

Chapter 14

Exchange Rates in the Long Run:
The Monetary Approach

Example

- Prices of US and Canadian CPI baskets
 - ◆ 1970 $P_{CAN}=C\$ 100$ 1990 $P_{CAN}=C\$392$
 - ◆ 1970 $P_{US}=\$100$ 1990 $P_{US}=\$336$
- Exchange rates (C\$/ $\$$)
 - ◆ 1970 $E_{C\$/\$}=1$ 1990 $E_{C\$/\$}=1.15$
- Prices of baskets in C\$
 - ◆ CAN 1970 100 1990 392 (up 292%)
 - ◆ US 1970 100 1990 386 (up 286%)
- The purchasing power of the C\$ was the same over the US and Canadian baskets in 1970.
- The purchasing power of the C\$ was ALSO virtually the same over US and Canadian baskets in 1990.
- Mere coincidence?
 - ◆ An important economic theory says:no
 - ◆ The theory of purchasing power parity

Intuition

- Arbitrage principles are applied yet again, this time not to currencies or interest rates, but to goods.
- Assume (suspend disbelief)
 - ◆ No transaction costs, no barriers to trade, frictionless markets, identical goods in each location,...
 - ◆ Then prices must be equal in all locations for any good when expressed in a common currency
 - ◆ (Or else there would be a profit opportunity from buying low and selling high)

The Law of One Price

- A microeconomic idea. Consider a single good (g) in two geographically different markets.
- The **law of one price (LOOP)** states that the price of the good in each market must be the same.
 - ♦ For prices to be commensurate they must be expressed in a common currency.

- Relative price ratio for g:

$$\underbrace{q_{E/US}^g}_{\substack{\text{relative price} \\ \text{of good } g \\ \text{in Europe} \\ \text{versus U.S.}}} = \underbrace{(E_{\$/\epsilon} P_E^g)}_{\substack{\text{European price} \\ \text{of good } g \\ \text{expressed} \\ \text{in \$}}} / \underbrace{P_{US}^g}_{\substack{\text{U.S. price} \\ \text{of good } g \\ \text{expressed} \\ \text{in \$}}}$$

- If LOOP holds then:
- Examples?

$$E_{\$/\epsilon} P_E^g = P_{US}^g$$

Purchasing Power Parity

- Macroeconomic counterpart to LOOP. If LOOP holds for every good in the CPI basket, then the prices of the entire baskets must be the same in each location.
- The **purchasing power parity (PPP)** states that these overall price levels in each market must be the same.
- Relative price level ratio:

$$\underbrace{q_{E/US}}_{\substack{\text{relative price} \\ \text{of basket} \\ \text{in Europe} \\ \text{versus U.S.}}} = \underbrace{(E_{\$/\epsilon} P_E)}_{\substack{\text{European price} \\ \text{of basket} \\ \text{expressed} \\ \text{in \$}}} / \underbrace{P_{US}}_{\substack{\text{U.S. price} \\ \text{of basket} \\ \text{expressed} \\ \text{in \$}}}$$

$$E_{\$/\epsilon} P_E = P_{US}, \text{ or } q_{E/US} = 1.$$

The real exchange rate

- Relative price level ratio q is an important concept.

$$\underbrace{q_{E/US}}_{\substack{\text{relative price} \\ \text{of basket} \\ \text{in Europe} \\ \text{versus U.S.}}} = \underbrace{(E_{\$/\epsilon} P_E)}_{\substack{\text{European price} \\ \text{of basket} \\ \text{expressed} \\ \text{in \$}}} / \underbrace{P_{US}}_{\substack{\text{U.S. price} \\ \text{of basket} \\ \text{expressed} \\ \text{in \$}}}$$

- Called the **real exchange rate (RER)**
 - ♦ Nominal exchange rate:
price of foreign currency in terms of domestic currency
 - ♦ Real exchange rate:
price of foreign basket in terms of domestic basket
- PPP holds if and only if real exchange rate is 1:

$$E_{\$/\epsilon} P_E = P_{US}, \text{ or } q_{E/US} = 1.$$

Real depreciation & appreciation

$$\underbrace{q_{E/US}}_{\substack{\text{relative price} \\ \text{of basket} \\ \text{in Europe} \\ \text{versus U.S.}}} = \underbrace{(E_{\$/\epsilon} P_E)}_{\substack{\text{European price} \\ \text{of basket} \\ \text{expressed} \\ \text{in \$}}} / \underbrace{P_{US}}_{\substack{\text{U.S. price} \\ \text{of basket} \\ \text{expressed} \\ \text{in \$}}}$$

- The real exchange rate is a new concept, but it has some terminology in common with the nominal exchange rate:
 - ◆ If the real exchange rate rises (more home goods are needed in exchange for foreign goods), we speak of a **real depreciation**.
 - ◆ If the real exchange rate falls (fewer home goods are needed in exchange for foreign goods), we speak of a **real appreciation**.

