projects at various stages of completion, it equals:

$$I_t = \sum_{j=1}^{4} \frac{1}{4} s_t^j$$

Shocks

Separate technology shock in each country, but can be correlated.

$$\begin{bmatrix} A_{ht+1} \\ A_{ft+1} \end{bmatrix} = \begin{bmatrix} \rho_{11} & \rho_{12} \\ \rho_{21} & \rho_{22} \end{bmatrix} \begin{bmatrix} A_{ht} \\ A_{ft} \end{bmatrix} + \begin{bmatrix} \varepsilon_{ht+1} \\ \varepsilon_{ft+1} \end{bmatrix}$$

Where epsilons have covariance matrix:

$$E_t \left[\begin{pmatrix} \varepsilon_{ht} \\ \varepsilon_{ft} \end{pmatrix} \begin{pmatrix} \varepsilon_{ht} & \varepsilon_{ft} \end{pmatrix} \right] = V$$

Correlations in technology are captured by off-diagonal elements of rho matrix and V matrices.

Equilibrium:

- We will assume that financial markets are complete. People in either country have access to a full set of conditional assets they can buy to insure against shocks.
- We could try to model explicitly all the assets and find the solution for the competitive equilibrium: solve for the two optimization problems, one for each country separately.
- But under complete markets, the solution will be a Pareto optimum. So we can also solve for the equilibrium as a single optimization problem of a social planner that maximizes the sum of utilities of the two countries.
- So solve following subject to the constraints above, along with their foreign counterparts.

$$\max \quad E_t \sum_{t=0}^{\infty} \beta^t \left[\frac{1}{2} U\left(C_{ht}, (1 - L_{ht})\right) + \frac{1}{2} U\left(C_{ft}, (1 - L_{ft})\right) \right]$$

Solution:

- Get first order conditions for consumption and capital accumulation, which are the same as previous models we've considered, with one additional one for labor supply:

$$-\frac{U'_L}{U'_C} = F'_L$$

- Combine these with the resource constraints.

- Solve for a deterministic steady state (dropping uncertainty)

- Take a log-Linear approximation around the steady state.

- Solve the linear system of equations, such as by method of Blanchard and Kahn (1980): find unstable roots of system by eigen values; imposing the associated eigenvectors.

- Calibration is standard:

