Part 1.
Preliminaries: National Income Accounting and Data

Let's agree on some definitions:

- GDP: Gross Domestic Product: Total value of all final goods and services produced within a country's borders.

- This can be measured as the value added: sales minus payments for intermediate inputs of all firms.

- Can decompose this into expenditure categories:

$$GDP = C + I + G + TB$$

  - C: consumption
  - I: investment
  - G: government consumption
  - TB: trade balance = exports - imports

- GNI: Gross national Income: total value of all income earned by a country's factors of production (without regard to location). This implies:

$$GNI = GDP + NFIA$$

- NFIA: net factor income from abroad = (foreign income payments to domestic factors of production) – (domestic income payments to foreign factors of production).

Ireland has high output (GDP) per person, but much lower income (GNI almost 20% lower)

Rank	GDP per capita	GNI per capita
1	Luxembourg	Luxembourg
2	United States	United States
3	Norway	Norway
4	**Ireland**	Switzerland
5	Switzerland	Canada
6	Canada	Denmark
7	Denmark	United Kingdom
8	Netherlands	Netherlands
9	Austria	Belgium
10	Iceland	Iceland
11	Australia	Austria
12	United Kingdom	Australia
13	Belgium	Japan
14	France	France
15	Sweden	Sweden
16	Japan	Finland
17	Finland	**Ireland**
18	Germany	Germany
19	Italy	Italy
20	Spain	Spain

- Gross national disposable income (GNDI):

$$GNDI = GNI + NUT$$

Includes net unilateral transfers (NUT) : international gifts, negative entry for giving country; positive for receiving country.
(balances exports bought with foreign aid)

- When we include unilateral transfers on the right hand side of our accounting equation…

$$GNDI = C + I + G + \underbrace{\{TB + NFI + NUT\}}_{CA}$$

- CA: current account: consists of all international transaction of goods, services, and income.

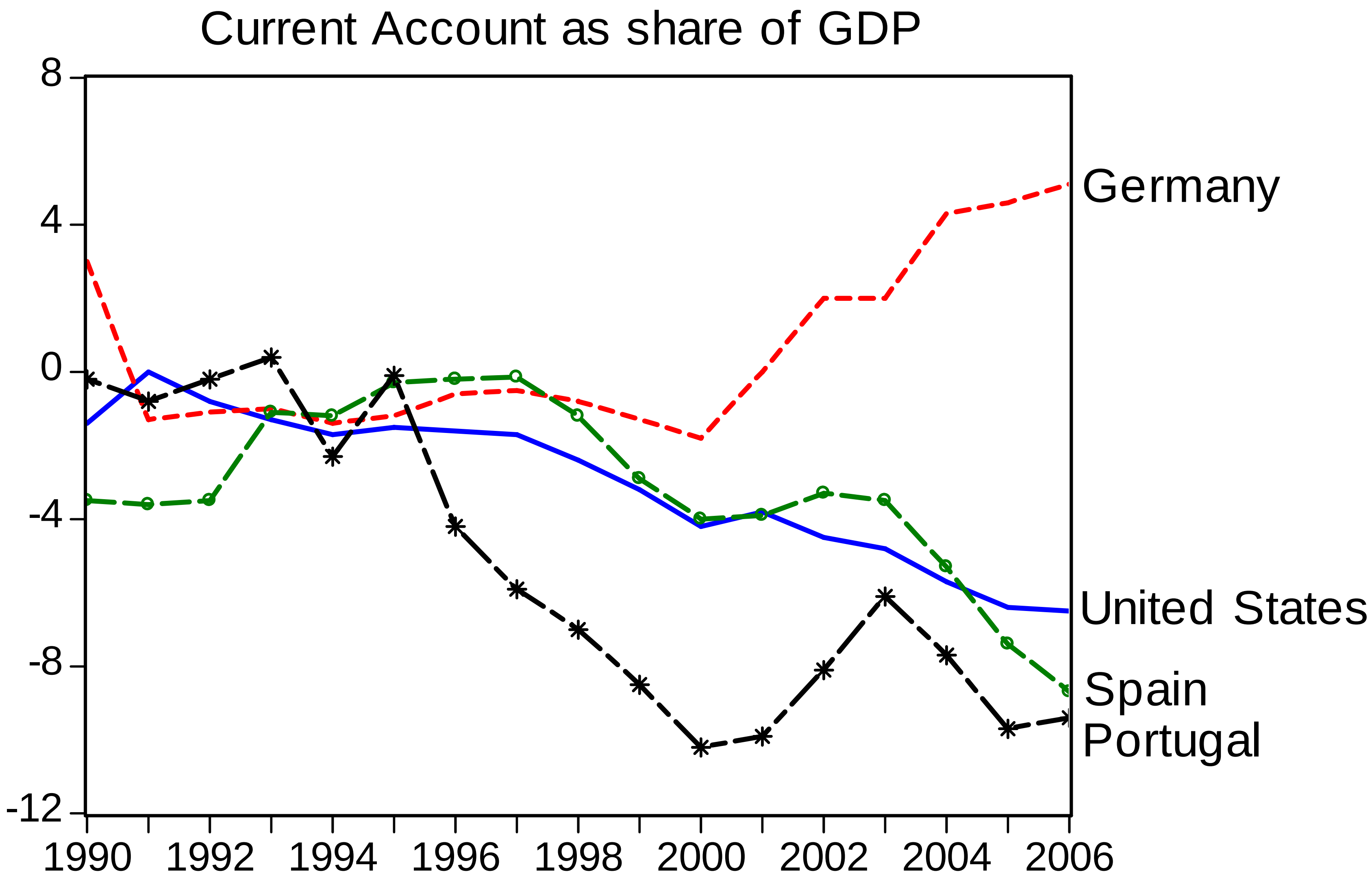
Balance of Payments accounts (BOP): constructed to measure all international transactions.

- Goods, services and income transactions measured by current account above.
- Asset transactions measured by Financial Account (FA): bonds, stocks, money, government foreign currency reserves, factories, land, ownership of bank accounts, etc.

BOP rule: each international transaction gives rise to two entries in some category of BOP accounts, one positive and one negative.

This implies the Balance of payments identity:

$$CA + FA = 0$$



Current Account as share of GDP
8
4
0
-4
-8
-12
1990
1992
1994
1996
1998
2000
2002
2004
2006
Germany
United States
Spain
Portugal

Source: OECD

Question: where are the large current account deficits coming from? One possibility…

- Twin deficits hypothesis: tendency for government budget deficits to cause current account deficits.

- To evaluate this claim, decompose total national saving (S) into two parts. Total saving (S) =

    - public saving by the government sector
      $S_g = T - G$, where T is taxes

    - private saving by households and firms
      $S_p = Y - T - C$

$$GNDI = C + I + G + CA$$

$$CA = GNDI - C - G - I$$

$$= (GNDI - T - C) + (T - G) - I$$

$$= s^p + s^g - I$$

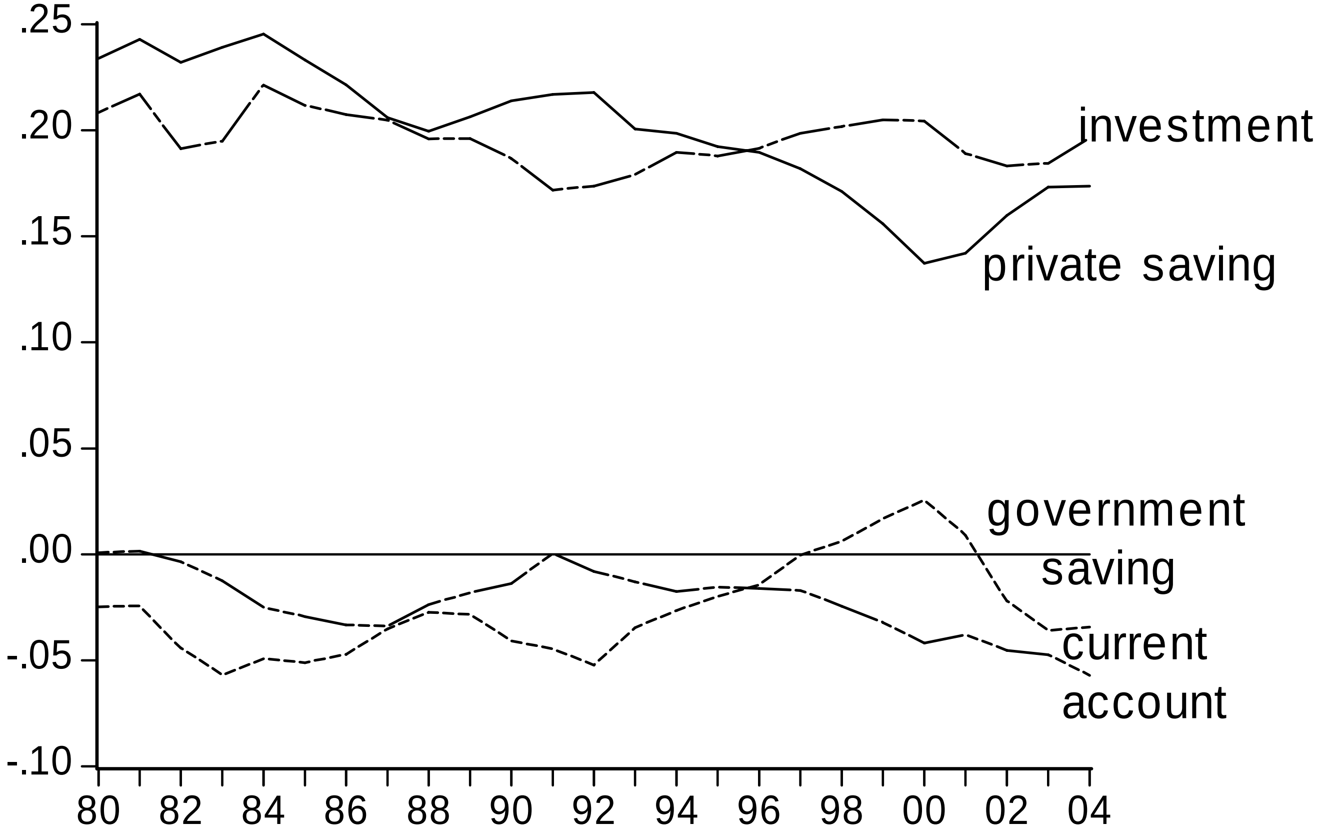
$$= \text{private saving - government deficit - I}$$

- Implication: All else equal an increase in the government deficit causes an increase in the current account deficit. Is all else equal?

- In the US data below, which of these components contributes to the CA deficit?

Figure 2
U.S. current account and components

share of GNP



Source: IMF

Questions:

- When is it justified to run a current account deficit?
- How large a deficit is too large?

The simple accounting exercises above cannot answer these questions. We need a formal model.