Overvaluation & undervaluation

$$\underbrace{q_{E/US}}_{\substack{\text{relative price} \\ \text{of basket} \\ \text{in Europe} \\ \text{versus U.S.}}} = \underbrace{(E_{\$/\epsilon} P_E)}_{\substack{\text{European price} \\ \text{of basket} \\ \text{expressed} \\ \text{in \$}}} / \underbrace{P_{US}}_{\substack{\text{U.S. price} \\ \text{of basket} \\ \text{expressed} \\ \text{in \$}}}$$

- We use PPP-implied level of 1 as a benchmark or reference level for the real exchange rate. This leads naturally to more new terminology.
 - If the real exchange rate is above one (by x percent), then foreign goods are relatively expensive, and we speak of the foreign currency as being *overvalued* (by x percent).
 - If the real exchange rate is below one (by x percent), then foreign goods are relatively cheap, and we speak of the foreign currency as being *undervalued* (by x percent).

PPP as a theory of the exchange rate

- Rearrange the PPP equation:

$$\underbrace{E_{\$/\epsilon}}_{\text{exchange rate}} = \underbrace{P_{US} / P_E}_{\text{ratio of price levels}}$$

- One of the most important equations in this course. A clear prediction about exchange rates
- *Purchasing power parity implies that the exchange rate at which two currencies trade is equal to the relative price levels of the two countries.*
 - ♦ Example: A basket of goods costs \$1000 in the United States, and £800 in the UK. What is the exchange rate (\$/£) implied by the PPP theory?

PPP in levels & rates of change

- The PPP equation:
$$\underbrace{E_{\$/\epsilon}}_{\text{exchange rate}} = \underbrace{P_{US} / P_E}_{\text{ratio of price levels}}$$

- If this is true in levels it is true in rates of change.

- ◆ The rate of change of the LHS is the rate of increase of E.
 - ◆ This is the rate of depreciation:

$$\frac{\Delta E_{\$/\epsilon,t}}{E_{\$/\epsilon,t}} = \frac{E_{\$/\epsilon,t+1} - E_{\$/\epsilon,t}}{\underbrace{E_{\$/\epsilon,t}}_{\substack{\text{rate of depreciation} \\ \text{of the nominal exchange rate}}}}$$

- ◆ The rate of change of the RHS is rate of change of home prices minus rate of change of foreign prices (quotient rule).
 - ◆ This is the home-foreign inflation differential:

$$\frac{\Delta P_{US,t}}{P_{US,t}} - \frac{\Delta P_{E,t}}{P_{E,t}} = \underbrace{\left(\frac{P_{US,t+1} - P_{US,t}}{P_{US,t}} \right)}_{\substack{\text{rate of inflation in U.S.} \\ \pi_{US,t}}} - \underbrace{\left(\frac{P_{E,t+1} - P_{E,t}}{P_{E,t}} \right)}_{\substack{\text{rate of inflation in Europe} \\ \pi_{E,t}}}$$

PPP in levels & rates of change

- The PPP equation expressed in rates of change is therefore:

$$\underbrace{\frac{\Delta E_{\$/\epsilon,t}}{E_{\$/\epsilon,t}}}_{\substack{\text{rate of depreciation} \\ \text{of the nominal exchange rate}}} = \underbrace{\pi_{US,t} - \pi_{E,t}}_{\text{inflation differential}}$$