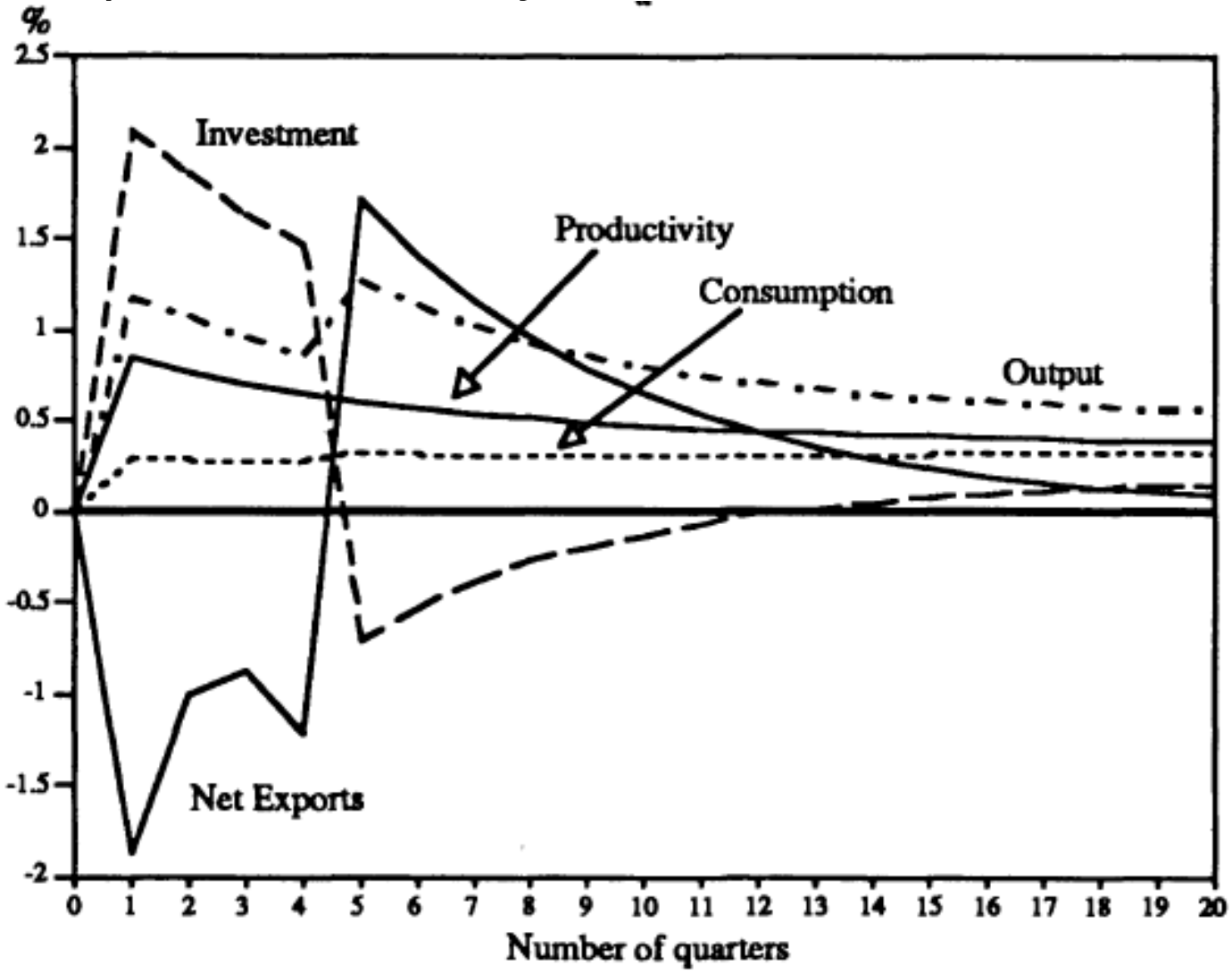
  - discount factor = 0.99 (assume quarterly period).
  - Intertemporal elasticity equals 0.5.
  - Technology shocks have persistence 0.9, and cross persistence of 0.09. Correlation of epsilons are 0.258.

- Simulate: 20 runs of 100 periods each.

- Hodrick-Prescott (HP) filter and compute same statistics as for actual data from the real economy.

- Compare the moments from simulated data to those from actual data.

d) Results: Consider a 1 % rise in A (positive epsilon for one period) in home country.



First do impulse responses:

Home:

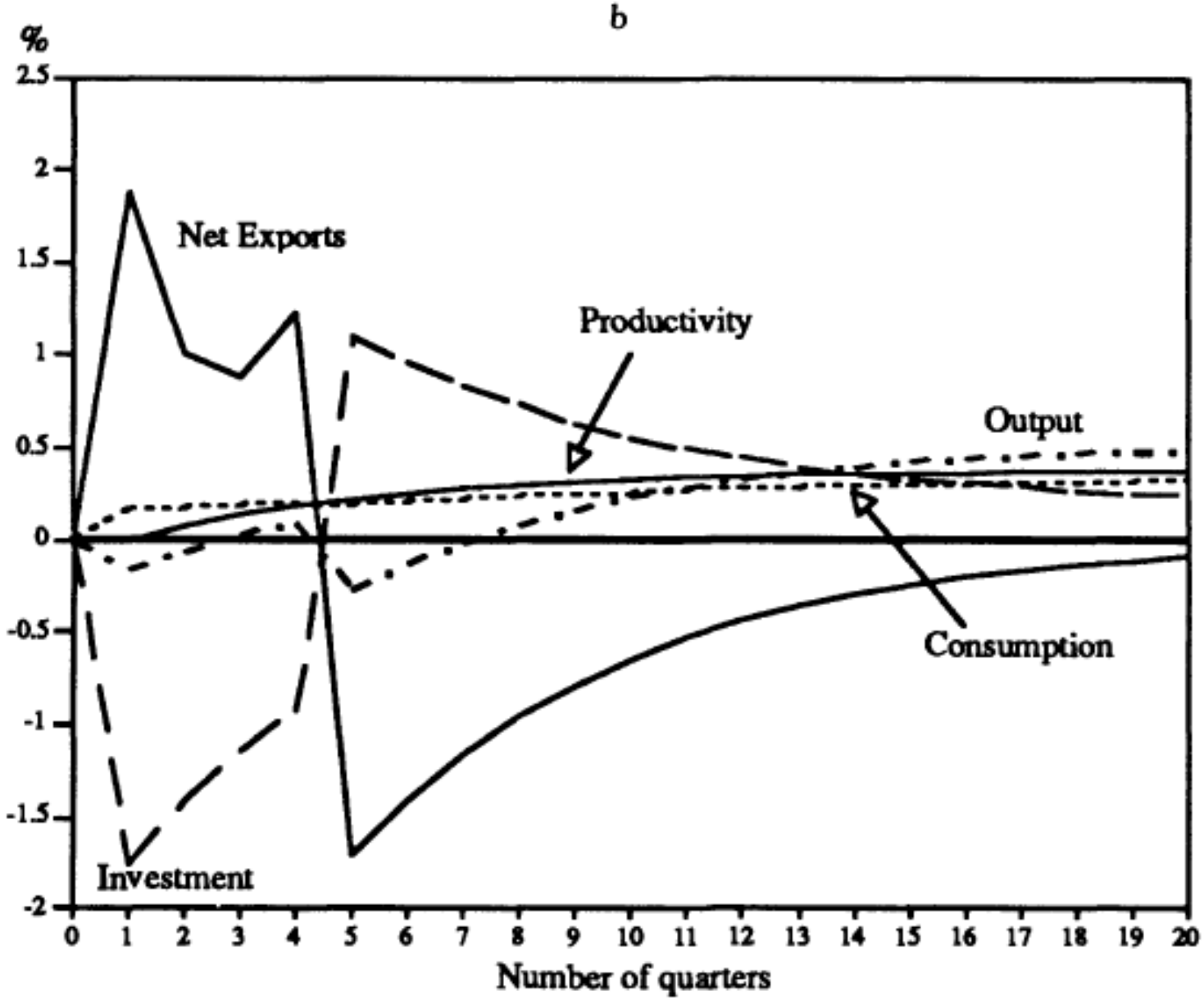
- Rise in productivity raises output.

- Also raises investment significantly because of marginal productivity of capital. Investment is very volatile in an open economy since it is easy to borrow from abroad to finance investment.

- This makes net exports go negative (not shown explicitly, but is apparent).

- Also raises consumption as smoothed.

Foreign response:



Foreign:

- Investment moves the opposite way because want to shift resources to where are the most productive. As a result output moves opposite as well. Falls at first.

- But consumption moves very similarly. Even though output falls, consumption rises like in home country.

- The reason is that they have assets that insure them. For example, if foreigners own half of shares in domestic firms, they get paid get half of the extra output produced in the home country. So their consumption rises.

Moments:

A. Business Cycle Properties

Economy	Standard Deviation (%)		Ratio of Standard Deviation to That of y				Autocorr.
	y	nx	c	I	n	A	y
U.S. data	1.92	0.52	.75	3.27	.61	.68	.86
Benchmark	1.50	3.77	.42	10.99	.50	.67	.62
Transport cost	1.35	0.37	.47	2.91	.47	.75	.61
Autarky	1.26		.54	2.65	.91	.99	.62

B. International Comovements

Economy	*Correlation of Foreign and Domestic Variables*				
	y	*c*	*i*	*n*	*a*
U.S. data	.66	.51	.53	.33	.56
Benchmark	−.21	.88	−.94	−.94	.25
Transport cost	−.05	.89	−.48	−.48	.25
Autarky	.08	.56	−.31	−.31	.25

Correlations:

- Also see in correlations: output correlation is less than in data (-0.21 versus 0.66), but consumption correlation is higher than in data (.88 versus .51).

- Otherwise match things pretty well. (Investment a bit too volatile)

- Main problem is **consumption correlation puzzle**: consumption is less correlated in data than is output. Model says the opposite.

One Possible solution: Transport costs:

- Try a version of model with costs of trading goods.
- In a world budget constraint, impose a cost that is a quadratic function of net exports. So if try to import goods to raise investment, becomes expensive.

$$Y_{ht} + Y_{ft} = C_{ht} + C_{ft} + I_{ht} + I_{ft} + G_{ht} + G_{ft} + \tau nx_t^2$$

$$where \quad nx_t \equiv Y_{ht} - C_{ht} - I_{ht} - G_{ht}$$

- Mechanically, just add this term on to budget constraint before do first order conditions. Calibrate, so on average cost is only about 1%.
- Result: lower response of net exports, and thereby investment response to technology shock. But not affect consumption or output correlation much. Output cor rise from -.21 to -.05. Consumption cor rise from 0.88 to 0.89.

