Updated 1/21/08

Lecture 2: Investment in an intertemporal CA model
Part1: basic puzzle

a) Motivation:

- Investment is volatile and is the cause of much of the short-run fluctuations in the CA. So it is important to include in our intertemporal CA model a theory of what determines investment.

Some stylized facts:

- cor(CA,I) = -0.4 on average for G7 in post ‘75 period
- cor(S,I) = 0.6 on average for G7 (about 0.9 for US)
- Note that our simple model of the previous lecture suggests the latter correlation would be zero.

b) <u>Cross-sectional evidence: Feldstein and Horioka (EJ-1980)</u>

This was the first paper to document and popularize the saving-investment correlation puzzle: a high cor(S,I).

This is often taken as evidence of lack of capital mobility. It appears that changes in national saving pass through almost completely to investment in the country.

Data: The authors begin by computing the saving rate and investment rate for each country, averaged over a 15 year period. One sees below that the saving rate in each country is very close to the investment rate in that country. Here are some examples:

Examples:

country	S/Y	I/Y	(average rates over 1960-74)
USA	0.186	0.186	(lowest in sample)
Japan	0.372	0.368	(highest)
Germany	0.271	0.264	

Test: Run cross-sectional regressions, using the 15-year averages for 16 OECD countries.

$$(I/Y)_i = \alpha + \beta(S/Y)_i + u_i \quad \text{for country i}$$

- If capital is mobile (and other assumptions) then beta should be close to zero (depending on their share in the world capital market).
- If no capital mobility, then beta would be close to unity.

Results: estimate of beta:

Full sample (60-74): 0.887 (std error = 0.074)

Robustness: try a few variations:

3 sub-periods:

0.909 for 1960-64,
0.872 for 1965-69
0.871 for 1970-74

(One might expect it would decrease with time as capital markets become more integrated.)

- Divide total saving into three categories (household, corporate, government)

- Find some differences between them, but not statistically significant.

Part2: Theoretical Explanations

a) Explanation 1: Global shocks

- A more careful examination of intertemporal model suggests two main types of reasons why S might move voluntarily with I.

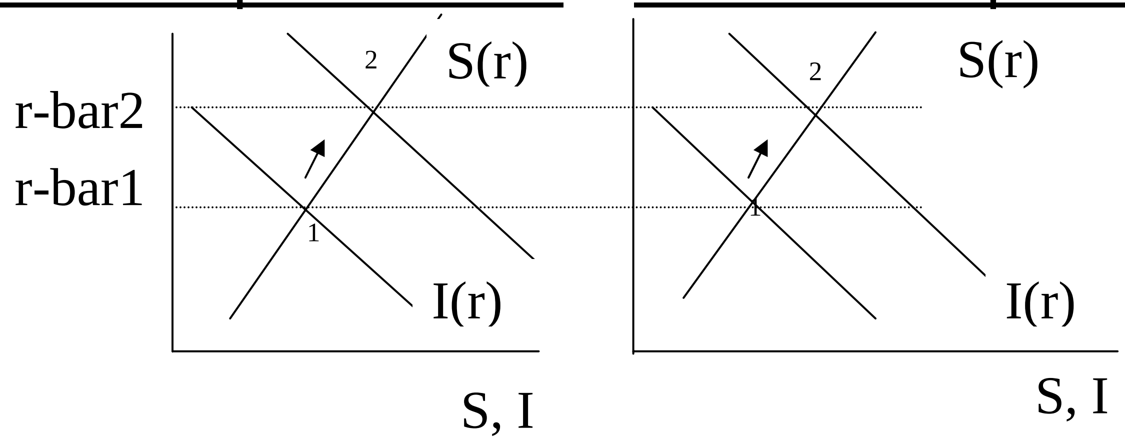
- First explanation focuses on global nature of shocks. Perhaps want to borrow but can't because rest of the world want to do same thing. All want to borrow and no one willing to lend.

Consider the market for loanable funds in a small open economy, where assume:

- Open: Capital is perfectly mobile: can borrow all want at going market world interest rate
- Small: domestic economy does not affect interest rate; this is determined in world capital market.

Case of world shock:

World capital market   Domestic capital market



Consider shock that causes domestic economy to want to increase investment. Want to smooth consumption, so try to borrow. But if the shock driving this is global in nature, demand for loanable funds is rising in the global market, and driving up the interest rate to point where people are content not borrowing.