Part 2.
A Two-period model of the current account

Assumptions:

- **Open**: can borrow freely at the world real interest rate (*r*)
- **Small**: actions of domestic agents do not affect the world capital market. So the world interest rate is exogenous. We assume here it is fixed.
- **One world good used** for consumption (*C*).
- **Endowment economy**, with output levels (*Y*) exogenous.
- Government spending and investment also exogenous (No role for G in utility or I in production)
- **Riskless bond** is only asset (*B*)
- **Representative agent** lives two periods and chooses consumption for each period.
- Discounts future at rate β. Assume $\beta = 1/(1+r)$.
- **No uncertainty:** perfect foresight

<u>Problem</u>: maximize discounted sum of utility subject to the budget constraints.

$$\max_{C_1, C_2} \quad U\left(C_1\right) + \beta U\left(C_2\right)$$

$$s.t.\, Y_1 - I_1 - G_1 - C_1 = B \quad \text{period 1 budget constraint}$$

$$Y_2 + \left(1 + r\right) B - I_2 - G_2 - C_2 = 0 \quad \text{period 2 budget constraint}$$

$$\text{where } \mathrm{U}\left(\mathrm{C_t}\right) \equiv \frac{1}{1-\sigma} C_t^{1-\sigma}$$

Note that the budget constraints reflect the national income and balance of payments identities.

Period 2 budget constraint may be rewritten:

$$B = \frac{C_2 - NO_2}{1 + r}$$

$$\text{where} \quad NO_t \equiv Y_t - I_t - G_t$$

Substitute this into the period 1 constraint to find the intertemporal budget constraint

$$C_1 + \frac{C_2}{1+r} = NO_1 + \frac{NO_2}{1+r}$$

An easy way to take the maximum is to use the intertemporal budget constraint to substitute out for period 2-consumption in the objective:

$$C_2 = -(1+r)C_1 + (1+r)NO_1 + NO_2$$

so

$$\max_{C_1} \quad \frac{1}{1-\sigma}(C_1)^{1-\sigma} + \beta\frac{1}{1-\sigma}\left[-(1+r)C_1 + (1+r)NO_1 + NO_2\right]^{1-\sigma}$$

Find the maximum by setting derivative equal to zero:

$$C_1^{-\sigma} + \beta\left[(1+r)C_1 - (1+r)NO_1 - NO_2\right]^{-\sigma}(1+r) = 0$$

$$C_1 + \left(\beta(1+r)\right)^{\frac{-1}{\sigma}}\left[(1+r)C_1 - (1+r)NO_1 - NO_2\right] = 0$$

Simplifies if impose our assumption that $\beta = 1/(1+r)$

$$(2+r)C_1 - (1+r)NO_1 - NO_2 = 0$$

$$\frac{2+r}{1+r}C_1 = NO_1 + \frac{NO_2}{1+r}$$

or

$$C_1 = \left(\frac{1+r}{2+r}\right)NO_1 + \left(\frac{1}{2+r}\right)NO_2$$

Note the Consumption smoothing behavior:

from above:

$$\frac{2+r}{1+r}C_1 = NO_1 + \frac{NO_2}{1+r}$$

If we substitute this back into the intertemporal budget constraint:

$$C_1 + \frac{C_2}{1+r} = NO_1 + \frac{NO_2}{1+r}$$

We get:

$$C_1 + \frac{C_2}{1+r} = \frac{2+r}{1+r}C_1$$

So

$$\frac{C_2}{1+r} = \frac{C_1}{1+r} \quad \text{or} \quad C_1 = C_2$$

Interpretation: household wishes to smooth consumption across time periods.

Deriving Current Account behavior:

In this context, the current account becomes:

$$CA_1 = NO_1 - C_1$$

Substitute in our solution for consumption above:

$$C_1 = \left(\frac{1+r}{2+r}\right)NO_1 + \left(\frac{1}{2+r}\right)NO_2$$

$$CA_1 = NO_1 - \left[\left(\frac{1+r}{2+r}\right)NO_1 + \left(\frac{1}{2+r}\right)NO_2\right]$$

To get:

$$CA_1 = \frac{1}{2+r}\left(NO_1 - NO_2\right)$$

Or equivalently

$$CA_1 = \frac{\beta}{1+\beta}\left(NO_1 - NO_2\right)$$

Interpretation of $CA_1 = \frac{\beta}{1+\beta}\left(NO_1 - NO_2\right)$

Current account depends on how output is expected to change over time.

Consider:

If $NO_1 > NO_2$, run CA surplus in period 1 as save for future in order to smooth consumption.

If $NO_1 < NO_2$, run CA deficit in period 1 as borrow from future in order to smooth consumption.

This logic applies to all the components of NO: output, investment, and government consumption.

Implications for the Twin Deficits Hypothesis:

To show the role of government budget deficit explicitly, need to introduce lump-sum taxes and government borrowing into the two-period model.

Define: T lump-sum taxes

B^G government issue of bonds

Household budget constraints become:

$$Y_1 - I_1 - T_1 - C_1 = B \quad \text{period 1 budget constraint}$$

$$Y_2 + (1+r)B - I_2 - T_2 - C_2 = 0 \quad \text{period 2 budget constraint}$$

$$C_1 + \frac{1}{1+r}C_2 = (Y_1 - I_1 - T_1) + \frac{1}{1+r}(Y_2 - I_2 - T_2) \text{ intertemporal constraint}$$

Government has its own budget constraints:

$$B^G = G_1 - T_1 \quad \text{period 1}$$

$$(1+r)B^G = T_2 - G_2 \quad \text{period 2}$$

$$G_1 - T_1 = -\frac{1}{1+r}(G_2 - T_2) \quad \text{intertemporal}$$

Combine household and government constraints:

$$C_1 + \frac{1}{1+r}C_2 = (Y_1 - I_1 - G_1) + \frac{1}{1+r}(Y_2 - I_2 - G_2)$$

This constraint is the same as for the case we solved above, hence the optimal consumption path is the same and current account is the same.

Interpretation:

- Solution for C and CA above still holds: high government spending implies current account deficit.
- Under the assumptions in this model, the timing of the taxes does not affect consumption or the current account (Ricardian model).

<u>Does Twin deficits hypothesis hold</u>:

It depends: If the government deficit results from high government spending ($G1>G2$), then will imply a current account deficit. If it results just from low taxes ($T1<T2$) alone, then does not imply a current account deficit.

Homework will ask you to demonstrate this to yourself in an example.

Part 3.
An infinite horizon intertemporal current account model

Now we generalize the model to a representative agent that lives more than two periods (infinite), and to stochastic endowments.

$$Max \;\; E_t \sum_{s=t}^{\infty} \beta^{s-t} U\left(C_s\right)$$

$$s.t. \;\; B_{s+1} - B_s = Y_s + rB_s - C_s - I_s - G_s \equiv CA_s$$

- Where Y, I and G are subject to shocks that are independently and identically distributed (i.i.d.) in each period.
- Note the role of the expectations operator.
- The budget constraint implies the BOP identity: –FA= CA.
- And note that it coincides with our national income accounting, where Y is GDP, and Y + rB is GNI in the context of this model (= GNDI since no NUT).

The <u>intertemporal budget constraint</u> can be computed

- by recursively substituting the single-period budget constraint into itself (as we did in two-period model)
- and imposing the condition that the present value of wealth goes to zero in the long run (transversality condition): $\lim_{s\to\infty}\left(\frac{1}{1+r}\right)^{s-t}\left(B_s\right)=0$,
- which rules out Ponzi schemes, where borrower rolls over debt forever without repayment.

The resulting Intertemporal budget constraint:

$$\sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}\left(C_s+I_s+G_s\right)=\left(1+r\right)B_t+\sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}\left(Y_s\right)$$

Interpretation: present value of total expenditure equals present value of total income plus initial wealth.

We need to use the tools of dynamic programming to solve for infinite horizon case…

We make use of the recursive nature of the problem facing the consumer, where we maximize the current consumption choice, conditional on the assumption that we will face the same optimization decision in all future periods.

Define a value function: the maximized value of the objective function, the discounted sum of all future utilities, given some initial value of bond holdings.

$$V\left(B_t\right) = \max_{C_s, B_{s+1}} \sum_{s=t}^{\infty} \beta^{s-t} U\left(C_t\right)$$

Then $V\left(B_{t+1}\right)$ is the value of utility that can be obtained with a beginning level of wealth in period $s = t+1$, and $\beta V\left(B_{t+1}\right)$ would be this discounted back to period $s=t$.
So rewrite the problem as:

$$V\left(B_t\right) = \max_{C_t, B_{t+1}} \left[U\left(C_t\right) + \max_{C_{t+1}, B_{t+2}} \sum_{s=t+1}^{\infty} \beta^{s-t} E_t U\left(C_s\right) \right]$$

$$= \max_{C_t, B_{t+1}} \left[U\left(C_t\right) + \beta E_t V\left(B_{t+1}\right) \right]$$

$$s.t.\ B_{s+1} - B_s = Y_s + rB_s - C_s - I_s - G_s \equiv CA_s$$

Incorporate the constraint by a Lagrangian. This is the Bellman equation.

$$V_t\left(B_t\right) = \max \left\{ U\left(C_t\right) + \beta E_t \left[V_{t+1}\left(B_{t+1}\right) \right] \right\} +$$

$$\lambda_t \left(Y_t + \left(1+r\right) B_t - C_t - I_t - G_t - B_{t+1} \right)$$

Take derivatives to find the first order conditions:

$$C_t : U'(C_t) = \lambda_t$$

$$B_{t+1} : \beta E_t \left[\frac{\partial V_{t+1}}{\partial B_{t+1}} \right] = \lambda_t$$

So: $$U'(C_t) = \beta E_t \left[\frac{\partial V_{t+1}}{\partial B_{t+1}} \right]$$

This equates the marginal utility of consuming current output to the marginal utility of allocating it to bonds and enjoying augmented consumption next period.

Now, to find $\frac{\partial V_{t+1}}{\partial B_{t+1}}$, take the derivative of the original problem (Lagrangian) with respect to B_t.

Recall $$V_t\left(B_t\right)=\max\ \left\{U\left(C_t\right)+\beta E_t\left[V_{t+1}\left(B_{t+1}\right)\right]\right\}+$$
$$\lambda_t\left(Y_t+\left(1+r\right)B_t-C_t-I_t-G_t-B_{t+1}\right)$$

So the derivative is: $$\frac{\partial V_t}{\partial B_t}=\lambda_t\left(1+r\right)$$

Update this one period

$$\frac{\partial V_{t+1}}{\partial B_{t+1}}=\lambda_{t+1}\left(1+r\right)$$

Combining with the FOC $U'\left(C_t\right)=\lambda_t$ we find the envelope condition:

$$\frac{\partial V_{t+1}}{\partial B_{t+1}}=U'\left(C_{t+1}\right)\left(1+r\right)$$

Combine with FOC $U'\left(C_t\right)=\beta E_t\left[\frac{\partial V_{t+1}}{\partial B_{t+1}}\right]$ to get

$$U'\left(C_t\right)=\beta\left(1+r\right)E_t\left[U'\left(C_{t+1}\right)\right]$$

Or under our assumption $\beta = \dfrac{1}{1+r}$

$$U'(C_t) = E_t\left[U'(C_{t+1})\right]$$

The optimal behavior is to smooth marginal utility of consumption in expectation.