Alternative 2: Autarky

- Perhaps puzzle is due to fact that effects of the shock are spread evenly to the other country because income is pooled through asset market.

- So consider model with no trade in assets or goods. Two separate economies. Only link is that have technology shocks that are correlated.

- This reduces consumption correlation to level similar to data though still a little high (0.56 versus 0.51). But output correlation is still too much too low.

- Reason consumption still correlated: agents know technology shock abroad will affect them: through

nondiagonal element of rho matrix. So they try to smooth consume part of extra output they know is coming.)

- Know if want to increase output correlation, could increase the correlation of technology shocks. But if do that, this increases the correlation of consumption as well. So can match or the other, but not both at same time.

- But this is an extreme example: complete autarky. More realistic if in middle. Need introduce partial asset markets, so need to be specific about nature of assets and model the assets themselves.

e) **Other papers**:

- One type of asset structure that we already have seen is easy to model is the one-period non-contingent bond. It pays off a certain interest rate in the next period. It does not help in insuring against unforeseen shocks, but just to borrow to smooth out the effects of shocks.

- Kollmann (JEDC 1996) considers this.

- The model is like the one above, but it introduces a bond in the household budget constraint, just like we did in past lectures. This variable is needed to keep track of relative wealth position of the countries.

- Result: can't spread risk, but can borrow to smooth out consumption almost as well, depending on how persistent the shock is.

- For example, if there is a negative home technology shock that lowers output, you can borrow abroad to boost consumption. Since you are a large country, your desire to borrow raises the world interest rate, and this induces foreign country to lower its consumption and lend to you.

- So consumption falls in the foreign country, and this helps offset part of the fall in consumption in home country. So consumption levels still move together.

- Whether this happens or not depends on the persistence of the technology shock. Story above applies for a temporary shock, where want to borrow to smooth consumption.

- But most technology shocks are thought to be pretty persistent, so permanent income falls more with current income. In this case incomplete markets can help lower the consumption correlation.

- Kollman finds that for rho = 0.95, the cross-country consumption correlation falls from 0.72 to 0.38. This is a big change.

Part2: Puzzle of Relative Prices

Stylized facts

- Define: Terms of Trade (TOT) = price of exports / price of imports.

- The relative price data reported here is the inverse of the usual definition of the TOT given above.

- Regularities: Look at table 11.5

- The ‘tems of trade’ is highly variable: Standard deviations are usually 2-3 times that of output.

- It is also highly persistent, with an autocorrelation near 0.8.

Table 11.5
Properties of the Terms of Trade in OECD Economies

Country	SD p(%)	Autocorr. p	Correlation of: (p, nx)	(p, y)
Australia	5.78	.82	−.10	−.27
Austria	1.73	.46	−.24	.04
Canada	2.99	.85	.05	−.05
France	3.52	.75	−.50	−.13
Germany	2.66	.85	−.08	−.11
Italy	3.50	.78	−.66	.38
Japan	7.24	.86	−.56	−.22
Switzerland	2.85	.88	−.61	.41
United Kingdom	3.14	.80	−.58	.09
Europe	3.68	.83	.30	−.20

Model

- To describe relative prices, we need two types of goods. Assume each country produces a distinctive good. Home produces good 1; foreign good 2. Households in each country consume both goods.

- Changes in model: Use star to indicate foreign variable, H and F to indicate good.

- Two goods market clearing conditions:

$$Y_{ht} = C_{ht} + C^*_{ht} + I_{ht} + I^*_{ht} + G_{ht} + G^*_{ht}$$

$$Y^*_{ft} = C_{ft} + C^*_{ft} + I_{ft} + I^*_{ft} + G_{ft} + G^*_{ft}$$

- Budget constraint is (using home goods as a numeraire)

$$Y_{ht} + p_t Y^*_{ft} = C_{ht} + C^*_{ht} + p_t \left(C_{ft} + C^*_{ft} \right) + \ldots$$

where p_t is the relative price of foreign goods in terms of home goods (p_f/p_h), or from the perspective of the home country, it is the relative price of imported goods in terms of exported goods

So p_t here is the inverse of the terms of trade as conventionally defined above.