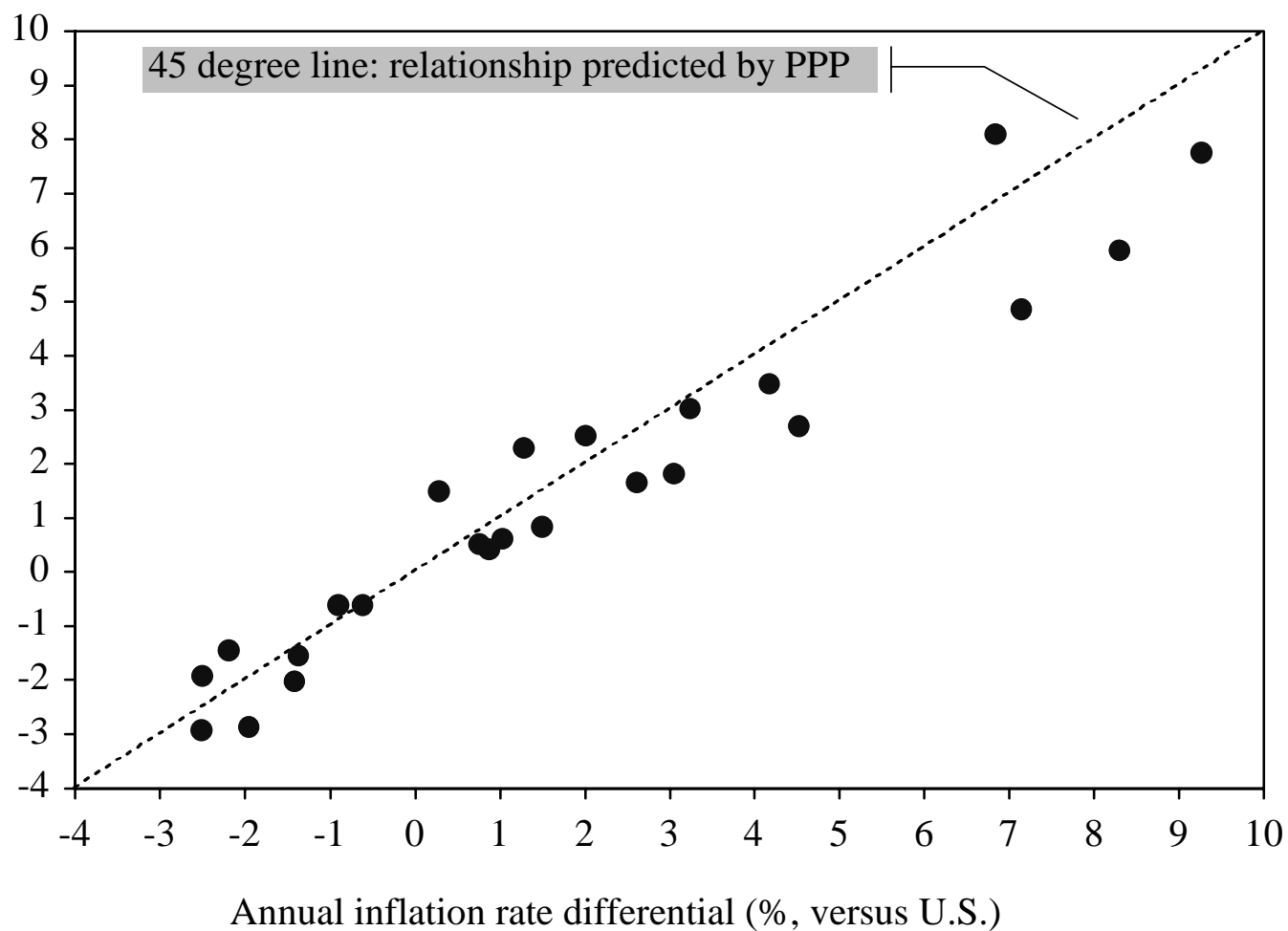
- This expression is known to as **Relative PPP**.
- Relative PPP implies that the rate of depreciation of the nominal exchange rate equals the inflation differential.*
- Example: Refer back to Canada and US from 1970 to 1990.
 - Canadian price level went from 100 to 392: inflation was 6.8% pa
 - US price level went from 100 to 336: inflation was 6.1% pa
 - Can. Exch. rate went from 1 to 1.15: depreciation was 0.7% pa
 - This was equal to the inflation differential (6.8 minus 6.1 equals 0.7)

Notes on Absolute & Relative PPP

- Unlike Absolute PPP, Relative PPP predicts a relationship between changes in prices and changes in exchange rates, rather than a relationship between their levels.
- Relative PPP was derived from Absolute PPP; i.e., the latter implies the former. *If Absolute PPP holds then Relative PPP must hold also.*
- *Now for some evidence in the various versions of PPP...*