Under our assumed utility function: $U(C_t) = C_t - \dfrac{1}{2}C_t^2$

This is
$$1 - C_t = E_t\left[1 - C_{t+1}\right]$$
$$or \quad C_t = E_t\left[C_{t+1}\right]$$

This implies the same intertemporal consumption smoothing as found in the two-period model.

Next: we wish to derive the <u>current account implications</u>:

Recall that the intertemporal budget constraint states:

$$\sum_{s=t}^{\infty} \beta^{s-t} \left(C_s + I_s + G_s\right) = \left(1+r\right) B_t + \sum_{s=t}^{\infty} \beta^{s-t} \left(Y_s\right)$$

Regroup and impose expectations, since the constraint must hold ex-ante as well as ex-post:

$$\sum_{s=t}^{\infty} \beta^{s-t} E_t \left(C_s\right) = \left(1+r\right) B_t + \sum_{s=t}^{\infty} \beta^{s-t} E_t \left(Y_s - I_s - G_s\right)$$

Substitute the Euler equation $C_t = E_t\left[C_{t+1}\right]$ recursively for expected consumption, and rearrange:

$$\sum_{s=t}^{\infty} \beta^{s-t} C_t = \left(1+r\right) B_t + \sum_{s=t}^{\infty} \beta^{s-t} E_t \left(Y_s - I_s - G_s\right).$$

$$C_t = rB_t + (1-\beta)\sum_{s=t}^{\infty}\beta^{s-t}E_t(NO_s)$$

Substitute back into the single-period budget constraint:

$$CA_t = Y_t + rB_t - I_t - G_t - C_t = NO_t + rB_t - C_t$$

$$= (NO_t) - (1-\beta)\sum_{s=t}^{\infty}\beta^{s-t}E_t[NO_s]$$

or $$CA_t = \beta(NO_t) - (1-\beta)\sum_{s=t+1}^{\infty}\beta^{s-t}E_t[NO_s]$$

This indicates that most of a temporary rise in net output will be saved: the country will run a positive current account. A permanent rise in net output, however, will lead to no increased saving, and no change in the current account.

Can rewrite:

Define permanent value of a net output $\tilde{N}O_t$:

Want present value of this constant value at *t* to be equal to the present value of the real variable as it varies over time (NOs):

$$\sum_{s=t}^{\infty} \beta^{s-t} \tilde{N}O_t = \sum_{s=t}^{\infty} \beta^{s-t} NO_s$$

This means that the term on the RHS of equation on previous page equals $\tilde{N}O_t$

$$(1-\beta)\sum_{s=t}^{\infty} \beta^{s-t} E_t\left[NO_s\right] = \tilde{N}O_t$$

So write current account equation:

$$CA_t = NO_t - \tilde{N}O_t$$

Interpret: if net output rises above its usual level, this unusual income will be saved rather than consumed, so the current account rises.

Conclusions:

So effect of shock to net output on consumption and hence CA depends on if shock is temporary or permanent:

If **temporary**: just affect NO_t, then C rises by (1-beta) times this, and rest is saved and raises CA.

$$\Delta C_t = (1-\beta)\Delta NO_t$$

$$\Delta CA_t = \beta \Delta NO_t$$

If **permanent**, NO rises for current and all future periods, then:

$$\Delta C_t = \Delta NO_t$$

$$\Delta CA_t = 0$$

Consider an <u>intermediate degree of permanence</u>.

Say: $NO_t - \overline{NO} = \rho\left(NO_{t-1} - \overline{NO}\right) + \varepsilon_t$

Where shock is serially uncorrelated disturbance,

$E_t\left[\varepsilon_s\right] = 0,\ 0 \le \rho \le 1$

Means: $E_t\left[NO_s - \overline{NO}\right] = \rho^{s-t}\left(NO_t - \overline{NO}\right)$

Derivation: can skip in class:

$$CA_t = \beta\left(NO_t\right) - \left(1-\beta\right)\sum_{s=t+1}^{\infty}\beta^{s-t}E_t\left[NO_s\right]$$

$$= \beta\left(NO_t - \overline{NO}\right) - \left(1-\beta\right)\sum_{s=t+1}^{\infty}\beta^{s-t}E_t\left(NO_s - \overline{NO}\right)$$

$$= \beta\rho\left(NO_{t-1} - \overline{NO}\right) + \beta\varepsilon_t - \left(1-\beta\right)\sum_{s=t+1}^{\infty}\beta^{s-t}\rho^{s-t}\left[\rho\left(NO_{t-1} - \overline{NO}\right) + \varepsilon_t\right]$$

$$= \beta\rho\left(NO_{t-1} - \overline{NO}\right) + \beta\varepsilon_t$$

$$-\left(1-\beta\right)\rho\frac{\beta\rho}{1-\beta\rho}\left(NO_{t-1} - \overline{NO}\right) - \left(1-\beta\right)\frac{\beta\rho}{1-\beta\rho}\varepsilon_t$$

$$= \frac{\beta\rho\left(1-\rho\right)}{1-\beta\rho}\left(NO_{t-1} - \overline{NO}\right) + \frac{\beta\left(1-\rho\right)}{1-\beta\rho}\varepsilon_t$$

result:

$$CA_t = \frac{\beta\rho\left(1-\rho\right)}{1-\beta\rho}\left(NO_{t-1} - \overline{NO}\right) + \frac{\beta\left(1-\rho\right)}{1-\beta\rho}\varepsilon_t$$

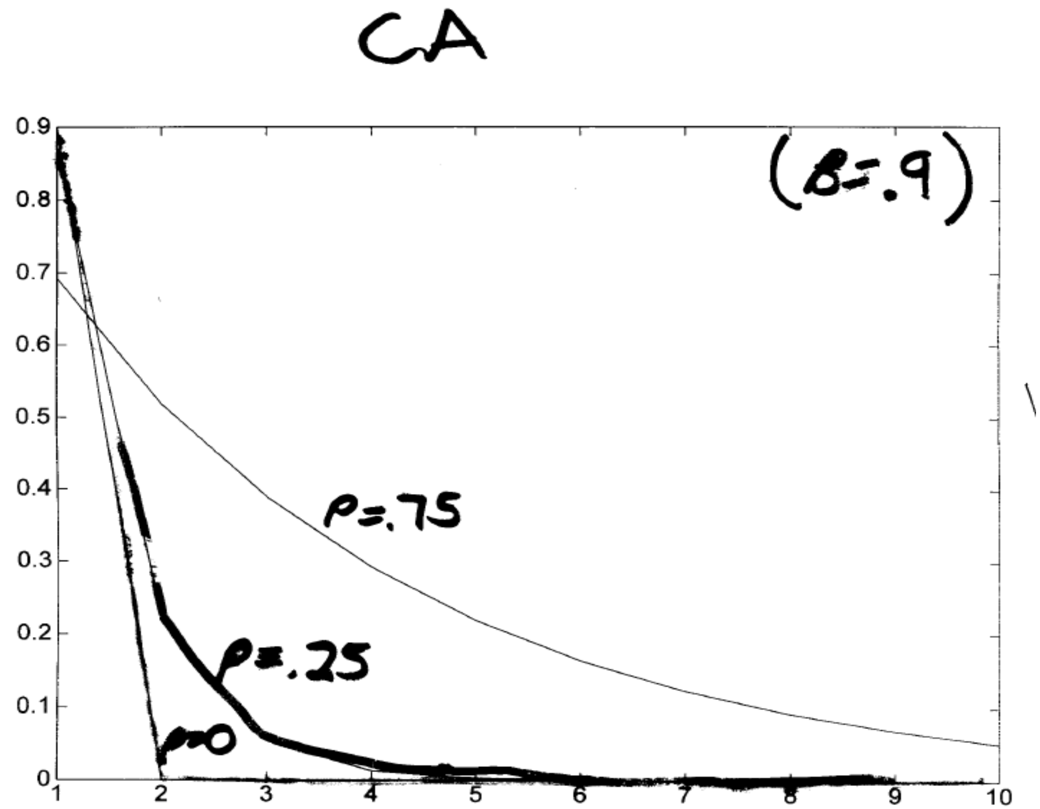
So there is a predictable component to CA, which disappears if rho equals 1 or zero: either extreme. But in middle range, a shock leads to predictable deviations in current account in future periods.

In case of rho=0: get same result as before: CA rises by Beta*shock & no effect in future periods.

In case of rho=1: get same result as before: no change in CA & no effect in future periods.

In between: shock has partial effect on CA in t, and has some effect to raise CA in future periods as well.

Simulations:



Where is ρ=1?

Part 4: Empirical Tests

How useful is this theory? Is it true?

Sheffrin and Woo (JIE 1990) were first to adapt for the intertemporal theory of the CA an estimation strategy used by Campbell to test consumption theory.

Idea: Present value test: take basic prediction of ICA model and superimpose over a VAR.

Basic prediction (Present-value restriction)

Recall:

$$CA_t = \beta\left(NO_t\right) - \left(1-\beta\right)\sum_{s=t+1}^{\infty}\beta^{s-t}E_t\left[NO_s\right]$$

$$= -\sum_{s=t+1}^{\infty}\beta^{s-t}E_t\left[NO_s - NO_{s-1}\right]$$

- To make operational in test, need proxy for expectations of change in net output. One way is to use lags of net output.
- But households have more information at date t on change in net output. So regress change in net output on current CA as well, because should also contain information on what households expect.

- So run a VAR to determine what households best forecast is for change in net output.

$$\begin{bmatrix} \Delta NO_s \\ CA_s \end{bmatrix} = \begin{bmatrix} \psi_{11} & \psi_{12} \\ \psi_{21} & \psi_{22} \end{bmatrix} \begin{bmatrix} \Delta NO_{s-1} \\ CA_{s-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_{1s} \\ \varepsilon_{2s} \end{bmatrix}$$

Get consumers' forecasts:

$$E_t \begin{bmatrix} \Delta NO_s \\ CA_s \end{bmatrix} = \begin{bmatrix} \psi_{11} & \psi_{12} \\ \psi_{21} & \psi_{22} \end{bmatrix}^{s-t} \begin{bmatrix} \Delta NO_t \\ CA_t \end{bmatrix}$$

- Can get forecast of CA alone by premultiplying rhs by vector [0 1], or forecast of NO by premultiplying by [1 0]. Represent coefficient matrix with: Ψ.

  And get rhs of present-value condition using this VAR:

$$\hat{C}A_t = -\begin{bmatrix}1 & 0\end{bmatrix}\beta\Psi\left(I-\beta\Psi\right)^{-1}\begin{bmatrix}\Delta NO_t \\ CA_t\end{bmatrix}$$

$$\equiv\begin{bmatrix}\Phi_{\Delta NO} & \Phi_{CA}\end{bmatrix}\begin{bmatrix}\Delta NO_t \\ CA_t\end{bmatrix} = K\begin{bmatrix}\Delta NO_t \\ CA_t\end{bmatrix}$$

- Note that CA in t is in info set we use to test present value condition. So test is whether the CA hat produced using condition is close to data on CA at t. That is, if coefficient vector above equals [0 1].