Some support in data: S-I correlation larger for larger economies (US and Japan)

b) <u>Explanation 2: Technology shocks: Simple version:</u>

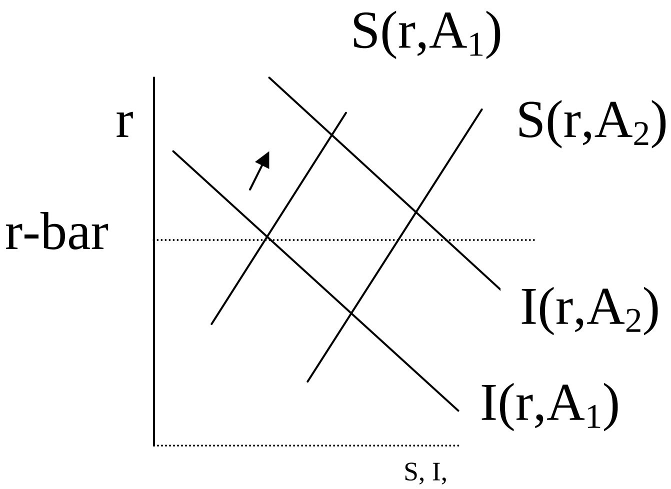
Second approach says to consider reasons why I might move. One possibility is that arise in technology also raises marginal product of capital.

If ouput is an increasing function of capital and technology with decreasing marginal product of capital. If technology rises temporarily, this would do several things:

- Raise output.
- Raise saving, if the rise in output is temporary.
- Raise investment because the marginal product of capital higher.

So saving and investment may move together.

Consider this in a small open economy context, supposing we start at CA=0 balance:



Shows case where S and I move roughly together.

Illustrate explanation 2: Small open economy with Invest.

features:

As before: fixed world interest rate, real bond is only asset

New: output is a function of capital and technology,

(no depreciation or adjustment cost on investment)

Abstract away from government spending.

Problem:

$$Max \ \ E_t \sum_{s=t}^{\infty} \beta^{s-t} U\left(C_s\right)$$

$$s.t. \ \ B_{t+1} - B_t = Y_t + rB_t - C_t - I_t \equiv CA_t$$

$$Y_t = A_t F\left(K_{t-1}\right)$$

$$I_t = K_t - K_{t-1}$$

Implications:

- Consumption smoothing under quadratic utility (as before)

$$U'_t = (1+r)\beta E_t\left[U'_{t+1}\right]$$

$$C_t = E_t\left[C_{t+1}\right]$$

- Implies usual current account equation:

$$CA_t = Y_t - I_t - C_t$$

$$= \beta\left(Y_t - I_t\right) - (1-\beta)\sum_{s=t+1}^{\infty}\beta^{s-t}E_t\left[Y_s - I_s\right]$$

- We will focus on two cases: completely temporary shocks or completely permanent shocks. In these two extremes, all future periods will be the same as each other, so we can simplify the condition above:

$$CA_t = \beta\left(Y_t - I_t\right) - \beta E_t\left(Y_{t+1} - I_{t+1}\right)$$

- So to find current account, we need to trace out what happens to output and investment now and in the future:

What determines investment and output:

- First order condition governing capital accumulation:

$$1 = E_t\left[\left(1 + A_{t+1}F'\left(K_t\right)\right)\beta\frac{U'_{t+1}}{U'_t}\right]$$

(skip)

$$= E_t\left[\left(1 + A_{t+1}F'\left(K_t\right)\right)\right]E_t\left[\beta\frac{U'_{t+1}}{U'_t}\right] + \text{cov}\left[\left(1 + A_{t+1}F'\left(K_t\right)\right), \beta\frac{U'_{t+1}}{U'_t}\right]$$

- Use consumption smoothing equation, and abstract from covariance term for now:

$$E_t\left[A_{t+1}F'(K_t)\right] = r$$

So accumulate capital until the *expected future* marginal product equals world real interest rate.

- Under a Cobb-Douglas production function, $Y_t = K^{\alpha}{}_{t-1}$

  This says: $K_t = \left(\frac{\alpha}{r} E_t \left[A_{t+1} \right] \right)^{\frac{1}{1-\alpha}}$

  So $Y_t = A_t \left(\frac{\alpha}{r} E_{t-1} \left[A_t \right] \right)^{\frac{\alpha}{1-\alpha}}$ This depends on last period's expectations.