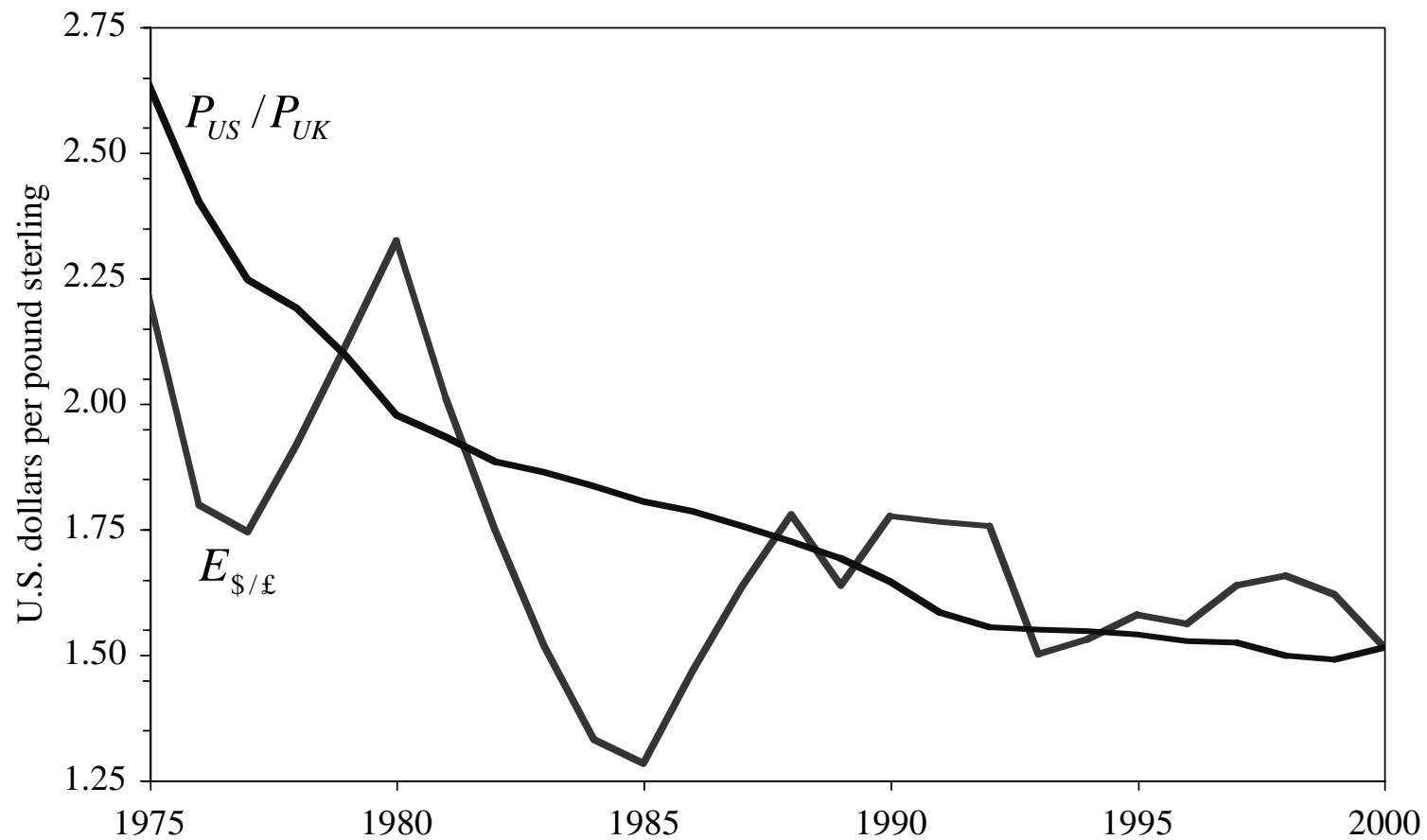
PPP in the long run: not bad

Relative PPP, ~30 years, annual averages 1972–2000:



PPP better in long run than short run

Absolute PPP, 25 years, US/UK annual data 1975–2000:



Summing up PPP

- The evidence suggests that the theory of purchasing power parity works best in the long run.
- May take a long time for long-run relationship to emerge: estimates suggest they may die out at a rate of about 15% per year: after one year 85% of an initial deviation or gap persists; after two years 72% still persists; after four years 52%.
- There are several reasons why PPP holds less well in the short run.

Why PPP fails in the short run

- Transaction costs. Trade is not costless because costs of international transportation are significant, and there may be tariffs and duties when they cross national borders.
- Nontraded goods. Some goods are inherently nontradable; one can think of them as having infinitely high transaction costs.

Why PPP fails in the short run

- Imperfect competition. Many goods are differentiated products, often with brand names, copyrights, and legal protection. So firms have some market power to set their own price.
- Price stickiness. One of the most common assumptions of macroeconomics is that prices are “sticky” in the short run. So they may not move in response to the frequent changes in the exchange rate.

Something to chew on...

- For 20 years *The Economist* newspaper has applied the idea of PPP to evaluate whether currencies are undervalued or overvalued.
 - ◆ Recall, home currency is $x\%$ overvalued/ undervalued when the home basket costs $x\%$ more/less than the foreign basket.
- Actually, it is really a LOOP based test.
- *The Economist* uses a very simple “basket” consisting of just one globally uniform, standardized product.
- The Big Mac

Burgernomics

- Invented in 1986 by economics editor Pam Woodall, based in London.
 - ◆ She asks correspondents around the world to visit McDonalds and get the prices and she then computes the \$ price in each location relative to the US:

$$\text{“Big Mac index”} = q^{\text{Big Mac}} - 1 = \frac{E_{\$/\text{local currency}} P_{\text{local}}^{\text{Big Mac}}}{P_{\text{US}}^{\text{Big Mac}}} - 1$$

- ◆ The % deviation (+/–) from the US price measures the over/under valuation of the local currency using the burger basket.

Latte parity

- In 2004 *The Economist* tried using a new globally uniform, standardized product.
- The Starbucks tall latte

“Big Mac index” =

$$q^{\text{Big Mac}} - 1 = \frac{E_{\$/\text{local currency}} P_{\text{local}}^{\text{Big Mac}}}{P_{\text{US}}^{\text{Big Mac}}} - 1$$

“tall-latte index” =

$$q^{\text{tall latte}} - 1 = \frac{E_{\$/\text{local currency}} P_{\text{local}}^{\text{tall latte}}}{P_{\text{US}}^{\text{tall latte}}} - 1$$

Pick your poison

Our hot tips

Local currency under (-)/over (+) valuation against the dollar, %, using:

	Starbucks tall-latte index	McDonald's Big Mac index
Australia	-4	-17
Britain	+17	+23
Canada	-16	-16
China	-1	-56
Euro area	+33	+24
Hong Kong	+15	-45
Japan	+13	-12
Malaysia	-25	-53
Mexico	-15	-21
New Zealand	-12	-4
Singapore	+2	-31
South Korea	+6	0
Switzerland	+62	+82
Taiwan	-5	-21
Thailand	-31	-46
Turkey	+6	+5

