

Solutions to Final Exam

1a) Bellman form:

$$V_t(B_t) = \max \left\{ \left(\frac{\mathbf{s}}{\mathbf{s}-1} C_t \frac{\mathbf{s}-1}{\mathbf{s}} \right) + \mathbf{b} [V_{t+1}(B_{t+1})] \right\} +$$

$$I_t(Y_t + (1+r_t)B_t - C_t - I_t - G_t - B_{t+1})$$

$$C_t : U'_{C_t} = I_t$$

First order conditions:

$$B_{t+1} : \mathbf{b} \frac{\partial V_{t+1}}{\partial B_{t+1}} = I_t$$

Envelope condition:

$$B_t : \frac{\partial V_t}{\partial B_t} = I_t(1+r_t) \rightarrow \frac{\partial V_{t+1}}{\partial B_{t+1}} = I_{t+1}(1+r_{t+1})$$

Euler equation:

$$U'_{C_t} = \mathbf{b}(1+r_{t+1}) [U'_{C_{t+1}}]$$

or:

$$C_t^{-\frac{1}{\mathbf{s}}} = \mathbf{b}(1+r_{t+1}) C_{t+1}^{-\frac{1}{\mathbf{s}}}$$

$$C_t = \mathbf{b}^{-\mathbf{s}} (1+r_{t+1})^{-\mathbf{s}} C_{t+1}$$

$$C_{t+1} = \mathbf{b}^{\mathbf{s}} \frac{1}{1+r_{t+1}} C_t$$

Define time-varying discount factor:

$$R_{t,s} = \left[\prod_{v=t+1}^s (1+r_v) \right]^{-1} \quad \text{for } s > t; \quad R_{t,t} = 1$$

Write intertemporal budget constraint:

$$\sum_{s=t}^{\infty} R_{t,s} (C_s) = (1+r_t)B_t + \sum_{s=t}^{\infty} R_{t,s} (NO_s)$$

Substitute recursively for C to get consumption function:

$$C_t = \left[\sum_{s=t}^{\infty} (R_{t,s})^{\mathbf{s}} (R_{t,s})^{-\mathbf{s}} \right]^{-1} \left[(1+r_t)B_t + \sum_{s=t}^{\infty} R_{t,s} (NO_s) \right] \quad (1)$$

$$C_t = \left[\sum_{s=t}^{\infty} (R_{t,s})^{1-\mathbf{s}} \mathbf{b}^{\mathbf{s}(s-t)} \right]^{-1} \left[(1+r_t)B_t + \sum_{s=t}^{\infty} R_{t,s} (NO_s) \right]$$

Put back in single-period budget constraint to get CA:

$$CA_t = r_t B_t + NO_t - C_t$$

So

$$CA_t = r_t B_t + NO_t - \left[\sum_{s=t}^{\infty} (R_{t,s})^{1-\mathbf{s}} \mathbf{b}^{\mathbf{s}(s-t)} \right]^{-1} \left[(1+r_t)B_t + \sum_{s=t}^{\infty} R_{t,s} (NO_s) \right]$$

Note that under the special case of $r_t = \frac{1}{b} - 1$, this becomes the familiar form:

$$C_t = \left[\sum_{s=t}^{\infty} (b^{s-t})^{1-s} b^{s(s-t)} \right]^{-1} \left[(1+r)B_t + \sum_{s=t}^{\infty} b^{s-t} (NO_s) \right]$$

$$C_t = \left[\sum_{s=t}^{\infty} b^{s-t} \right]^{-1} \left[(1+r)B_t + \sum_{s=t}^{\infty} b^{s-t} (NO_s) \right]$$

$$C_t = (1-b) \left[(1+r)B_t + \sum_{s=t}^{\infty} b^{s-t} (NO_s) \right]$$

$$C_t = rB_t + (1-b) \sum_{s=t}^{\infty} b^{s-t} (NO_s)$$

$$\text{So } CA_t = (NO_t) - (1-b) \sum_{s=t}^{\infty} b^{s-t} E_t [NO_s]$$

- b) i. Consumption rises by $(1-\beta)$ unit, the current account rises by β unit.
 ii. Consumption falls by 1 unit, no change in the current account.
 iii. According to the model above and its definition of the current account, consumption rises by $r = \frac{1}{b} - 1$ unit, and there is no change in the current account. The answer can depend on who the current account and capital account handle capital gains.