- Model home consumption as an aggregation (a function "g") over home and foreign good:

$$C_t = g(C_{ht}, C_{ft}) = \left[C_{ht}^{\varpi} C_{ft}^{1-\varpi} \right]$$

Where start off using Cobb-Douglas for the aggregation function. (Can use same aggregation function for investment and government demands.)

- Put this in utility function, and derive optimal choice between the two goods based on relative price. Intratemporal substitution again:

$$\frac{U'_{cf}}{U'_{ch}} = p_t$$

- So using the chain rule over the utility function, this allows us to express pt (the inverse TOT) as the ratio of derivatives of the aggregation function over the two types of goods.

$$p_t = \frac{\partial g(C_{ht}, C_{ft})}{\partial C_{ft}} / \frac{\partial g(C_{ht}, C_{ft})}{\partial C_{ht}} = \frac{1-\varpi}{\varpi}\left(\frac{C_{ht}}{C_{ft}}\right)$$

- Can compute net exports (in units of home goods):

$$nx_t = C_{ht}^* - p_t C_{ft}$$

Results

Calibrate:

- Intratemporal elasticity = 1.5
- Share of imports in GNP = 0.15

Simulation results for TOT:

- Persistence: 0.83, similar to data. Inherit persistence from technology shock.

- Correlation of TOT with NX is negative, similar to what is in data.

- Volatility: Sdev of TOT is much less in model than in data (data is 7X larger).

Table 11.6
Properties of the Terms of Trade in Theoretical Economics

Country	SD p(%)	Autocorr. p	Correlation of: (p, nx)	Correlation of: (p, y)
U.S. data	3.68	.83	.30	−.20
Benchmark	0.48	.83	−.41	.49
Two shocks (technology and government spending)	0.57	.67	−.05	.39
Large import share	0.66	.83	−.41	.55
Small elasticity	0.76	.77	−.80	.51

Puzzle:

So have a "**relative price puzzle**."

It is clear we can't resolve this puzzle in this model just by varying parameter values.

Discuss Ideas of how to resolve?

Recall that the intratemporal optimality condition shows that the relative price is directly related to the ratio of imports to consumption of domestic goods.

$$p_t = \frac{\partial g(C_{ht}, C_{ft})}{\partial C_{ft}} \Big/ \frac{\partial g(C_{ht}, C_{ft})}{\partial C_{ht}}$$

$$= \frac{1-\varpi}{\varpi}\left(\frac{C_{ht}}{C_{ft}}\right)$$

in percent changes

$$\tilde{p}_t = \tilde{C}_{ht} - \tilde{C}_{ft}$$

- The model matches the volatility of the import ratio pretty well. But there is no way to increase the volatility of the terms of trade greater than this, because there is a tight connection between quantities and relative prices here.

- If there is a negative technology shock abroad that raises the relative price of imported goods in the home country, there is a fall in the quantity of imported goods.

- The tight link between price and quantities implies that technology shocks that lead to moderate swings in quantities cannot generate big swings in prices.

Alternative

- Consider using a different aggregator, with an intratemporal elasticity different from unity:

$$C_t = g(C_{ht}, C_{ft}) = \left[C_{ht}^{1-\frac{1}{\psi}} + (1-\omega) C_{ft}^{1-\frac{1}{\psi}} \right]^{\frac{\psi}{\psi-1}}$$

where ψ is the elasticity of intratemporal substitution.

- This alters the intratemporal condition (in percent changes):

$$\tilde{p}_t = \frac{1}{\psi}\left(\tilde{C}_{ht} - \tilde{C}_{ft}\right)$$

- Idea: Says there is a tight link between relative price and quantities. If make intratemporal elasticity (ψ) small, then

the change in p will be big for any given change in import share.

- That is, if goods are not very substitutable, when there is a fall in the supply of importable good, it will take a very big rise in the price of importables to make everyone satisfied consuming a smaller quantity of them.

- But empirical estimates imply a range of .5 to 5 for the elasticity; even a small value of 0.5 is not small enough to generate the observed price volatility.