Source: *The Economist*

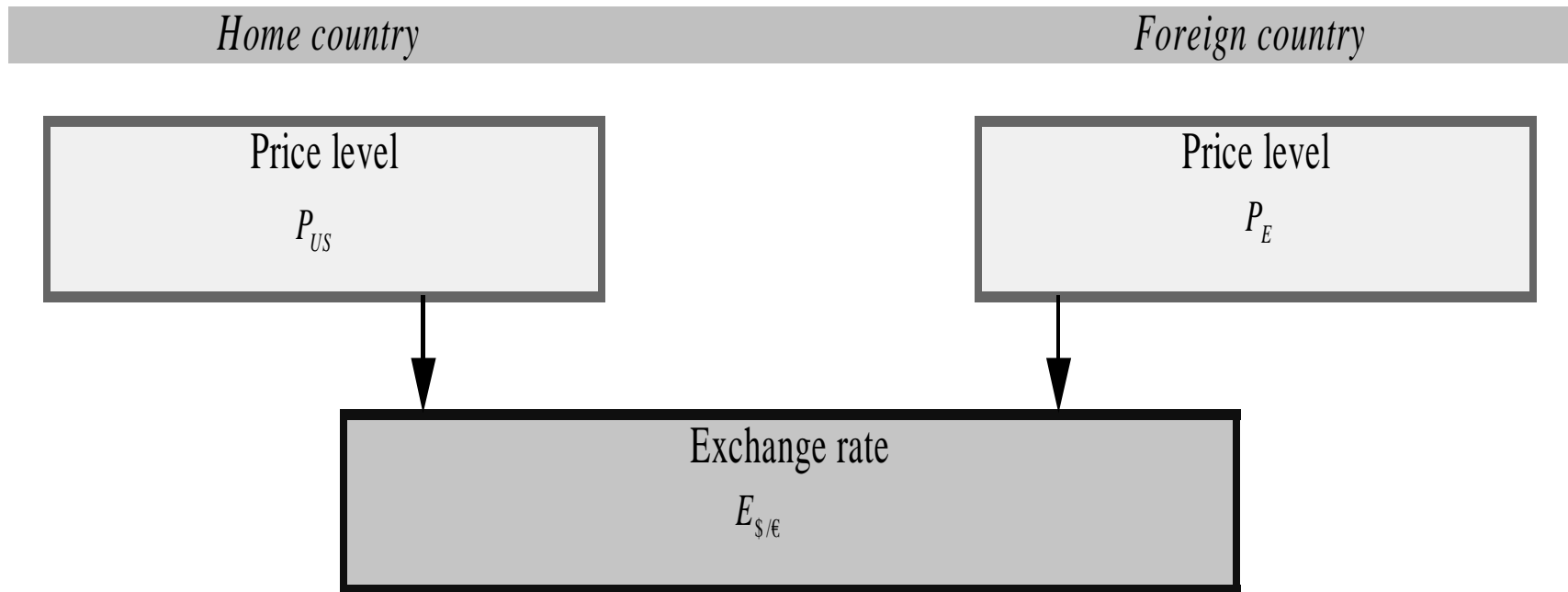
PPP as a theory of the exchange rate

- Rearrange the PPP equation:

$$\underbrace{E_{\$/\epsilon}}_{\text{exchange rate}} = \underbrace{P_{US} / P_E}_{\text{ratio of price levels}}$$

- One of the most important equations in this course. A clear prediction about exchange rates
- *Purchasing power parity implies that the exchange rate at which two currencies trade is equal to the relative price levels of the two countries.*
 - ♦ *Begs the question: where do price levels come from?*
 - ♦ *Answer: in the long run, price levels are determined by monetary forces*

Building block



Preamble on money

What is money?

- ◆ It serves 3 functions

1. Store of value

- ◆ Money is an asset, but it pays no interest; has value in the future too.

2. Unit of account

- ◆ How prices are expressed

3. Medium of exchange

- ◆ Has value today and in the future because it is the most liquid form of payment: an asset that is easily converted into goods and services

Measurement of money

- Many definitions of the amount or stock of money in the economy, ranging from narrow to broad
 - ◆ M0 currency
 - ◆ M1 M0 + demand deposits, traveler's checks
 - ◆ M2 M1 + savings and small time deposits
 - ◆ M3 M2 + large time deposits, money market funds, repurchase agreements
- In this course when we refer to money and use the symbol M, we will mean M1
 - ◆ Money that is most liquid and earns no interest

Money supply

- We assume that the nominal money supply $M=M1$ is controlled by the central bank
 - ◆ In fact, the central bank directly controls only part of M , namely $M0$, or currency (notes and coins)
 - ◆ However, central banks can indirectly control $M1$ by using interest rate policies and other tools (such as reserve requirements) to influence the total amount of bank deposits created $M1-M0$.
 - ◆ Hence, for simplicity, we assume that the central bank controls $M=M1$