- So investment is:

$$I_t = K_t - K_{t-1} = \left(\frac{\alpha}{r} E_t \left[A_{t+1} \right] \right)^{\frac{1}{1-\alpha}} - \left(\frac{\alpha}{r} E_{t-1} \left[A_t \right] \right)^{\frac{1}{1-\alpha}}$$

<u>Specify the shock process</u>:

Shock: $A_s - \overline{A} = \rho\left(A_{s-1} - \overline{A}\right) + \varepsilon_s$

Where $0 \leq \rho \leq 1$ indicates persistence.
ε_s is a serially uncorrelated shock with $E_{s-1}\left[\varepsilon_s\right] = 0$

We begin by studying the two extreme cases, where analytical solution is possible: $\rho = 0$ and $\rho = 1$.

Consider case 1: Temporary shock to productivity:

Shock: $A_s - \overline{A} = \rho\left(A_{s-1} - \overline{A}\right) + \varepsilon_s$

$\rho = 0, \quad \varepsilon_t > 0 \quad so \quad A_t > \overline{A}$ in period t

Steady state: $\overline{CA} = 0, \ \overline{I} = 0, \quad \overline{K} = \left(\frac{\alpha}{r}\overline{A}\right)^{\frac{1}{1-\alpha}}, \ \overline{Y} = \overline{A}\overline{K}^{\alpha}$

Use equations above to find investment and output:

$$I_t = K_t - K_{t-1} = \left(\frac{\alpha}{r}E_t\left[A_{t+1}\right]\right)^{\frac{1}{1-\alpha}} - \left(\frac{\alpha}{r}E_{t-1}\left[A_t\right]\right)^{\frac{1}{1-\alpha}} = \left(\frac{\alpha}{r}\overline{A}\right)^{\frac{1}{1-\alpha}} - \left(\frac{\alpha}{r}\overline{A}\right)^{\frac{1}{1-\alpha}}$$

$= 0$

$$E_t\left[I_{t+1}\right]=E_t\left[K_{t+1}\right]-K_t=\left(\frac{\alpha}{r}E_{t+1}\left[A_{t+2}\right]\right)^{\frac{1}{1-\alpha}}-\left(\frac{\alpha}{r}E_t\left[A_{t+1}\right]\right)^{\frac{1}{1-\alpha}}=\overline{K}-\overline{K}=0$$

$$Y_t=A_t\overline{K}^{\alpha}>\overline{Y}$$

$$E_t\left[Y_{t+1}\right]=\overline{A}\overline{K}^{\alpha}=\overline{Y}$$

Note that the capital stock is unaffected in period t.

Plug into current account equation from above:

$$CA_t=\beta\left(Y_t-E_tY_{t+1}-I_t+E_tI_{t+1}\right)=\beta(Y_t-\overline{Y})$$

This is just like the effect of a temporary endowment shock in previous models.

Consider case 2: Permanent shock to technology:

Shock: $\rho = 1, \quad \varepsilon_t > 0 \quad so \quad A_t > \overline{A}$ in period t

and $E_t\left[A_{t+1}\right] = A_t > \overline{A}$ in period t+1

Use equations above to find investment and output:

$$I_t = K_t - K_{t-1} = \left(\frac{\alpha}{r}A_t\right)^{\frac{1}{1-\alpha}} - \left(\frac{\alpha}{r}\overline{A}\right)^{\frac{1}{1-\alpha}} > 0$$

$$E_t\left[I_{t+1}\right] = 0$$

$$Y_t = A_t \overline{K}^{\alpha} > \overline{Y} \quad (Same \ value \ as \ in \ temporary \ case)$$

$$E_t\left[Y_{t+1}\right] = A_t\left(\frac{\alpha}{r}A_t\right)^{\frac{\alpha}{1-\alpha}} > Y_t > \overline{Y}$$

Plug into current account equation from above:

$$CA_t = \beta\left(Y_t - E_t\left[Y_{t+1}\right] - I_t + E_t\left[I_{t+1}\right]\right) < 0$$

The current account now falls. This is for two reasons:

- First output in future periods is higher than the current period, so consumption smoothing makes consumption higher than current income, so saving falls.