- Do this as a Wald test: Restriction is that K = [0 1]

$$\left[K-\hat{K}\right]\left(\partial k/d\psi \;\; V \;\; \partial k/d\psi\,'\right)^{-1}\left[K-\hat{K}\right]'$$

where use derivative of K estimate with respect to parameter estimates from VAR and covariance matrix of these parameter estimates,V.

- Can extend to have more lags in the VAR.

- Results: Mixed for Sheffrin and Woo on annual data for four countries. Found that it worked reasonably well for Belgium and Denmark but fails very badly for Canada and UK.

Predicted CA implied by the K test, compared to the CA data:

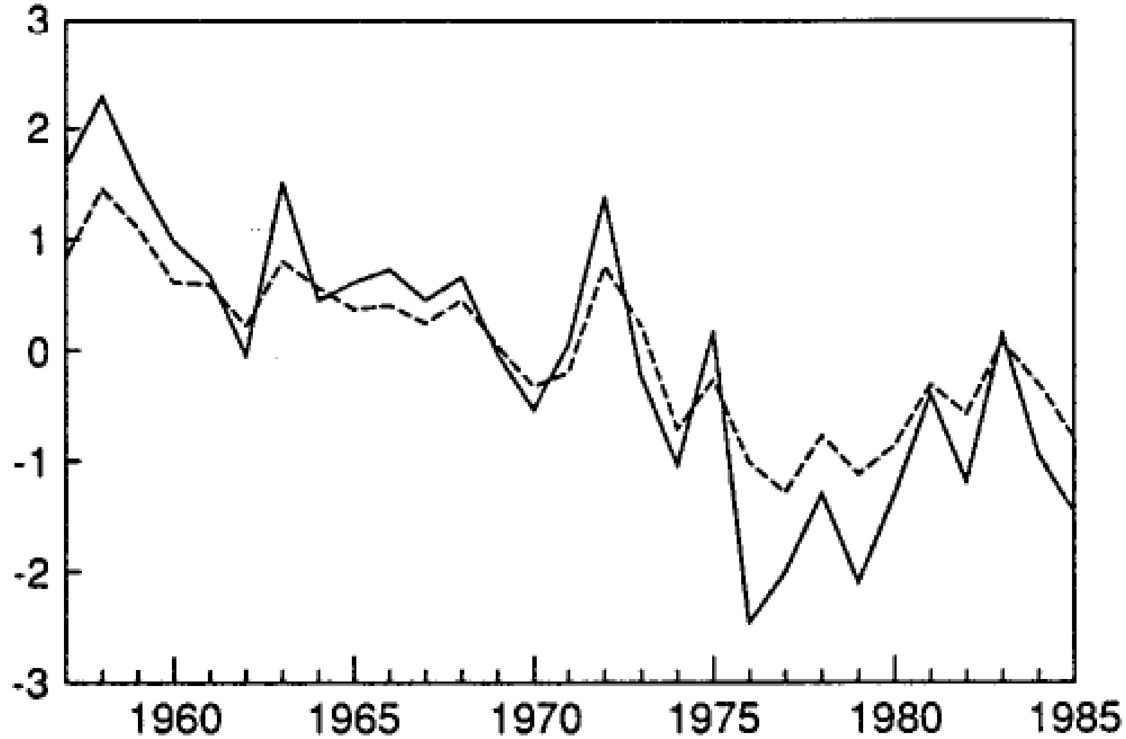


Fig. 2. Denmark. The solid line is the actual current account while the dotted line is the forecasted current account from the VAR, 1957–85.

There is an <u>additional way to test</u> this theory, sometimes used in the literature.

Start with the original model condition:

$$CA_t = -\sum_{s=t+1}^{\infty} \beta^{s-t} E_t\left[\Delta NO_s\right]$$

Subtract ΔNO_t from both sides:

$$CA_t - \Delta NO_t = -\sum_{s=t}^{\infty} \beta^{s-t} E_t\left[\Delta NO_s\right]$$

Now subtract from both sides the following:

$$(1+r)CA_{t-1} = \frac{1}{\beta}\left(-\sum_{s=t}^{\infty} \beta^{s-t+1} E_{t-1}\left[\Delta NO_s\right]\right) = -\sum_{s=t}^{\infty} \beta^{s-t} E_{t-1}\left[\Delta NO_s\right]$$

So we have:

$$CA_t - \Delta NO_t - (1+r)CA_{t-1} = -\sum_{s=t}^{\infty} \beta^{s-t} \left\{ E_t\left[\Delta NO_s\right] - E_{t-1}\left[\Delta NO_s\right] \right\}$$

Then If we define:

$$R_t \equiv CA_t - \Delta NO_t - \left(1+r\right)CA_{t-1},$$

the restrictions above imply that this R variable should be unforecastable on the basis of information dated t-1 or before, like lags of NO and CA.

- To test this, regress this R variable on lagged values of net output and the current account, and see if these regressors are significant.

Example from Sheffrin-Woo (1990):
Statistical R and K tests:

Table 4

Denmark.

	VAR		Tests on R_t				K vector	
			I_{t-1}		I_{t-2}		(4%)	(14%)
	ΔNO_{t-2}	CA_t	4%	14%	4%	14%	$\hat{C}A_{t-1} = KZ_{t-1}$	
ΔNO_{t-1}	−0.43	−0.87	−0.43	−0.44			0.11	0.19
	(0.20)	(0.27)	(0.24)	(0.24)			(0.22)	(0.14)
ΔNO_{t-2}	−0.24	−0.05	0.19	0.19	0.53	0.62	0.18	0.16
	(0.21)	(0.20)	(0.26)	(0.26)	(0.23)	(0.24)	(0.14)	(0.11)
CA_{t-1}	−0.05	0.71	−0.27	−0.38			0.52	0.34
	(0.17)	(0.22)	(0.20)	(0.20)			(0.33)	(0.19)
CA_{t-2}	−0.15	0.002	0.15	0.15	0.017	−0.05	0.13	0.10
	(0.16)	(0.22)	(0.20)	(0.20)	(0.124)	(0.12)	(0.13)	(0.09)
R^2	0.25	0.64	0.38	0.45	0.17	0.20		
F-statistic			3.77	4.94	2.74	3.27		
P-value			0.02	0.01	0.08	0.05	0.00	0.00
χ^2-statistic							40.2	70.3

Notes: Standard errors in parentheses; constants omitted in VAR; regressions are for 1957–85.

Predicted CA implied by the K test, compared to the CA data:

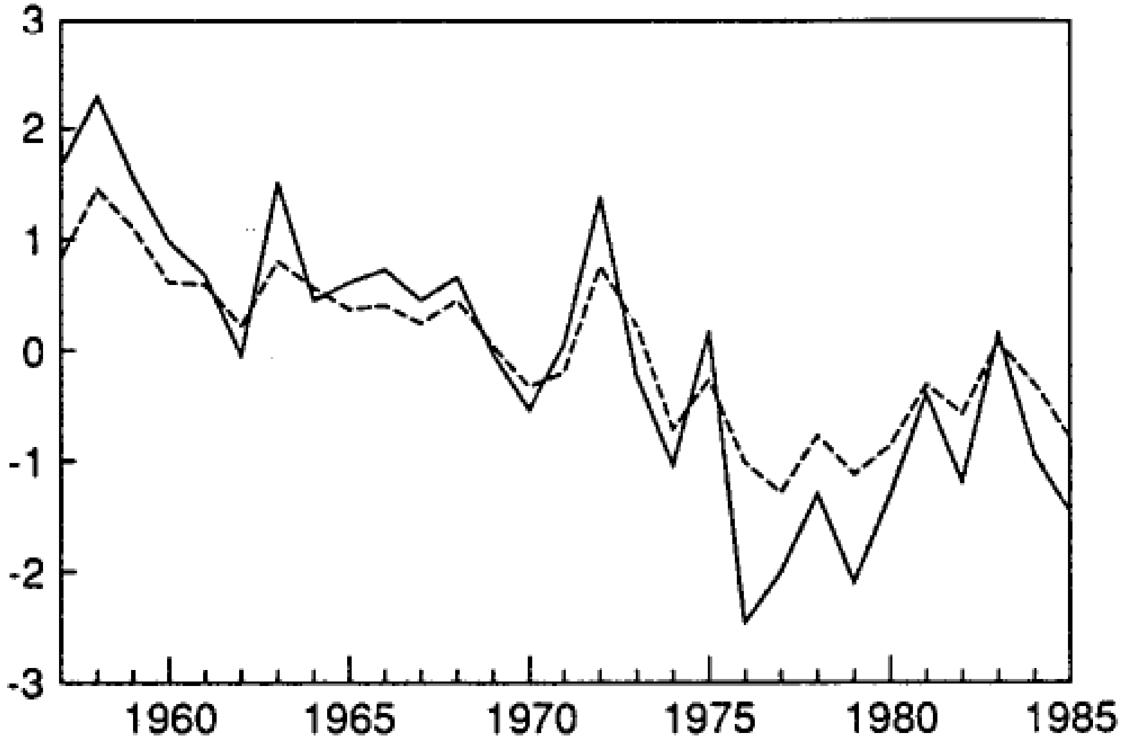


Fig. 2. Denmark. The solid line is the actual current account while the dotted line is the forecasted current account from the VAR, 1957–85.

Conclusions from large empirical literature on the simplest version of the intertemporal CA model:

- Works only for some countries, fails for majority
- Works worst for small countries (ironic)
- Main failing is that it underpredicts the volatility of CA fluctuations. Some interpret this to indicate that there is excessive capital mobility. (surprising?)

A useful extension: Habits

- Empirical results become very good if the underlying preferences are extended to be non-time separable.