Money demand, nominal

- We assume that the demand for nominal money is driven by the need to use money to undertake transactions
 - ◆ In the simplest model, the **quantity theory**: the amount of transactions assumed proportional to the dollar value of nominal income $P \times Y$ (where real income is Y)
 - ◆ Hence, money demand is proportional to the dollar value of nominal income, where the proportionality is given by a constant L .

$$\underbrace{M^d}_{\text{demand for money (\$)}} = \underbrace{P \times Y}_{\text{nominal income (\$)}} \times \underbrace{\bar{L}}_{\text{a constant}}$$

- Example (US):
Nominal income = \$12 trillion, $L=0.1$, $M=\$1.2$ trillion
- Annually, each \$1 allows economy to transact GDP worth \$10.
- We say the **velocity** of money (V) is 10 where $V = 1/L$

Money demand, real

- Rearrange to get an expression for the demand for real money balances (the nominal value of money deflated by the price level P):

$$\underbrace{\frac{M^d}{P}}_{\text{demand for real money}} = \underbrace{\bar{L}}_{\text{a constant}} \times \underbrace{Y}_{\text{real income}}$$

- The demand for real money is a constant multiple of the real income level Y .

Money market equilibrium

LONG RUN

- If money demand M_d equals money supply M then we can replace M_d with M in last two equations

- Nominal:

$$M = \bar{L}PY$$

- Real:

$$\frac{M}{P} = \bar{L}Y$$

- In the long run, we assume prices P are fully flexible and adjust so this equilibrium is attained.
- So this is also known as the **flexible price model**.

The monetary theory of the price level

LONG RUN

- Now just rearrange and solve for the price level P .
- Do this for each economy (e.g. US, euroland)

fundamental equations of the monetary model of the price level

$$P_{US} = \frac{M_{US}}{\bar{L}_{US} Y_{US}}$$

$$P_E = \frac{M_E}{\bar{L}_E Y_E}$$

- Intuition: These expressions say that the price level P is determined by the ratio of nominal money supplied M to nominal money demanded LY .
 - ♦ Prices rise if there is “more money chasing fewer goods”

The monetary theory of the exchange rate

LONG RUN

- Now use another piece of theory that is only suited to the long run, PPP.
- PPP says E equals the ratio of the price levels. So just plug in the price levels:

fundamental equation of the monetary approach to exchange rates

$$\underbrace{E_{\$/\epsilon}}_{\text{exchange rate}} = \underbrace{\frac{P_{US}}{P_E}}_{\text{ratio of price levels}} = \frac{\left(\frac{M_{US}}{\bar{L}_{US} Y_{US}} \right)}{\left(\frac{M_E}{\bar{L}_E Y_E} \right)} = \underbrace{\frac{(M_{US} / M_E)}{(\bar{L}_{US} Y_{US} / \bar{L}_E Y_E)}}_{\substack{\text{relative nominal money supplies} \\ \text{divided by} \\ \text{relative real money demands}}}$$

The monetary theory of the exchange rate LONG RUN

- The monetary theory can also be expressed in terms of rates of growth.
 - ◆ rate of growth of a product = sum of growth rates
 - ◆ rate of growth of a quotient = difference of growth rates
- Define
 - ◆ growth rate of nominal money supply M
 - ◆ growth rate of real income Y :

$$\mu_{US,t} = \underbrace{\left(\frac{M_{US,t+1} - M_{US,t}}{M_{US,t}} \right)}_{\text{rate of money supply growth in U.S.}}$$

$$g_{US,t} = \underbrace{\left(\frac{Y_{US,t+1} - Y_{US,t}}{Y_{US,t}} \right)}_{\text{rate of real output growth in U.S.}}$$

- ◆ and similarly for Europe

The monetary theory of the exchange rate

LONG RUN

- The levels equation

$$P_{US} = \frac{M_{US}}{\bar{L}_{US} Y_{US}}$$

- The same equation in differences (remember L is a constant)

$$\pi_{US,t} = \mu_{US,t} - g_{US,t}$$

- *Important monetarist result: inflation equals the excess of money growth over real output growth.*
- Same for Europe

$$\pi_{E,t} = \mu_{E,t} - g_{E,t}$$

The monetary theory of the exchange rate LONG RUN

- Putting it all together in growth rates
 - ◆ Use relative PPP rather than absolute PPP

$$\underbrace{\frac{\Delta E_{\$/\epsilon,t}}{E_{\$/\epsilon,t}}}_{\text{rate of depreciation of the nominal exchange rate}} = \underbrace{\pi_{US,t} - \pi_{E,t}}_{\text{inflation differential}} = (\mu_{US,t} - g_{US,t}) - (\mu_{E,t} - g_{E,t})$$