- Second, this is compounded by the fact that there is a rise in investment, dragging the current account down further.

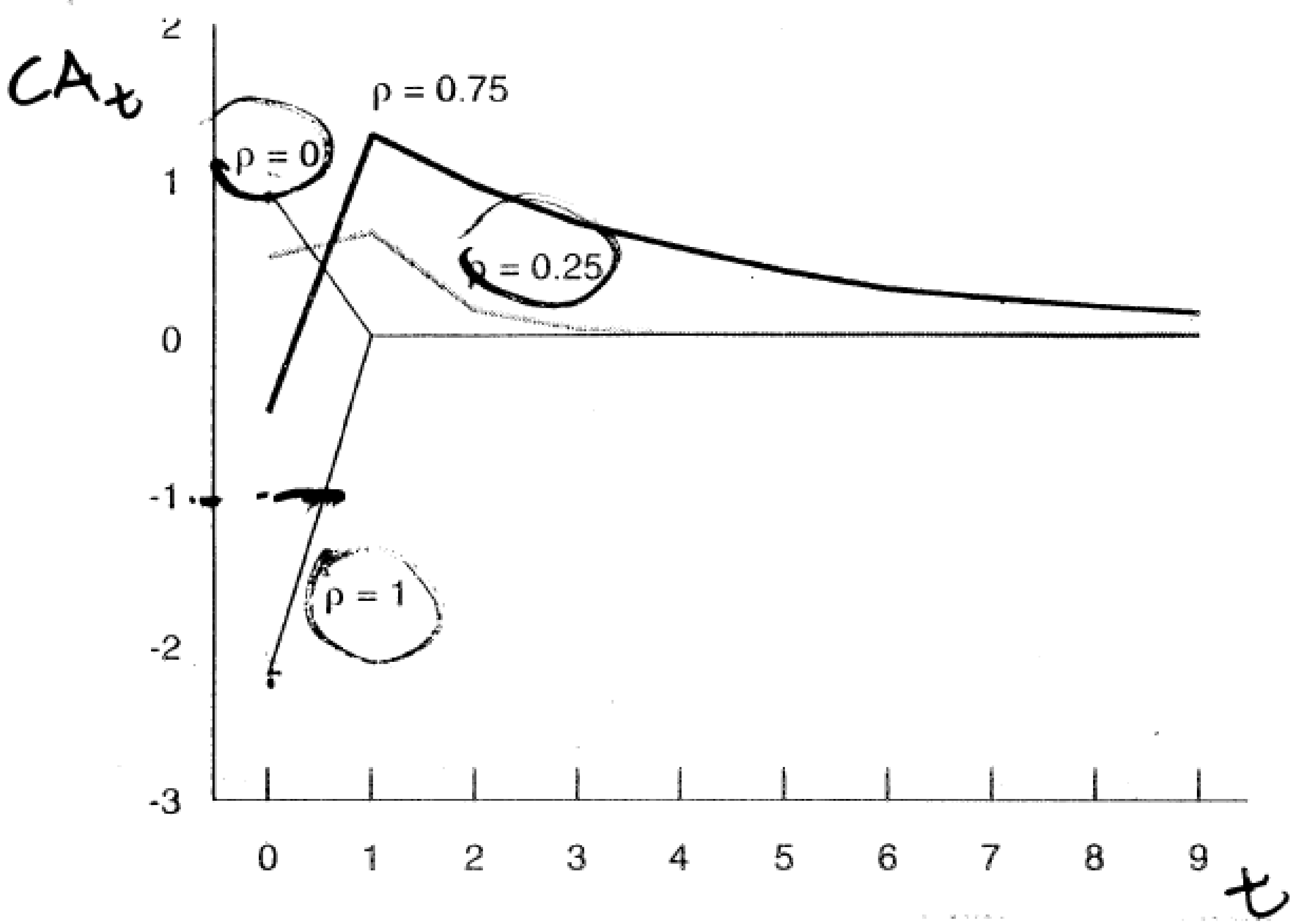
- Note that since $\Delta CA_t = \Delta S_t - \Delta I_t$ and saving is falling while investment is rising, this implies that the **fall in the current account is larger than the change in investment:** $\left|\Delta CA_t\right| > \left|\Delta I_t\right|$

To summarize: A temporary positive technology shock leads to a current account surplus; a permanent positive technology shock leads to a large deficit.