- Gruber (2000) re-derives model under the specification of preferences with habits:

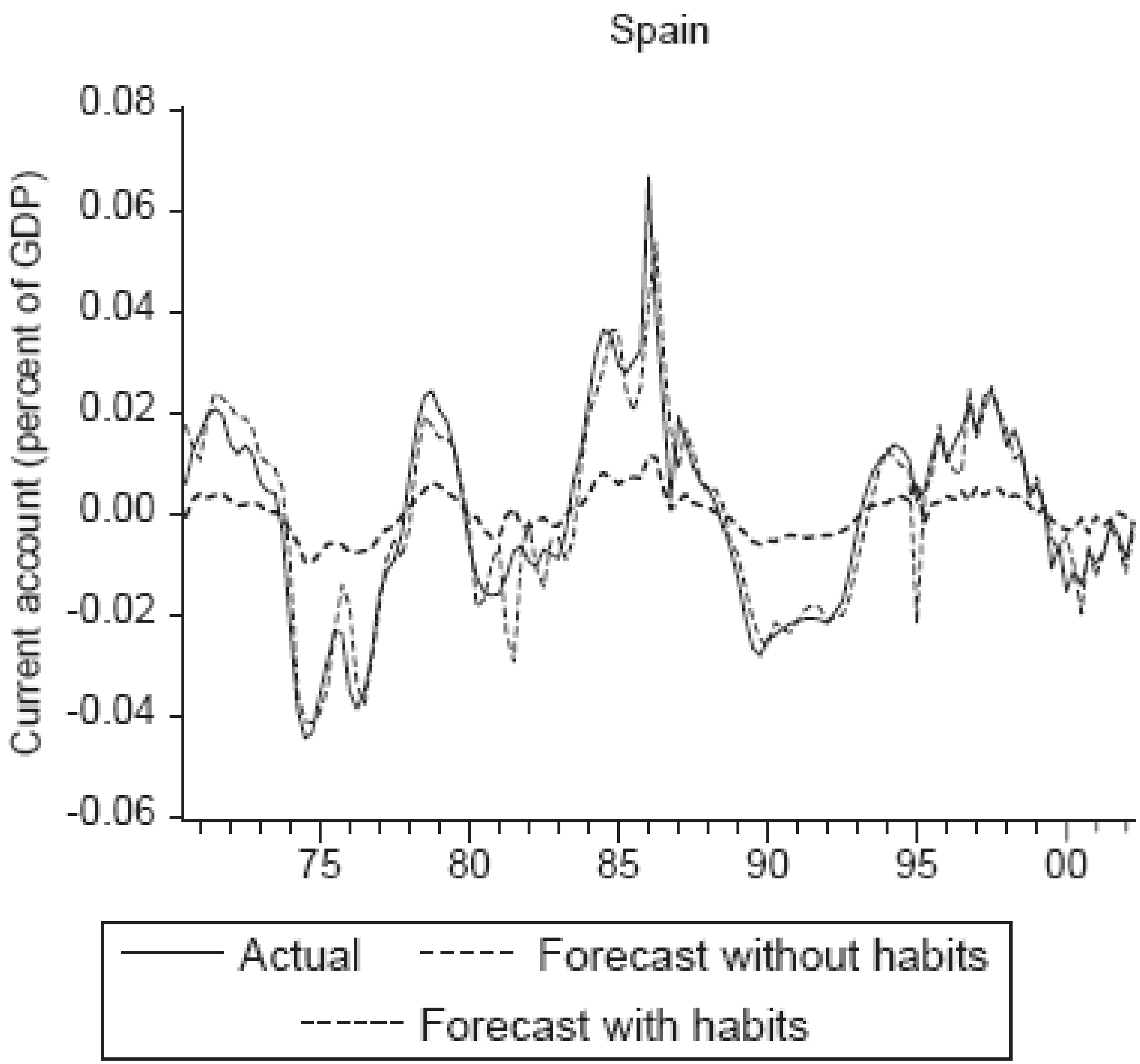
$$U_s = u\left(C_s - \gamma C_{s-1}\right) = u\left(\left(1-\gamma\right)C_s + \gamma\left(\Delta C_s\right)\right)$$

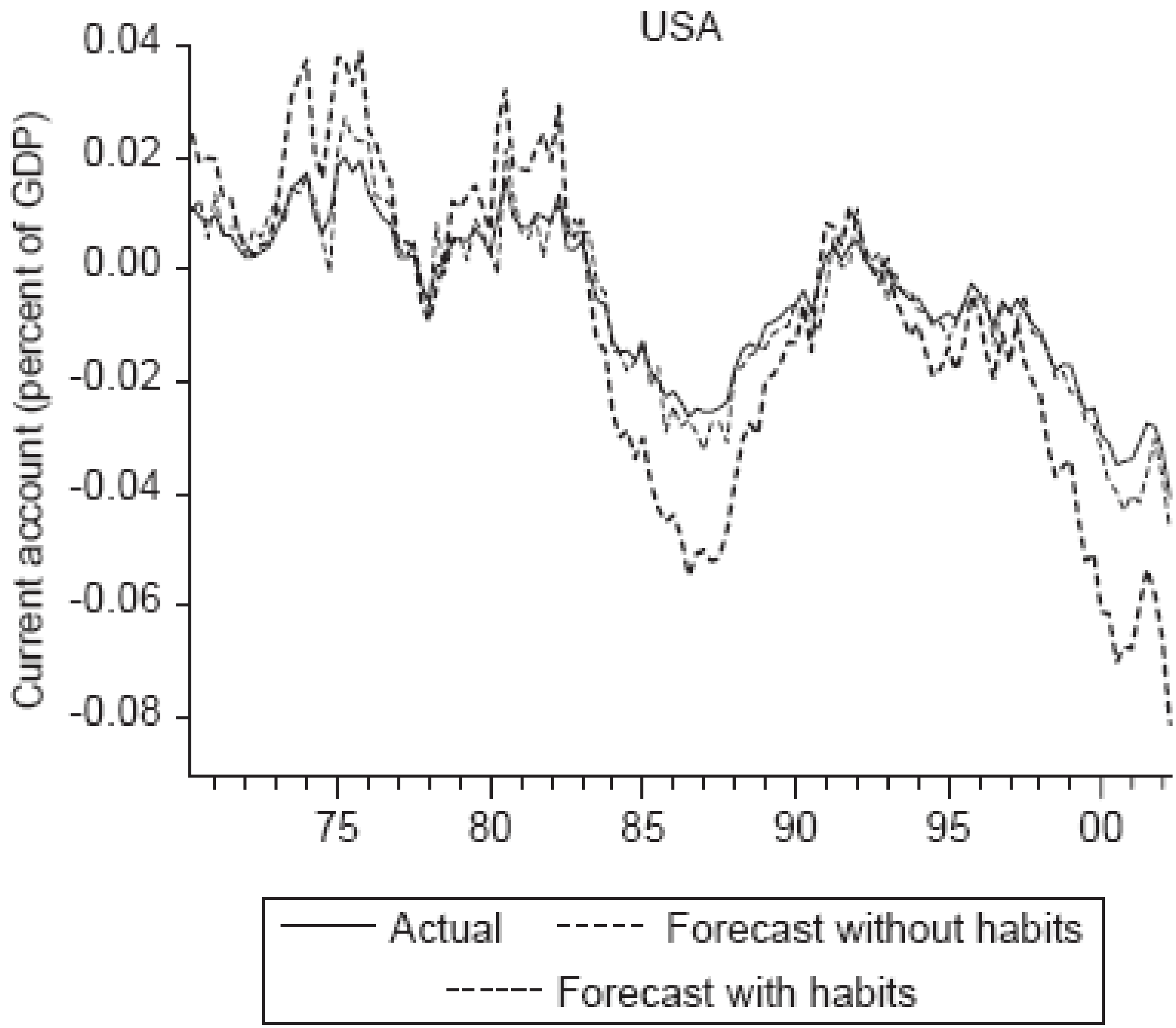
  where γ shows the role of habits.

- This will imply intertemporal smoothing of consumption changes rather than of consumption levels.

- So that permanent shocks will not translate fully into immediate consumption change; they will pass in part into saving, and greater current account fluctuations.

Results…


Spain
Current account (percent of GDP)
0.08
0.06
0.04
0.02
0.00
-0.02
-0.04
-0.06
75
80
85
90
95
00
Actual
Forecast without habits
Forecast with habits


USA
0.04
0.02
0.00
-0.02
-0.04
-0.06
-0.08
Current account (percent of GDP)
75
80
85
90
95
00
Actual
Forecast without habits
Forecast with habits

Part 5: Extensions to the intertemporal model

- Another way to improve the empirical performance of the model is to allow additional sources of shocks: interest rates and relative prices.
- This theory follows Dornbusch JPE 1983. This is the theory behind the paper by Campa and Gavilan (2006)
- Problem: household consumes two types of goods, one tradable (T) and the other nontradable (N). The price of NT in terms of T is p. (model with perfect foresight).

$$Max \sum_{s=t}^{\infty} \beta^{s-t} U\left(C_{T,s}, C_{N,s}\right)$$

$$s.t.\ Y_s + r_s B_s - \left(C_{T,s} + p_s C_{N,s}\right) - I_s - G_s = B_{s+1} - B_s$$

$$where\ U\left(C_{T,s}, C_{N,s}\right) = \frac{\sigma}{\sigma - 1}\left(C^{\theta}{}_{T,s} C^{1-\theta}{}_{N,s}\right)^{\frac{\sigma-1}{\sigma}}$$

- Where we measure everything in units of traded goods, including interest rate.
- Here the intertemporal elasticity of substitution is σ, and the intratemporal elasticity between traded and nontraded goods is unity (because of Cobb-Douglas form).

FOCs give intertemporal and intratemporal condition
Intratemporal tradeoff between two types of goods within the period

$$\frac{U'_{Nt}}{U'_{Tt}} = p_t$$

Intertemporal tradeoff between periods

$$1 = \beta\left(1 + r_{t+1}\right)\frac{U'_{Nt+1}}{U'_{Nt}}\frac{p_t}{p_{t+1}}$$

We will find it easier if write the problem and conditions, not in terms of two separate goods, but in terms of a composite consumption good, combining the two types of goods in a single consumption basket.

Define consumption basket using aggregator in utility, C:

$$C_t = C^{\theta}{}_{T,t}C^{1-\theta}{}_{N,t}$$

This is separate from measure of total expenditure on consumption goods, Z, where use traded goods as the numeraire:

$$Z_t = C_{Tt} + p_t C_{Nt}$$

- Define an aggregate consumption price index, P, as the amount of expenditure it takes to get one unit of the consumption index.