Conclusion: We need a good way to break the tight link between relative prices and quantities. This is a topic being pursued in subsequent literature. We will discuss this topic further in later lectures.

Part 3:
Recent contribution: Burstein, Kurz, Tesar (2006)

Issue: To explain the wide variation in cross-country output correlations, in terms of different types of trade relationships.

Production sharing: as international integration increases, it is becoming for common for different stages of the production process to take place in different countries.

Observation: Countries that engage in internationalized production have higher output correlations.

Objective: Paper tries to model internationalized production in terms of intermediate inputs being complementary, and sees if it can replicate the higher output correlations.

Table 1) Share in Trade (%)

		Import from	Export to
US	EU15	18.6	22.5
	Eastern Europe	0.3	0.3
	Canada	18.9	21.1
	Mexico	8.1	9.6
EU15	US	19.5	18.9
	Eastern Europe	6.5	8.1
	Canada	1.9	2.0
	Mexico	0.6	1.1

EU trades much with US and with Eastern Europe, but it trades different things with each of these regions…

Figure 1) Trade pattern

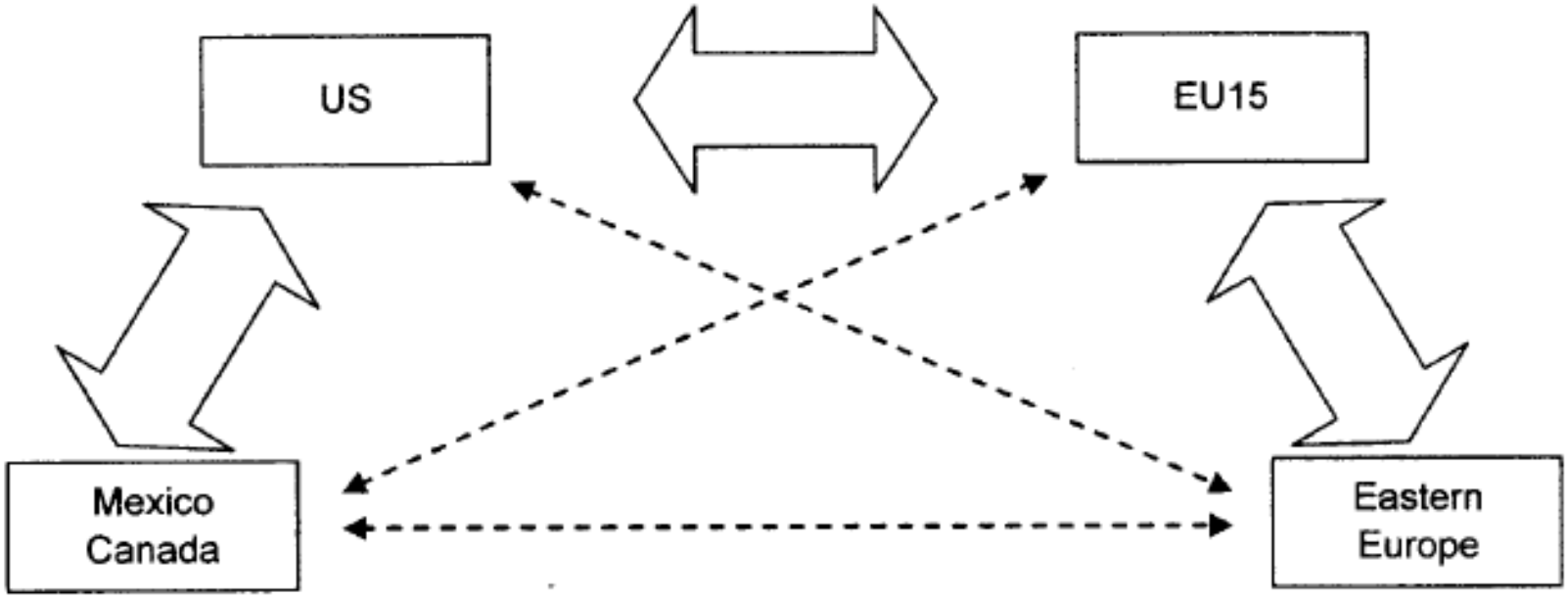


Table 2) Output Correration

US &	EU15	Canada	Mexico
	0.06	0.74	0.19

EU15 &	US	Eastern Europe
	0.06	0.43

Table 3) Sales of US affiliates to US
(as a share of total sales, %)