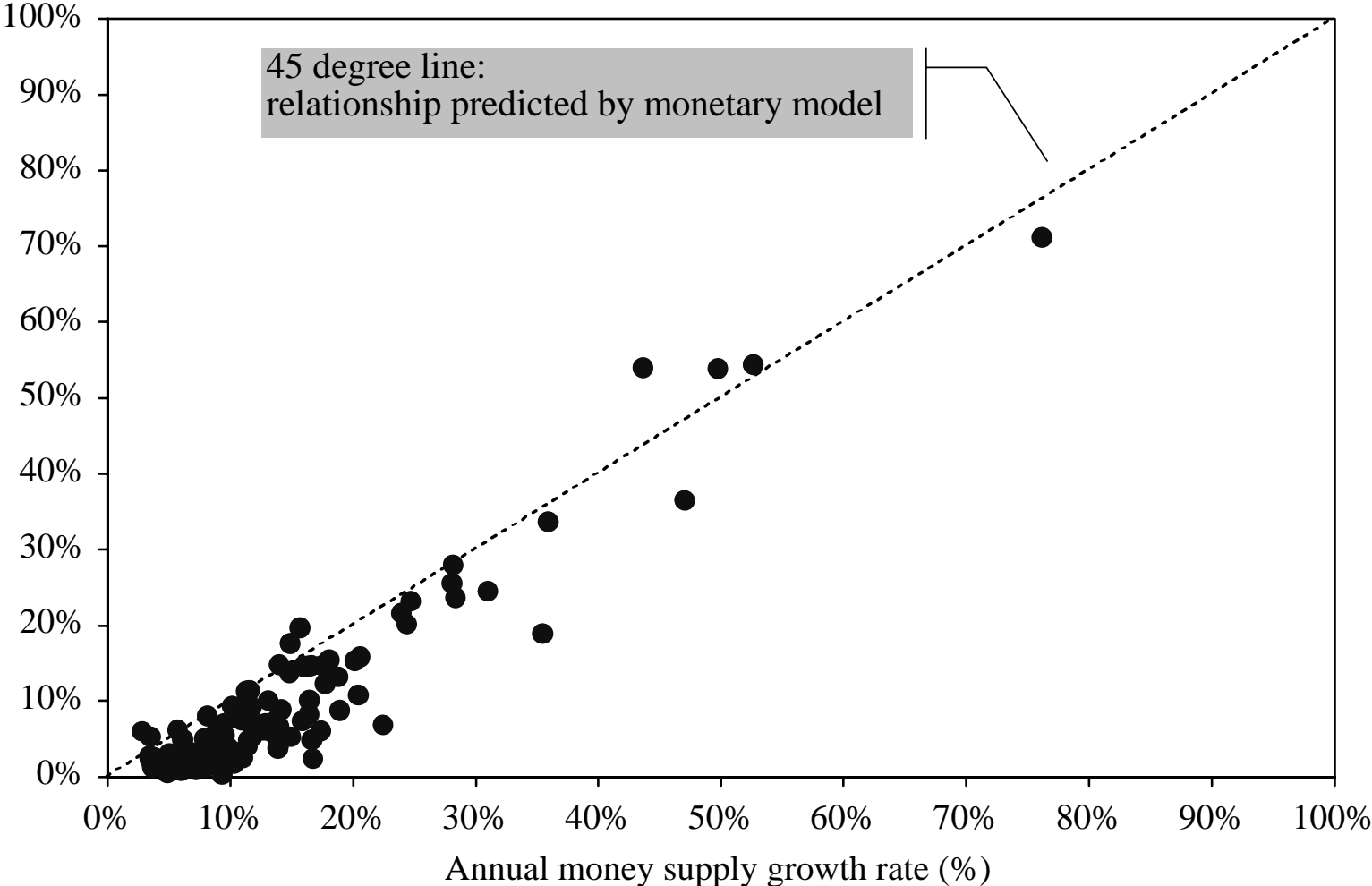
$$= \underbrace{(\mu_{US,t} - \mu_{E,t})}_{\text{differential in nominal money supply growth rates}} - \underbrace{(g_{US,t} - g_{E,t})}_{\text{differential in real output growth rates}}.$$

- ◆ We have a number of important and testable implications
- ◆ Let's have a look at what theory predicts and the evidence

