Solutions to Final Exam

1a) Much of this problem can be answered by noting it is the same as the habit persistence problem explored in Gruber (2000). The only difference is that the habit persistence parameter (γ) is replaced by the negative of the deprecation rate of durables ($1-\delta$). So the derivations and solutions of Gruber all apply here with this substitution. The effects of durable goods on the current account are identical to the effects of habits, but in reverse direction.

If one works through the problem from scratch, the steps are shown below. For brevity, I will sometimes use C_t^* to refer to $C_t + C_{t-1}$.

Bellman form:

$$V_t\left(B_t, C_{t-1}\right) = \max \left\{\left(C_t + C_{t-1}\right) - \frac{1}{2}\left(C_t + C_{t-1}\right)^2 + \beta E_t\left[V_{t+1}\left(B_{t+1}, C_t\right)\right]\right\} + \lambda_t\left(NO_t + \left(1+r\right)B_t - C_t - B_{t+1}\right)$$

First order conditions:

$$C_t : \lambda_t = 1 - \left(C_t + C_{t-1}\right) + \beta E_t\left[\frac{\partial V_{t+1}}{\partial C_t}\right]$$

$$B_{t+1} : \beta E_t\left[\frac{\partial V_{t+1}}{\partial B_{t+1}}\right] = \lambda_t$$

Envelope conditions:

$$B_t : \quad \frac{\partial V_t}{\partial B_t} = \lambda_t\left(1+r\right) \quad \rightarrow \quad \frac{\partial V_{t+1}}{\partial B_{t+1}} = \lambda_{t+1}\left(1+r\right)$$

$$C_{t-1} : \frac{\partial V_t}{\partial C_{t-1}} = 1 - \left(C_t + C_{t-1}\right) \quad \rightarrow$$

$$\frac{\partial V_{t+1}}{\partial C_t} = 1 - \left(C_{t+1} + C_t\right)$$

Combine these into an Euler equation:

$$\lambda_t = \beta\left(1+r\right)E_t\left[\lambda_{t+1}\right]$$

$$where \quad \lambda_t = 1 - \left(C_t + C_{t-1}\right) + \beta - \beta E_t\left(C_{t+1} + C_t\right)$$

$$so \quad \left(C_t + C_{t-1}\right) + \beta E_t\left(C_{t+1} + C_t\right) = E_t\left[\left(C_{t+1} + C_t\right) + \beta\left(C_{t+2} + C_{t+1}\right)\right]$$

$$so \quad C_{t-1} + \beta C_t = E_t\left[C_{t+1} + \beta C_{t+2}\right]$$

This Euler equation is an unstable second order difference equation, with the stable solution:

$$E_t C_{t+1} = C_{t-1}$$

This shows that households do not necessarily smooth the level of consumption expenditure. If it is expected that future consumption will be higher than last period's consumption, the household will want to raise current consumption expenditure an extra amount beyond what it expects consumption to be next period. This is because the household wishes to smooth marginal utility over time, which requires it to account for the utility of lagged consumption expenditure. Note that this works in the opposite way to habit persistence, which makes people adjust consumption levels more gradually.

b) The intertemporal budget constraint in expectational form is:

$$\sum_{s=t}^{\infty} \beta^{s-t} E_t(C_s) = (1+r)B_t + \sum_{s=t}^{\infty} \beta^{s-t} E_t(NO_s)$$

Substitute the Euler equation recursively for expected consumptions on the left hand side:

$$C_t + \beta C_{t-1} + \beta^2 C_t + \beta^3 C_{t-1} + \beta^4 C_t + ... = (1+r)B_t + \sum_{s=t}^{\infty} \beta^{s-t} E_t(NO_s)$$

$$\left(\frac{1}{1-\beta^2}\right) C_t + \left(\frac{\beta}{1-\beta^2}\right) C_{t-1} = (1+r)B_t + \sum_{s=t}^{\infty} \beta^{s-t} E_t(NO_s)$$

$$C_t = (1-\beta^2)\left[(1+r)B_t + \sum_{s=t}^{\infty} \beta^{s-t} E_t(NO_s)\right] - \beta C_{t-1}$$

This looks like our condition from homework #1, except $(1-\beta^2)$ replaces $(1-\beta)$ {and there is an extra $-\beta C_{t-1}$ }.

Substitute back into the single-period budget constraint:
Again, this looks a lot like the solution for the current account from homework #1, except

$$CA_t = Y_t + rB_t - I_t - G_t - C_t = NO_t + rB_t - C_t$$

$$= NO_t + rB_t - \left[-\beta C_{t-1} + (1-\beta^2)\left[(1+r)B_t + \sum_{s=t}^{\infty} \beta^{s-t} E_t(NO_s)\right]\right]$$

$$= \beta^2 NO_t - (1-\beta^2) \sum_{s=t+1}^{\infty} \beta^{s-t} E_t(NO_s) - (1-\beta)B_t + \beta C_{t-1}$$

β^2 replaces β, and $(1-\beta^2)$ replaces $(1-\beta)$.

c) <u>Temporary rise in output by amount X</u>:

The formula above shows that the current account will rise by amount $\beta^2 X$, which is smaller than βX, which is the amount it would rise under the usual case. The reason is that with durable goods, you want to raise your current consumption expenditure more in the initial period, so you run a slightly smaller current account surplus.

d) <u>Permanent rise in output by amount X</u>:

The formula above shows that the current account will change by amount $\left[\beta^2 - (1-\beta^2)\left(\frac{\beta}{1-\beta}\right)\right] X = \beta^2 - \beta(1+\beta) X = -\beta X$. So the current account falls, as opposed to the usual case, where the current account does not change. Again the reason is that under durable goods, you want to increase you current consumption expenditure more in the initial period, so the current account is somewhat less than the usual case.