- So: $Z_t = P_t C_t$ This puts consumption index in units of traded goods (the numeraire)

- Recap notation: C and P, Z and p

- Under Cobb-Douglas utility, the intratemporal optimality condition implies: (can skip in class)

$$\frac{U'_{Nt}}{U'_{Tt}} = p_t$$

$$\frac{(1-\theta)C_T{}^{\theta}C_N{}^{-\theta}}{\theta C_T{}^{\theta-1}C_N{}^{1-\theta}} = \frac{1-\theta}{\theta}\frac{C_T}{C_N} = p$$

$$so \;\; C_T = p\frac{\theta}{1-\theta}C_N$$

$$use \;\; Z = C_T + pC_N = p\frac{\theta}{1-\theta}C_N + pC_N = \frac{p}{1-\theta}C_N$$

$$C_{Tt} = \theta Z_t$$

so

$$C_{Nt} = (1-\theta)\frac{Z_t}{p_t}$$

Substitute these into the definition of consumption index:

$$C_t = (\theta Z_t)^{\theta}\left((1-\theta)\frac{Z_t}{p_t}\right)^{(1-\theta)}$$

Divide through by $C_t = Z_t/P_t$:

$$(\theta P_t)^{\theta}\left((1-\theta)\frac{P_t}{p_t}\right)^{(1-\theta)} = 1$$

Solve this for the price index:

$$P_t = p_t^{1-\theta}\left[\theta^{-\theta}(1-\theta)^{-(1-\theta)}\right]$$

This allows us to rewrite the household problem in terms of the consumption index (as if there were just one good):

$$Max \sum_{s=t}^{\infty} \beta^{s-t} \frac{\sigma}{\sigma - 1} \left(C_s \right)^{\frac{\sigma-1}{\sigma}}$$

$$s.t. \; Y_s + r_s B_s - P_s C_s - I_s - G_s = B_{s+1} - B_s$$

So the intertemporal Euler can be written (now it's similar to before):

$$\frac{U'_t}{P_t} = \beta \left(1 + r_{t+1}\right) \frac{U'_{t+1}}{P_{t+1}}$$

Assume CES utility, write in units of consumption index:

$$C_t = \beta^{-\sigma} \left(\left(1 + r_{t+1}\right) \frac{P_t}{P_{t+1}} \right)^{-\sigma} C_{t+1}$$

or in units of traded goods (for total consumption expenditure)

$$Z_t = \beta^{-\sigma}\left(1+r_{t+1}\right)^{-\sigma}\left(\frac{P_t}{P_{t+1}}\right)^{1-\sigma} Z_{t+1}$$

$$= \beta^{-\sigma}\left(1+r_{t+1}\right)^{-\sigma}\left(\frac{p_t}{p_{t+1}}\right)^{(1-\sigma)(1-\theta)} Z_{t+1}$$

$$= \beta^{-\sigma}\left(1+r^c{}_{t+1}\right)^{-\sigma} Z_{t+1}$$

where define consumption-based real interest rate:

$$1+r_{t+1}^c = \left(1+r_{t+1}\right)\left(\frac{p_t}{p_{t+1}}\right)^{-\frac{(1-\sigma)}{\sigma}(1-\theta)}$$

combining effect of interest rate paid in terms of traded goods, and expected changes in the price of non traded goods in terms of traded.

Interpret this intertemporal condition:

- A rise in the conventional interest rate (r) makes borrowing to finance extra consumption more expensive, so consumption expenditure today will fall relative to the future. By elasticity σ.

- An expected rise in the future relative price of tradables to nontradables (1/p) has a similar effect, since loans must be paid off in the future in terms of traded goods. If the price of traded goods is temporarily low and expected to rise, then the future repayment of a loan in traded goods has a higher cost on terms of the overall consumption bundle than it does today.

- In terms of the equations above, if the relative price of traded goods is expected to rise in the future (so that the relative price of nontraded goods, p, is expected to fall in the future), then ($\frac{p_t}{p_{t+1}}$) is high. Provided that sigma >1, this causes the consumption based interest rate ($1+r^c$) to rise above the conventional interest rate (1+ r) in the equation above. This lowers the current total consumption expenditure by elasticity $(\sigma - 1)(1 - \theta)$. Again, provided that σ >1, this means current consumption will fall.

- The effect of relative prices above can be decomposed into two separate effects: one intertemporal, the other intratemporal. Recall that sigma is the intertemporal elasticity here. A rise in the future price on traded goods makes total consumption expenditure fall by elasticity $\sigma(1-\theta)$.

- In addition, a change in the relative price of nontraded goods also induces intratemporal substitution between goods. If the price of traded goods is temporarily low relative to nontraded goods as assumed above, households will substitute toward traded goods today by the intratemporal elasticity, which is unity for a Cobb-Douglas form. This raises total current consumption expenditure, in units of traded goods, by elasticity $1*(1-\theta)$.

- The intertemporal effect will dominate over the intratemporal effect if the intertemporal elasticity, σ, is greater than unity.

If want to solve for CA implications:

Define time-varying discount factor:

$$R_{t,s} = \left[\prod_{v=t+1}^{s} (1 + r_v) \right]^{-1} \quad for \;\; s > t; \; R_{t,t} = 1$$

$$R^c{}_{t,s} = \left[\prod_{v=t+1}^{s} (1 + r^c{}_v) \right]^{-1} \quad for \;\; s > t; \; R^c_{t,t} = 1$$

Write intertemporal budget constraint:

$$\sum_{s=t}^{\infty} R_{t,s} \left(Z_s \right) = \left(1 + r_t \right) B_t + \sum_{s=t}^{\infty} R_{t,s} \left(NO_s \right)$$

Substitute recursively for Z to get consumption function:

$$Z_t = \left[\sum_{s=t}^{\infty} \left(R_{t,s} \right) \left(R^c{}_{t,s} \right)^{-\sigma} \beta^{\sigma(s-t)} \right]^{-1} \left[\left(1 + r_t \right) B_t + \sum_{s=t}^{\infty} R_{t,s} \left(NO_s \right) \right]$$

Put back in single-period budget constraint to get CA:

$$CA_t = r_t B_t + NO_t - Z_t$$

- Main effect of a rise in the consumption-based real interest rate: (Abstracting from effects on valuation of existing asset holdings, B_t).

- A rise in r^c_{t+1} lowers $R^c_{t,s}$ in the equations above, which tends to lower Z and raise CA. This reflects the fact that borrowing for current consumption becomes more expensive.

- Recall that such a rise in the consumption-based real interest rate could result from a rise in the conventional real interest rate (r_{t+1}), or it could result from a change in the relative price of nontradables (p_{t+1}/p_t), depending on the intratemporal elasticity.

Empirical tests of the extended model:

- Idea:Test equation with consumption based real interest rate: use world real interest rate data and real exchange rate as proxy for relative prices of goods.

Method:

- Log linearize IBC and impose linearized version of condition above: gives condition to test:

$$CA*_t = -E\sum_{i=1}^{\infty}\beta^i\left[\Delta no_{t+i} - \sigma r^c{}_{t+i}\right]$$

where CA* is a log-linearized version of CA components, and r^c is consumption based real interest rate from before.

- Similar to Sheffrin-Woo, where CA was function just of expected change in NO, now includes also r^c

- Do VAR on the three variables: Z is vector: CA*, ΔNO, and r^c.

- Again use CA condition to compute CA prediction using forecast of variables from VAR

$$\hat{C}A_t^* = KZ_t$$

where

$$K = -\left(\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} - \sigma \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right) \beta A \left[I - \beta A \right]^{-1}$$

Construct chi-sq stat just as with Sheffrin and Woo, but with 3 not 2 degrees of freedom

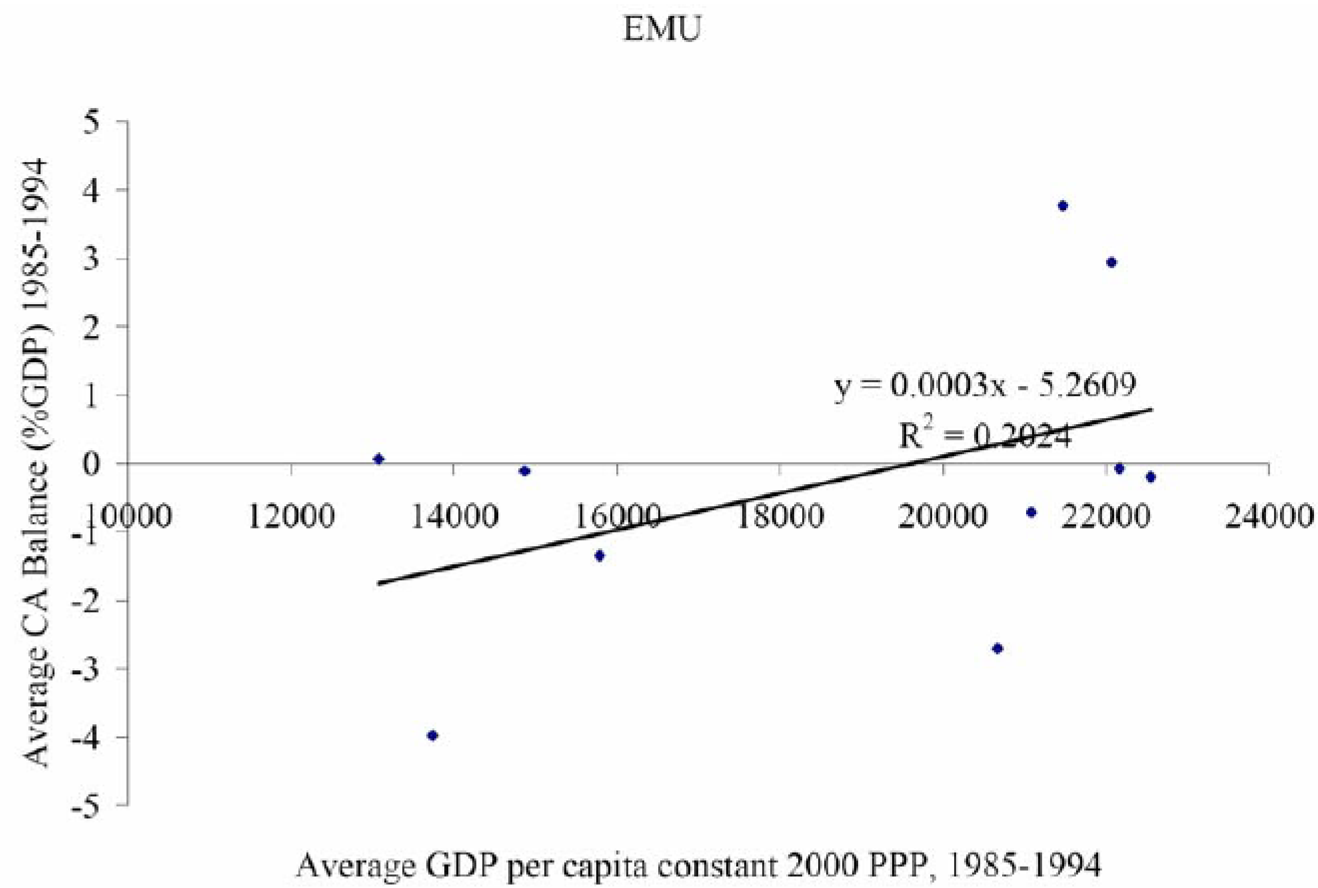
Application: Campa and Gavilan (2006) paper

Data:
- 10 EU countries
- quarterly, seasonally adjusted data from IFS
- compute world interest rate as average over the countries, adjusted for forecasted inflation