Canada	0.43
Mexico	0.35
Europe	0.06

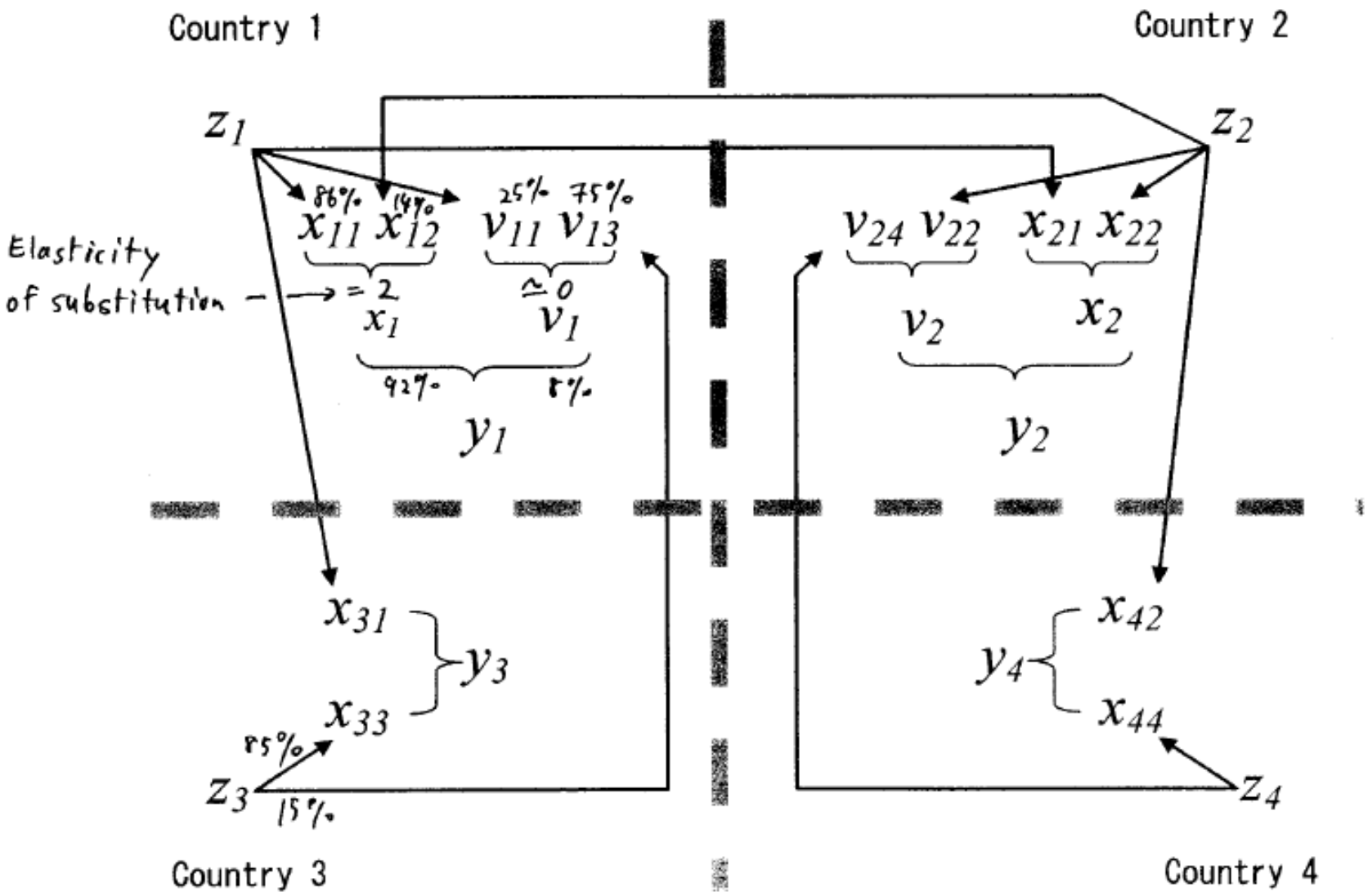
Model setup:

- Four countries:
  1 and 2 correspond to US and EU: Developed
  2 and 3 correspond to Mexico and E. Europe: developing

- Each country uses labor and capital to produce an intermediate good

- Country 1 combines its intermediate with that of country 2 as a substitute; and it combines it with that of country 3 as a complementary good

- So internationalized production is modeled as complementarity of intermediate inputs.