We can consider cases between these two extremes, where

$$\left(A_s - \overline{A}\right) = \rho\left(A_{s-1} - \overline{A}\right) + \varepsilon_s \qquad 0 < \rho < 1$$

These intermediate cases lead to current account balances closer to zero, with saving and investment moving more closely together. See figure.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Part3: Empirical Work Glick and Rogoff (JME 1995)

idea: Try some simple tests of the predictions of the theoretical models discussed above. Focus is on technology shocks, both domestic and global.

Reduced form regressions:

- Annual data for the G7 countries, 1961-1990, on investment, current account.
- Run time series regressions in first differences. For each country, regress the change in current account on the change in investment.

$$\Delta CA_t = a + b\Delta I_t$$

- Results: (see transparency of table 1) The results show that the correlation is significantly negative for all the countries, as predicted by the theory. This is especially true for a post 1974 sub-sample.

Table 1

Time-series regressions of current account on investment. $\Delta CA_t = a + b\Delta I_t$

Country	Sample period	b	R^2	$D.W.$
U.S.	1961–90	− 0.16 (0.07)**	0.18	1.44
Japan	1961–90	− 0.32 (0.07)**	0.40	1.27
Germany	1961–90	− 0.29 (0.11)**	0.21	1.94
France	1968–90	− 0.37 (0.11)**	0.34	1.82
Italy	1961–90	− 0.55 (0.08)**	0.61	1.95
U.K.	1961–90	− 0.53 (0.09)**	0.53	2.08
Canada	1961–90	− 0.31 (0.08)**	0.37	2.06

More structural regressions:

- Want to be more structural and look at the effects of technology shocks, as discussed in the theoretical model discussed earlier in the lecture.

- Get estimates of technology by computing a Solow residual

$$\log(A) = \log(Y) - \alpha \log(L)$$

  (Note that this measure ignores changes in capital input.)

  The labor share parameter is calibrated based on the OECD database, and it varies from a low of 0.48 for Italy up to 0.68 for the UK.

- The authors want to distinguish between the effects of world technology shocks (A^w) and country-specific technology shocks (A^c). Recall that this distinction matters for the current account: A rise in country-specific technology induces a rise in investment financed by a current account deficit. But a global shock cannot be financed by borrowing abroad.

- They define the global shock as the average over the G7 economies, and define the country-specific as the residual left over after subtracting this global shock from the Solow residual for that country.

- These technology shocks are very persistent. (see table 2) Dickey-Fuller tests show we cannot reject they are nonstationary. So the analysis of the “fully permanent” case of the model earlier in the lecture may apply here.

- The form of their regressions is as follows:

$$\Delta I_t = a_0 + a_1 \Delta A^c_t + a_2 \Delta A^w_t + a_3 I_{t-1}$$

$$\Delta CA_t = b_0 + b_1 \Delta A^c_t + b_2 \Delta A^w_t + b_3 I_{t-1}$$

Note: These regression equations presume that technology shocks follow a random walk. Otherwise they would involve some additional lagged terms representing dynamics.

<u>Intertemporal theory predicts:</u>

$a_1>0$, $b_1<0$ $\uparrow A^c \rightarrow \uparrow I$ and $\downarrow$ CA

$abs(b_1) > a_1$ because S falls if Ac is permanent

(and data can't reject that techno shocks are permanent)

$a_2>0$, $b_2=0$ $\uparrow A^w \rightarrow \uparrow I$, can't borrow, so no change in CA

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Results:

For the pooled regressions over all countries: (SUR) (See table 4)

a1 = 0.35, b1 = -0.17, a2>0 and significant

b2 not signif dif from zero

But reject remaining hypothesis: abs(b1) > a1: saving not fall in Ac shock.

- The regression results for each country are generally very consistent with main hypotheses.

- For the investment equation: the coefficients all the right sign, and significantly so in 80% of the cases. Questionable why world shocks have a bigger effect than country specific shocks.

- For the current account equation: right sign for persistent shocks: country-specific shocks lower the current account, world shocks have small effects not significantly different from zero (true for all countries except UK)

Conclusions:

- Generally supportive of main predictions of the theory.

- Why reject last one: must be that techno shock is not totally permanent. Data cannot reject unit root, but also cannot reject other values little below it.