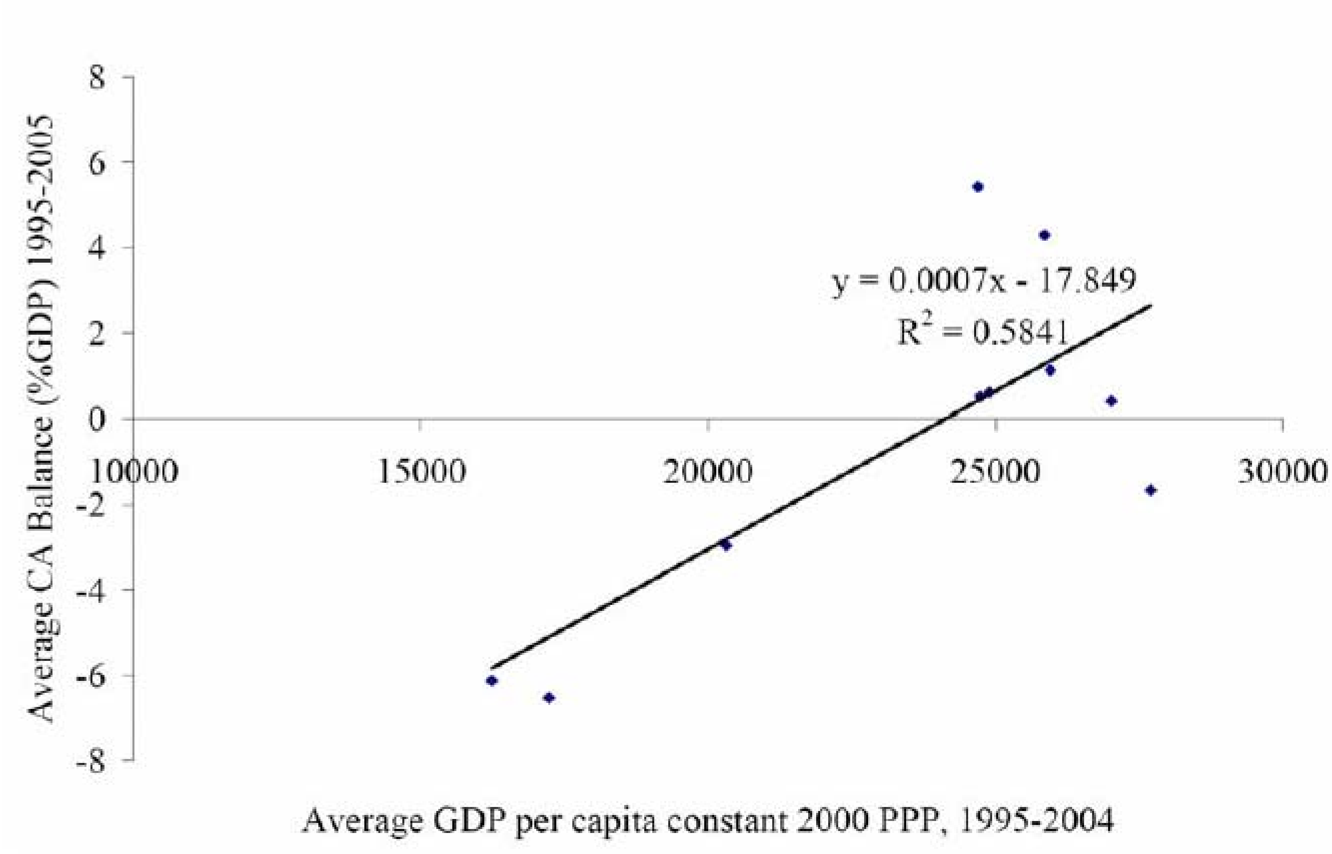
Preliminary observations:
- positive cross-sectional correlation of CA with income, consistent with theory, and has risen over time. (fig 1)
- increasing CA dispersion across countries may indicate increasing financial integration in Europe (fig 2)

Calibration of 3 parameters needed to conduct tests:
- share of nontraded goods , taken from outside estimates
- intertemporal elasticity: range of values <1 considered
- discount factor, taken from mean of 1/(1+r) data

Current account balances and GDP per capita in the Euro area



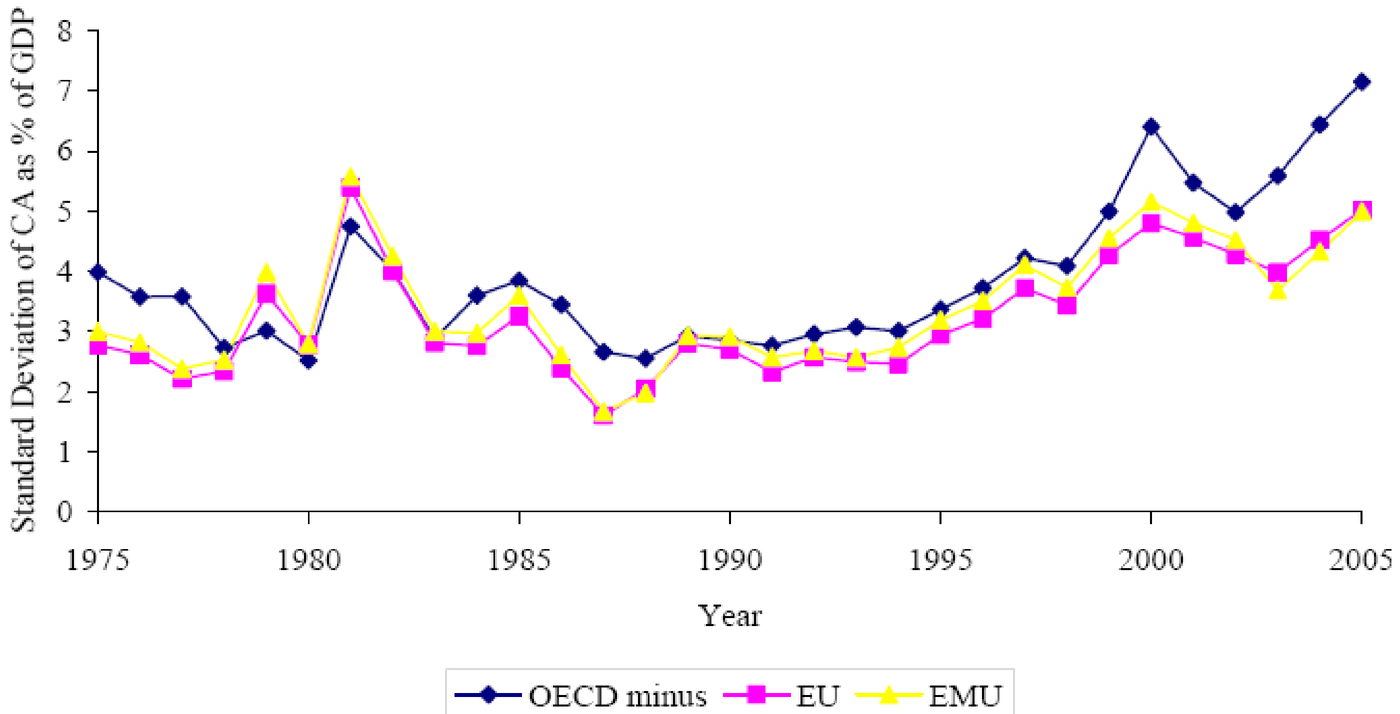
Note: positive correlation



Note: the positive correlation is higher in later subsample.

Figure 2

Standard deviation of current account balances 1975-2005

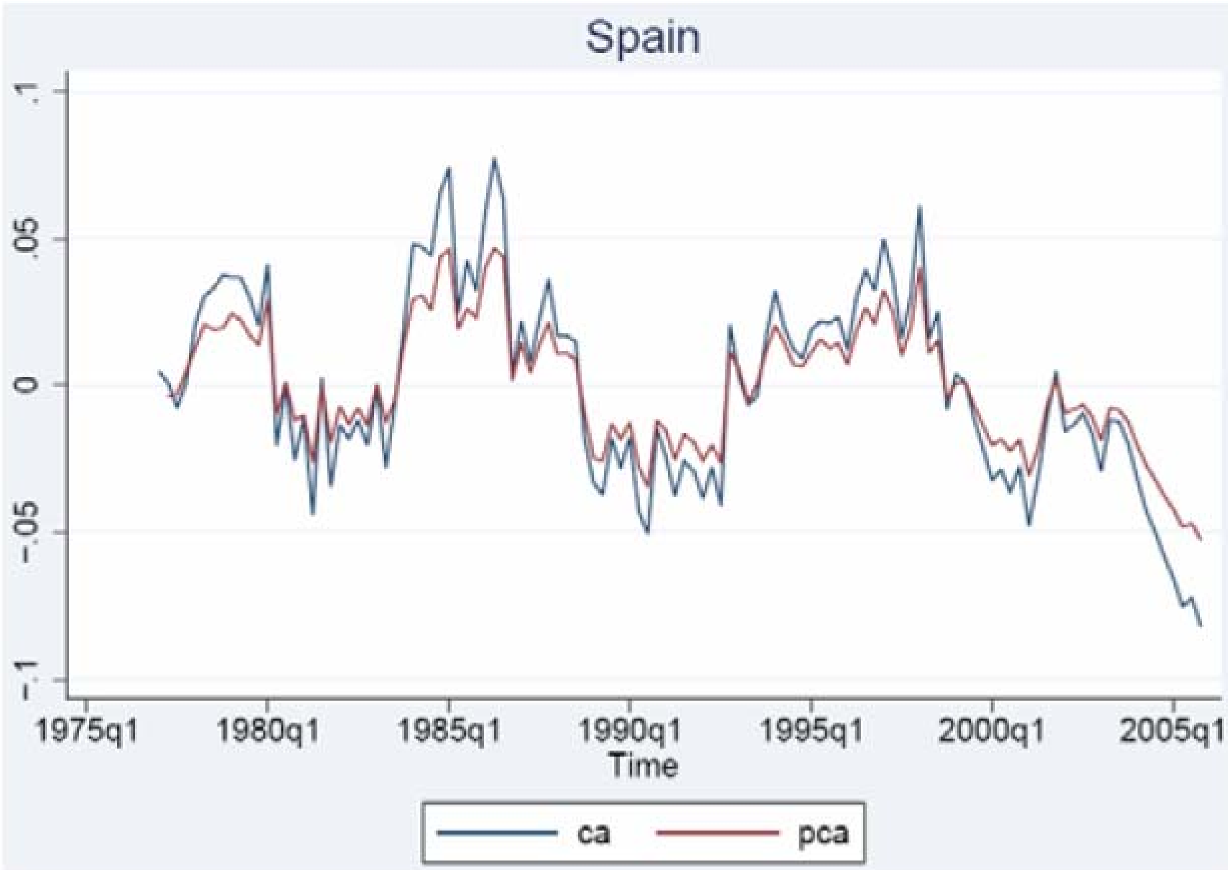


Note: dispersion of CA balances has risen over time, with both more borrowing and more lending.