- c) A rise in the current interest rate lowers the discount factor $R_{t,s} = \left[\prod_{v=t+1}^s (1+r_v) \right]^{-1}$, which is used for two things: for discounting the value of future earnings, and for determining the intertemporal tradeoff between consumption today and tomorrow. Both roles are seen in the math above, in line (1). The math shows that if $s > 1$, a rise in the interest rate lowers consumption and raises the current account. There are two effects at work. A rise in the interest rate shifts consumption away from the present toward the future. But it also makes the value of intertemporal wealth higher. Since s is the intertemporal elasticity, when this exceeds unity, the intertemporal substitution effect lowering consumption dominates.

2a) Feldstein and Horioka (1980) found in a cross-sectional study that saving and investment tended to be highly correlated within countries. Subsequent studies have confirmed the finding in time series data. This could indicate that countries were forced to finance investment out of domestic saving because of an inability to borrow in the world capital market. However, there are alternative explanations for the empirical finding. Investment might be driven by global technology shocks, which would drive up the world interest rate and prevent borrowing to finance investment. Another explanation is that the technology shocks driving investment are less than permanent, so consumption smoothing would imply a rise in saving.

b) Portfolio diversification puzzle: Poterba (1991) and Tesar and Werner (1995) have documented that equity portfolios tend to be heavily biased toward home assets, whereas a simple model would imply agents should hold a share of the world portfolio. In fact, Baxter

and Jermann (1997) showed that to diversify away from labor income risk, it may be optimal to hold a portfolio short in domestic assets. This probably does not indicate a higher cost to buying foreign assets, since the trade volume in foreign equities is high, even if the net holding is not. One alternative explanation is that shares in foreign nontraded goods that pay off in terms of traded goods are not useful for hedging against domestic nontraded goods consumption risk. To explain the magnitude of home bias in assets, one must assume a low degree of substitutability across goods. A related explanation is that costs of trading goods biases consumption toward home goods, so that home assets are a better hedge against consumption risk.

- c) Consumption correlation puzzle: consumption is less correlated across countries than is output. But under complete risk sharing or consumption smoothing, consumption should be highly correlated. One explanation (Baxter-Crucini or Kollmann) are bonds only asset markets with persistent output shocks. But another explanation is nontraded goods (Stockam-Tesar), or trade costs.
- d) Empirical tests nearly universally reject the relationship between country's interest rates implied by uncovered interest rate parity. See for example: Levich (1985), Mark (1985) or Backus, Gregory and Telmer (1993). VAR test by Eichenbaum and Evans (1995) also show failures of UIP conditional on money supply shocks. Because this parity requires people be willing to hold foreign assets rather than home assets if the expected return is higher, the failure of the condition could indicate a lack of integration in these asset markets. An alternative explanation is that there is a time-varying risk premium that induces fluctuations in the interest rate spread and makes uncovered interest parity fail.

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- 3) The literature has tended to argue that countries should focus on their own domestic objective rather than international variables. But there can be motivation for international coordination or exchange rate management under some conditions, such as local currency pricing, incomplete asset markets, or substitutability between consumption and leisure. Subsequent literature has pointed out many other market imperfections than can motivate policy dealing with international variables.

Obstfeld and Rogoff (2002) find that theoretically there may be gains from international coordination in a model with sticky wages and incomplete asset markets. They find that policy makers could manipulate exchange rates in the presence of sticky wages to make goods cheaper on one country, and hence shift consumption toward that country during bad times. However, quantitative exercises indicate that the gains from coordination are small relative to the gains from stabilization. So there does not appear to be good reason to promote international monetary policy coordination. The benefits of international coordination rise with incompleteness of asset markets or nontraded goods in goods markets, because that is when coordination can raise welfare by pooling risks.

Devereux and Engel (2003) find that there is no gain from coordination in a case where asset markets pool risk perfectly, because countries can replicate the flexible price equilibrium and achieve the optimum on their own. They find motivation for policy to fix the exchange rate if prices are sticky in the local currency, because then exchange rate movements do not serve a useful function of compensating for sticky prices.

Bacchetta and van Wincoop (2000) argue that it can be optimal to fix the exchange rate if consumption and leisure are substitutes in utility.

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