- The authors show by simulation that if rho on country-specific shocks is lowered even a small amount below unity (0.97), it counterbalances the effect of lagged output rise with capital. So income is higher on impact than pdv of future income, so saving rises and CA falls less than I rises. (See table 9)

Table 9
Rho is the AR coefficient,
Beta2 is the change in investment
Gamma2 is the change in current account

ρ	β_2	γ_2	$\lvert\gamma_2\rvert/\beta_2$
1.00	0.35	−0.97	2.76
0.99	0.35	−0.60	1.72
0.98	0.34	−0.35	1.04
0.97	0.32	−0.21	0.64
0.96	0.31	−0.04	0.13
0.95	0.30	0.05	0.18

Other significant papers on this topic:

Nason and Rogers (JMCB 2002): use structural VARs to identify the productivity shocks. Find that results depend heavily on the identifying restrictions.

Baxter and Crucini (AER 1993): (presented as time permits)

- Test theoretical explanations for saving-investment correlations by stochastic simulation of a business cycle model. These models are developed in the next section.
- They find that S-I correlations can be generated by the theories listed above and by large country size.

Part 4: Simulation Model of Baxter and Crucini (AER 1993)

Introduction

This paper makes use of both of the explanations above:

- technology shocks are correlated across countries, and
- technology shocks are of less than complete permanence, so consumption smoothing requires a rise in saving along with the rise in investment

The model also differs from those we have seen so far, in that it is a 2- country model. This means that there is yet a third reason why saving and investment might move together. A shock affecting one country may have effects on the world real interest rate. This factor becomes more important as a country becomes larger.

The case examined in this model is too complicated to solve analytically, so instead the authors rely on stochastic simulations to see if the model can replicate the correlation between saving and investment similar to that in the data.

Stylized facts:

The paper first documents the fact that saving and investment rates are highly correlated. They use time series evidence, whereas Feldstein and Horioka used cross-sectional evidence.

This table demonstrates that the puzzle is not just a long run phenomenon, but also applies to shorter run fluctuations, of the type we have analyzed in our models.

The table also indicates that the saving-investment correlation is higher for large countries (like the U.S. and Japan) than for small countries.

Table 1—Saving –investment correlations

Country	GNP (1985 U.S. $)	cor(S,I)
U.S	3,994	0.86
Japan	1,365	0.80
Germany	667	0.68
France	527	0.31
Italy	372	0.39
Canada	347	0.61
Australia	171	0.54
Switzerland	106	0.65

Model:

General description:

- Two countries (home and foreign-with a star) - new
- One good (so only reason for trade is to smooth consumption) -familiar
- Households value leisure as well as consumption. This is new, but leisure acts here like a particular nontraded good here, which we have seen before.
- Firms produce output using labor as well as capital -new
- The only shocks are to technology in the production function -familiar

- Household maximizes:

$$E_t \sum_{t=0}^{\infty} \beta^t \frac{1}{1-\sigma} \left(C_t^{\theta} L_t^{1-\theta} \right)^{1-\sigma}$$

where L is leisure not labor. Likewise abroad.

- Firm produces output using a constant returns technology:

$$Y_t = A_t K_t^{1-\alpha} \left(X_t N_t \right)^{\alpha}$$

where N is labor, K is capital, an X is labor-augmenting technical change (which permits the model to have a steady state growth rate per capita).

The productivity shocks are assumed to follow a Markov process (where ^ indicates a percent deviation from steady state).

$$\begin{bmatrix} \hat{A}_t \\ \hat{A}_t^* \end{bmatrix} = \begin{bmatrix} \rho & \upsilon \\ \upsilon & \rho \end{bmatrix} \begin{bmatrix} \hat{A}_{t-1} \\ \hat{A}_{t-1}^* \end{bmatrix} + \begin{bmatrix} \varepsilon_t \\ \varepsilon_t^* \end{bmatrix}$$

where $E(\varepsilon_t) = E(\varepsilon_t) = 0$

innovations that originate in one country are transimitted to the other country, depending on υ (a diffusion parameter).