2a)

i) Portfolio diversification puzzle: Poterba (1991) and Tesar and Werner (1995) have documented that equity portfolios tend to be heavily biased toward home assets, whereas a simple model would imply agents should hold a share of the world portfolio. In fact, Baxter and Jermann (1997) showed that to diversify away from labor income risk, it may be optimal to hold a portfolio short in domestic assets.

Several models have found explanations for this behavior in models assuming complete asset markets. The OR text and Tesar (1993) used nontraded goods, where contingent claims linked to nontraded goods endowments need to be paid off in terms of traded goods, so that if substitutability between traded and nontraded goods is lower than intertemporal substitutability, it is optimal to go short in such assets. Kollmann (2006) explained the puzzle in terms of home bias in preferences, assuming complete asset markets. Engel and Matsumoto (2006) used sticky prices in a model of complete asset markets.

ii) Consumption correlation puzzle

Puzzle:The correlation between the consumption levels of many country pairs is lower than their output correlations. This is a puzzle because simple models predict that consumption correlations should be very high, either due to risk sharing or consumption smoothing. In Backus, Kehoe, Kydlane (1992) the average consumption correlation among 10 industrial countries relative to the U.S. is found to be 0.51 and the average output correlation is 0.66.

Stockman and Tesar (AER 1995) considered a shock that lowers the supply of nontraded goods but not the supply of traded goods. Under the assumption that the intertemporal elasticity exceeds the intratemporal elasticity between the two goods, the consumer will prefers to consume the two types of goods together. So he cuts back on consumption of traded goods along with the fall in nontraded goods. A fall in traded goods consumption at home implies that foreign consumption of traded goods will rise. So foreign consumption should tend to rise overall at the same time that home consumption is falling, and this naturally helps generate a low consumption correlation across the countries.

b) Obstfeld and Rogoff (2002) show in a model with incomplete asset markets, sticky prices, and nontraded goods, that there are potential gains from coordination of macro policies. In short, this is due to the ability of policy makers to manipulate terms of trade to shift wealth to a country hit by a negative shock. But these are small, depending on the size of the coefficient or risk aversion, since this allows price movements to replicate risk sharing on their own. The benefits also rise with the share of nontraded goods.

Devereux –Engel (2003) show, assume complete asset markets, that there is no gain from choosing coordination macro policy rules, either under PCP or LCP pricing.

3a) Main facts about the real exchange rate :

i) It is volatile (4 times the volatility of output)

ii) especially during periods of flexible nominal exchange rate regimes.

iii) It's movements are highly persistent, as indicated by unit root tests. Estimates disagree on the half-life, ranging from 1 to 4 years.

iv) It is highly correlated with nominal exchange rates.

b) The cannonical model of Obstfeld and Rogoff (1995) does not explain deviations from PPP. Because home and foreign households have the same consumpiton index over home and foreign goods, their price indexes are the same aggregates over the same goods.

Home: $$P = \left[\int_0^1 p(z)^{1-\theta} \, dz \right]^{\frac{1}{\theta-1}} = \left[\int_0^n p(z)^{1-\theta} \, dz + \int_n^1 e \, p^*(z)^{1-\theta} \, dz \right]^{\frac{1}{\theta-1}}$$

Foreign: $$P^* = \left[\int_0^1 p^*(z)^{1-\theta} \, dz \right]^{\frac{1}{\theta-1}} = \left[\int_0^n \frac{p(z)^{1-\theta}}{e} dz + \int_n^1 p^*(z)^{1-\theta} \, dz \right]^{\frac{1}{\theta-1}}$$

If the price of each good is the same across countries, then PPP is implied by the indexes above.

However, if we allow the prices of a good to differ between countries, this no longer is true, and PPP might fail. Betts and Devereux (1996 EER) show this. With prices sticky in local currency, a movement in the nominal exchange rate opens a gap between the prices across countries when measured in a common currency. Further, Betts and Devereux also show that monetary shocks tend to generate even more movement in the nominal exchange rate under local currency pricing.

Note that this model cannot explain persistence in the real exchange rate much beyond the term of price contracts. Recent empirical evidence, as in Ibs et al (2005) inidicate that once problems like aggregation bias are accounted for, the real exchange rate persistence might not be so high.

c) Stockman solution for equilibrium nominal exchange rate: $e_t = \frac{U'_{2t}}{U'_{1t}} \frac{M^S{}_t}{M^{S*}{}_t} \frac{y_{2t}}{y_{1t}}$

where in this model $\frac{U'_{2t}}{U'_{1t}}$ equals the real exchange rate. This can explain persistence and correlation with the nominal exchange rate, in terms of persistent real endowment shocks. But it has a hard time explaining why the real exchange rate is more volatile than output levels, and why this increases during flexible exchange rate regimes.

Mussa would say it is due to monetary shocks which make the nominal exchange rate fluctuate, while price levels are sticky. So nominal exchange rate fluctuations are passed on to real exchange rates. This explains why volatility increased during flexible exchange rate periods. It has a harder time explaining persistence.

d) Liquidity models like Grilli and Roubini (1992), show that one can potentially get more volatility out of the nominal exchange rate with market segmentation between asset and goods markets in the presence of financial market shocks.

e) Answer may differ. It appears difficult to explain exchange rate behavior in terms of fundamentals. But Engel and West (2005) indicates there is a connection between the two, in that exchange rates can be used to predict future fundamentals.

(6/13/07)