Country 1
Country 2
z_1
z_2
86%
14%
25%
75%
x_{11} x_{12}
v_{11} v_{13}
Elasticity of substitution → = 2
≃ 0
x_1
v_1
92%
8%
y_1
v_{24} v_{22}
x_{21} x_{22}
v_2
x_2
y_2
x_{31}
x_{33}
y_3
85%
15%
z_3
x_{42}
x_{44}
y_4
z_4
Country 3
Country 4

Model equations:

Problem

$$\max L_1U_1 + L_2U_2 + L_3U_3 + L_4U_4$$

s.t.

$$U_i = E_0 \sum \beta^t \frac{\left[c_{it}^{\mu}(1-n_{it})^{1-\mu}\right]^{1-\sigma}}{1-\sigma}, i = 1,2,3,4$$

$$z_{it} = A_i e^{s_{it}} n_{it}^{\alpha} k_{it}^{1-\alpha}, i = 1,2,3,4$$

Production (1^{st} stage)

$$L_1 z_{1t} = L_1 x_{11t} + L_1 v_{11t} + L_2 x_{21t} + L_3 x_{31t}$$
$$L_2 z_{2t} = L_2 x_{22t} + L_1 v_{22t} + L_1 x_{12t} + L_4 x_{42t}$$
$$L_3 z_{3t} = L_3 x_{33t} + L_1 v_{13t}$$
$$L_4 z_{4t} = L_4 x_{44t} + L_2 v_{24t}$$

Distribution of goods through trade

$$x_{1t} = \left[\theta_1^{1-\rho} x_{11t}^{\rho} + (1-\theta)^{1-\rho} x_{12t}^{\rho}\right]^{\frac{1}{\rho}}$$

$$x_{2t} = \left[\theta_1^{1-\rho} x_{22t}^{\rho} + (1-\theta)^{1-\rho} x_{21t}^{\rho}\right]^{\frac{1}{\rho}}$$

$$x_{3t} = \left[\theta_3^{1-\rho} x_{33t}^{\rho} + (1-\theta)^{1-\rho} x_{31t}^{\rho}\right]^{\frac{1}{\rho}}$$

$$x_{4t} = \left[\theta_3^{1-\rho} x_{44t}^{\rho} + (1-\theta)^{1-\rho} x_{42t}^{\rho}\right]^{\frac{1}{\rho}}$$

$$v_{1t} = \left[\lambda^{1-\varsigma} v_{11t}^{\varsigma} + (1-\theta)^{1-\varsigma} v_{13t}^{\varsigma}\right]^{\frac{1}{\varsigma}}$$

$$v_{2t} = \left[\lambda^{1-\varsigma} v_{22t}^{\varsigma} + (1-\theta)^{1-\varsigma} v_{24t}^{\varsigma}\right]^{\frac{1}{\varsigma}}$$

Elasticity of substitution

$$\frac{1}{1-\rho} = 2$$

Production
(2nd stage)

$$\frac{1}{1-\varsigma} \simeq 0$$

$$y_{1t} = x_{1t}^{\omega} v_{1t}^{1-\omega}$$
$$y_{2t} = x_{2t}^{\omega} v_{2t}^{1-\omega}$$
$$y_{3t} = x_{3t}$$
$$y_{4t} = x_{4t}$$

Production
(3^{rd} stage, final goods)

$$y_{it} = c_{it} + i_{it}, i = 1,2,3,4$$
$$i_{it} = k_{it+1} - (1-\delta)k_{it}, i = 1,2,3,4$$

Distribution of final goods

Simulation results:

	Cor (y1, y2)	Cor (y1, y3)	Cor(y1, y3)-Cor(y1, y2)
Benchmark	-0.04	0.02	0.05
High elasticity of substitution	-0.04	-0.02	0.01
Higher v3/y3 (15% to 30%)	-0.04	0.07	0.11
Financial autarky	0.04	0.06	0.02

Conclude:

- Can generate higher output correlation for production sharing countries than for non-sharing countries.
- But it is hard to generate the very high output correlations observed in data.
- There might be better ways of modeling production sharing. Future work.