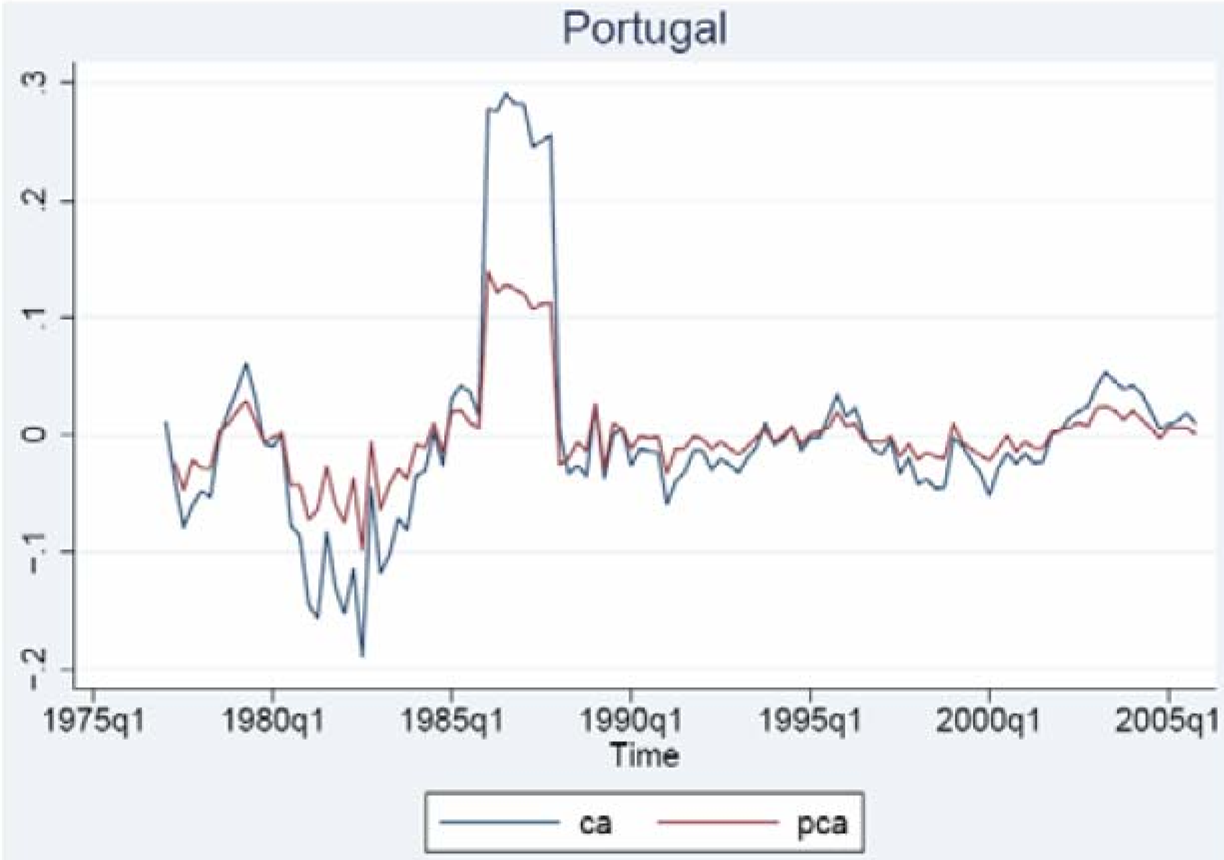
Results of statistical tests:

- K and R tests indicate that cannot reject for 6 of the ten countries (at the 10% significance level, for intertemporal elasticity <0.75.)

- CA prediction implied by K-test fits well for some countries of interest. See figures below.



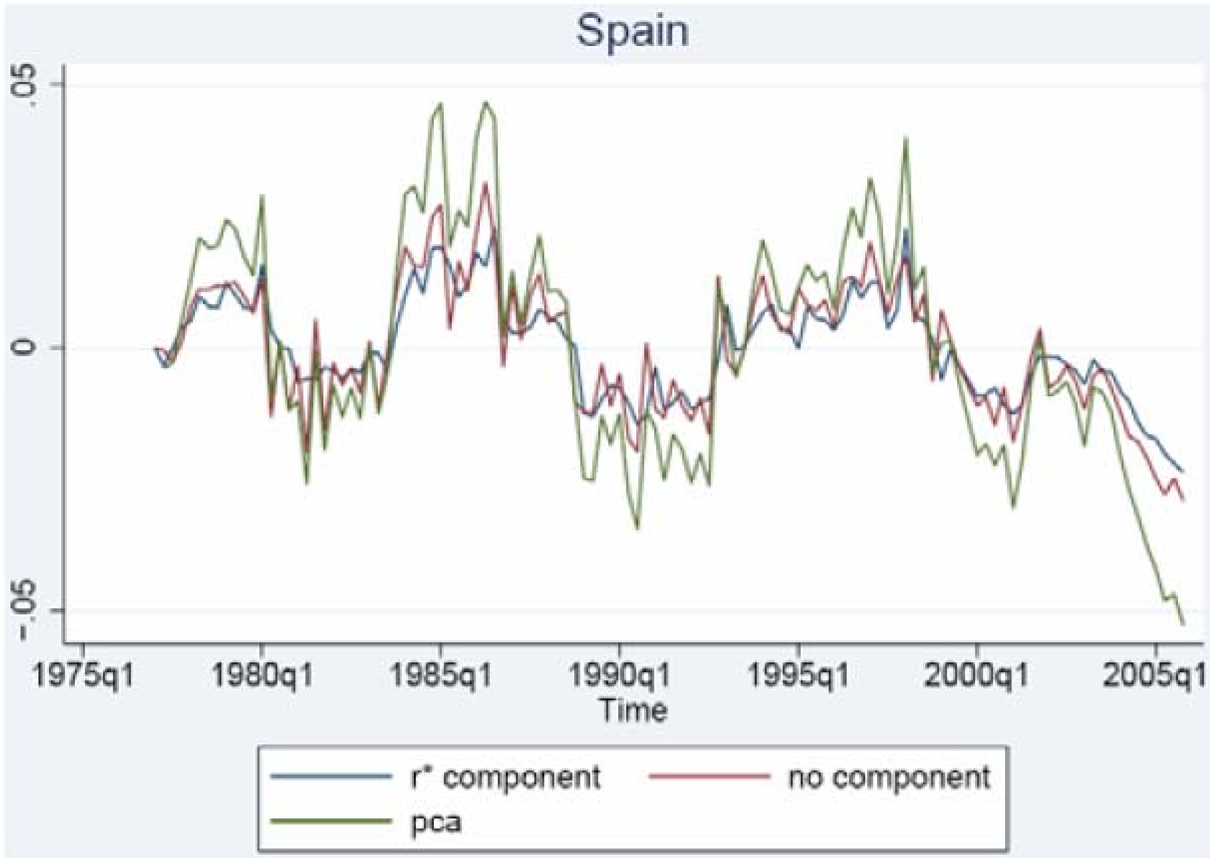
Spain
.1
.05
0
−.05
−.1
1975q1
1980q1
1985q1
1990q1
1995q1
2000q1
2005q1
Time
ca
pca

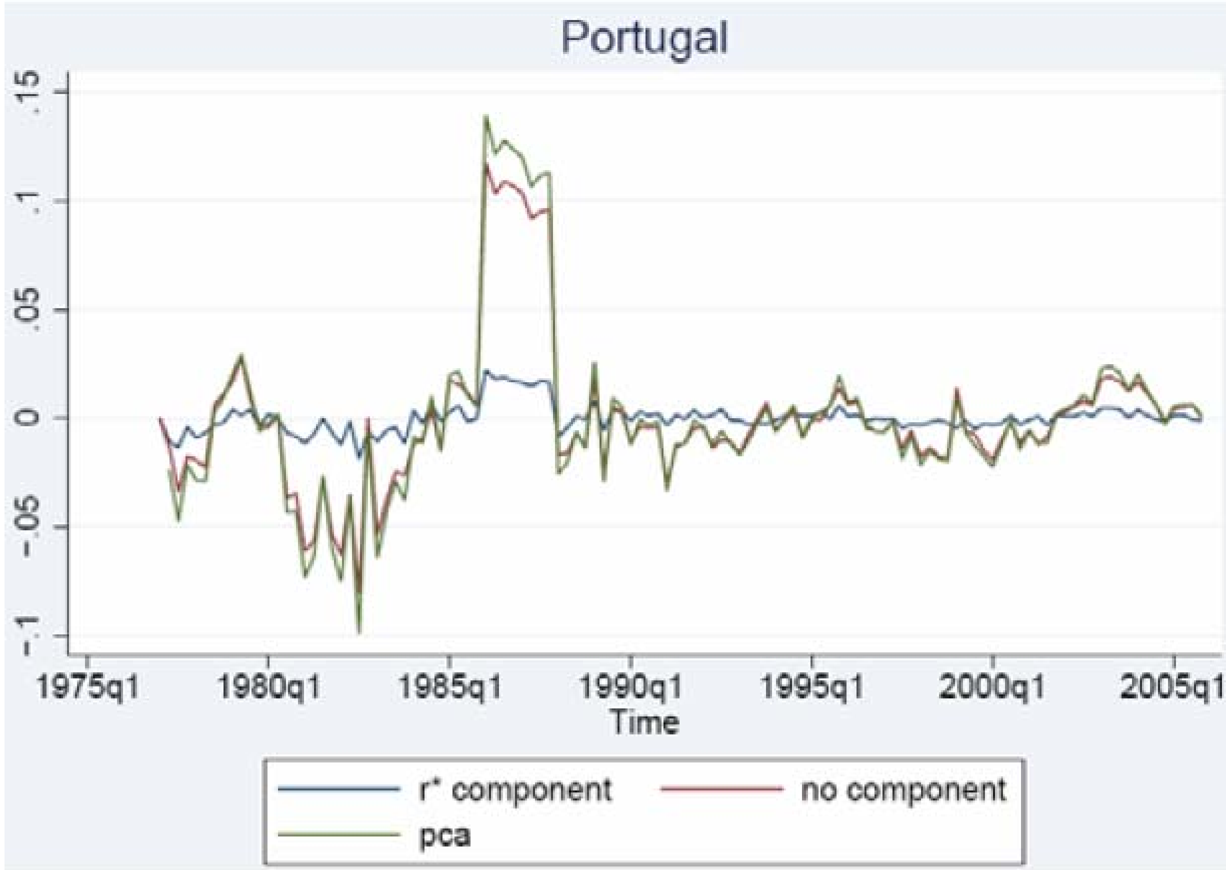

Portugal
.3
.2
.1
0
-.1
-.2
1975q1
1980q1
1985q1
1990q1
1995q1
2000q1
2005q1
Time
ca
pca


France
.05
0
-.05
-.1
1975q1
1980q1
1985q1
1990q1
1995q1
2000q1
2005q1
Time
ca
pca

Decompose CA into the movements due to Net output and movements in the relative price term.

- For Portugal and Spain, it is mainly expectations of rising net output that is driving CA deficit.


Spain
.05
0
-.05
1975q1
1980q1
1985q1
1990q1
1995q1
2000q1
2005q1
Time
r* component
no component
pca



Portugal
.15
.1
.05
0
-.05
-.1
1975q1
1980q1
1985q1
1990q1
1995q1
2000q1
2005q1
Time
r* component
no component
pca

Compute structural NO as the level implied by CA model and CA data: Spain has very high NO expectations compared to historical experience.

Questions for further discussion:

1) Do you think the CA deficits of Spain and Portugal reflect greater financial integration in the EU.

2) Are the CA deficits justified.

3) Are they beneficial to people in Spain? How about for people in Germany?

4) The US is also running historically high CA deficits. What might be the cause or justification there?

5) Consider the role of world interest rates and savings. (Engel and Rogers, 2006 JME)