The covariance matrix of the shocks allows the technology shocks to be correlated across countries.

$$E\left(\varepsilon_t, \varepsilon_t^*\right)\left(\varepsilon_t, \varepsilon_t^*\right)' = \begin{bmatrix} \sigma_\varepsilon^2 & \psi \\ \psi & \sigma_\varepsilon^2 \end{bmatrix}$$

- <u>Capital is accumulated</u> subject to an adjustment cost:

$$K_{t+1} = (1-\delta)K_t + \phi(I_t / K_t)K_t$$

where δ is the depreciation rate, and $\phi(I_t / K_t)$ is a function describing adjustment cost.

- The two countries are permitted to be different sizes, where π is the fraction of the world population in the home country. This means the world resource constraint is:

$$\pi(Y_t - C_t - I_t - G_t) + (1-\pi)\left(Y_t^* - C_t^* - I_t^* - G_t^*\right) = 0$$

This is like the single-period budget constraints we have seen before, but here it pools together the two countries.

- The resource constraint of time is that labor plus leisure must equal the endowment of one unit of time: $L_t + N_t = 1$. (analogously abroad)

- Finally, the model permits government purchases (here fixed) and transfers.

- Market structure: The authors assume a complete set of Arrow-Debreu securities can be traded, though these are not explicitly modeled here. This replaces the non-contingent bonds we were assuming in past models. (Lecture 4 will discuss in detail)

- The model will be solved as a log-linear approximation around the deterministic steady state.

Calibration:

Some parameters are chosen to make the model's steady state replicate long-run averages in the data:

- δ=2.5% per quarter
- α is set so that labor income is 0.58 of GNP
- θ is set so that 20% of time is set aside for work
- The rate of grown tin labor augmenting technology is set at 1.004 to reflect average quarterly steady state growth rates.

Some parameters are set to match econometric studies: σ=2 to reflect some past studies on risk aversion.

Some parameters are set to match dynamic properties of the economy.

- The variance and covariance of technology shocks are chosen so that the model replicates the variance and covariance of GDP across countries.

- A range of values are explored for the first and second derivative of the investment adjustment cost function, ϕ, and for the elasticity of I with respect to the first derivative of ϕ (here denoted as η). Values are used that allow the model to match moments in investment fluctuations.

Results:

Table 2:

Simulation Results for Baxter and Crucini (AER 1993):

Case	Cor(S,I)
1) Benchmark parameterization: ρ=0.93, ν=0.05, ψ=0.4, σ=2, η=15, two equal sized countries	0.93
2) intertemporal substitution: σ=10	0.93
3) correlation of shocks:	
ψ=0	0.93
ν=0	0.92

4) persistence of shocks	
ρ = 0.99	0.77
ρ = 0.70	0.84
ρ = 0.50	0.79
5) Cost of adjusting capital stock (inversely related to $\hbar$)	
η=1	0.45
η=5	0.79
η=100	0.64
η=100,000	-0.09
6) One large / one small country	
large	0.99
small	0.85

1) Benchmark parameterization gives a correlation between saving and investment that is quite close to that found in the data. This shows how easy it is to generate large correlations despite the fact that capital is fully mobile.

2) Consider role of intertemporal elasticity: consider a value of σ=10 instead of 2, so the intertemporal elasticity is 0.10, which would imply that households more aggressively smooth their consumption over time. This does not appear to affect the S-I correlation.

3) Consider role of correlated shocks: try zeroing out the correlation of shocks and the diffusion parameter. This has very little effect. Cor(S,I) not due to correlation of shocks.

4) Consider persistence of the technology shock: S-I correlation falls for a persistence parameter lower than the benchmark. But it also appears to fall for the case of extremely high persistence (ρ = 0.99) Recall from our previous theory that as technology shocks become extremely persistent, this means that output rises yet further in subsequent periods as the capital stock rises. In this context, consumption smoothing calls for a fall in saving, not a rise.

5) Consider the cost of adjusting the capital stock: As the importance of the cost of adjusting the capital stock falls, investment becomes more responsive to technology shocks. The implications may reflect a similar effect to that described above. If investment rises a lot, then future

output is higher than current output for persistent shocks. This would imply a fall in current saving compared to the large rise in investment. This is what likely generates the negative correlation reported in the table.

6) Consider countries of different sizes: if one country is 90% of total, with a small country being 10%. The result for the benchmark parameter setting is that the large country has a higher saving investment correlation than found before. This clearly is because if the large country wishes to invest and borrow for this, it affects the equilibrium real interest rate in the world market. This encourages higher domestic saving. This story is less true for a small country.