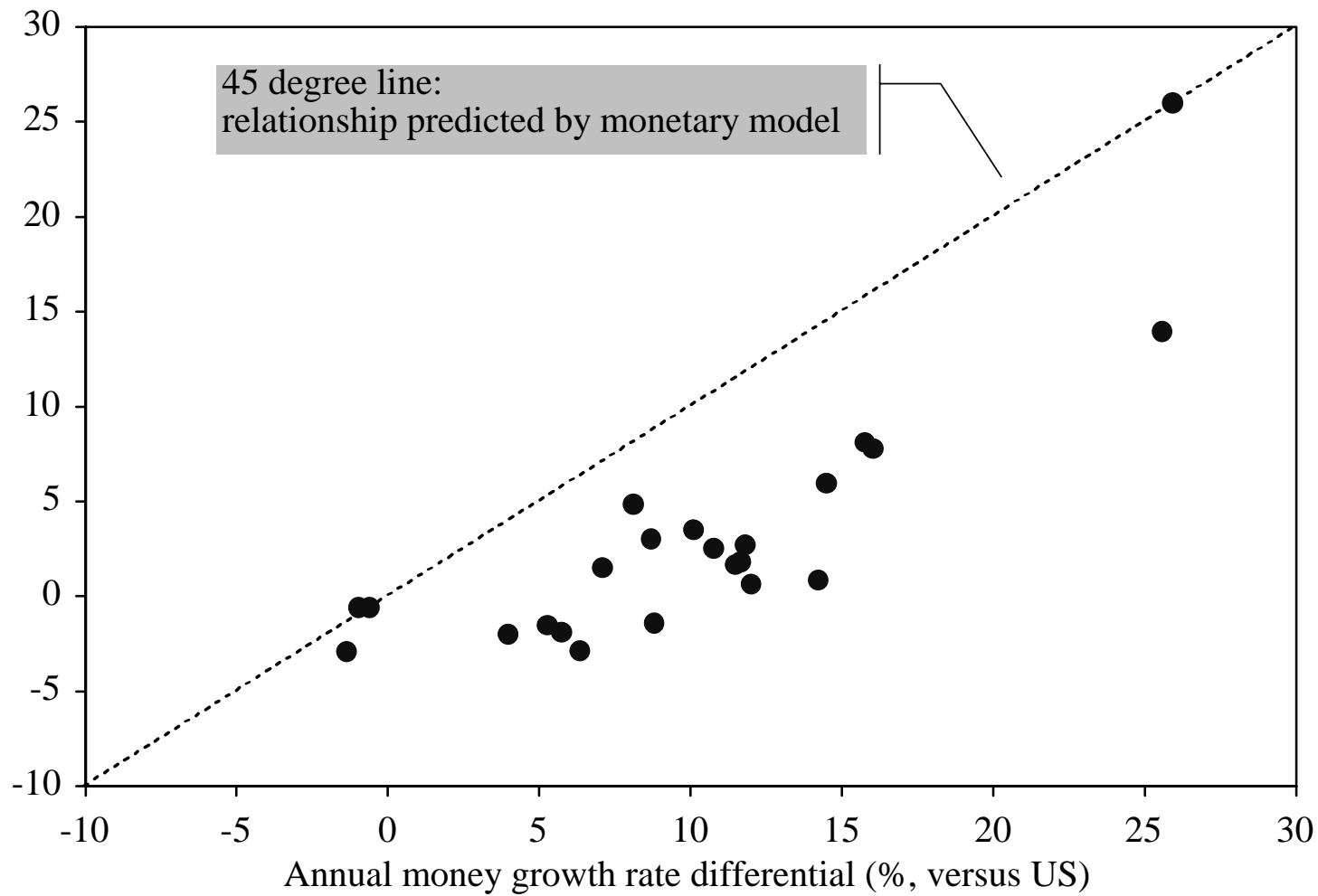
Policy predictions

- Both countries
 - ◆ Constant money growth rate μ , fixed level of output Y
- Foreign
 - ◆ Money growth μ is zero, inflation π is zero
- Home
 - ◆ Money growth μ is positive, inflation π is positive
- Policy change at time T
 - ◆ Home increases its rate of money growth μ by $\Delta\mu$
- What happens to all these variable according to the long run (flexible price) model?

Evidence: inflation and money growth



Evidence: exchange rates and money growth



Shortcomings of the simple model

- Quantity theory assumes that L is a constant
 - ◆ For a given level of real output Y , the level of real money balances M/P is assume constant
 - ◆ Not true in general
- Q: Why might people adjust their level of money balances?
- A: The more general theory assumes that L isn't constant, and depends inversely on the opportunity cost of holding money.
 - ◆ What is the opportunity cost of holding money?

Opportunity cost of holding money

- In nominal terms
 - ◆ Suppose the bank pays a nominal interest rate $i_{\text{bank}} = i$ on a interest bearing account
 - ◆ Cash pays a nominal interest rate of $i_{\text{cash}} = 0$
 - ◆ The difference is $i_{\text{bank}} - i_{\text{cash}} = i - 0 = i$
- *The opportunity cost of holding money is the nominal interest rate i .*

Standard model of money demand

- The standard model of money demand is motivated by two insights, the first of which carries over from our simple model presented earlier, the quantity theory.
- *Benefits:* As before, the benefit of money is that individuals can transact with it. As in the simple quantity theory, we continue to assume that transactions demand is in proportion to income, all else equal.
- *Costs:* The cost of holding money, instead of other assets, arises because money has a lower rate of return. In particular, the nominal interest rate on money is zero.

Standard model of money demand

- Moving from the individual or household level up to the aggregate or macroeconomic level, we can infer that the aggregate **money demand** will behave similarly:
- *All else equal, a rise in national dollar income (nominal income) will cause a proportional increase in transactions and, hence, in aggregate money demand.*
- *All else equal, a rise in the nominal interest rate will cause the aggregate demand for money to fall.*

Standard model of money demand

- Mathematically:

◆ Nominal

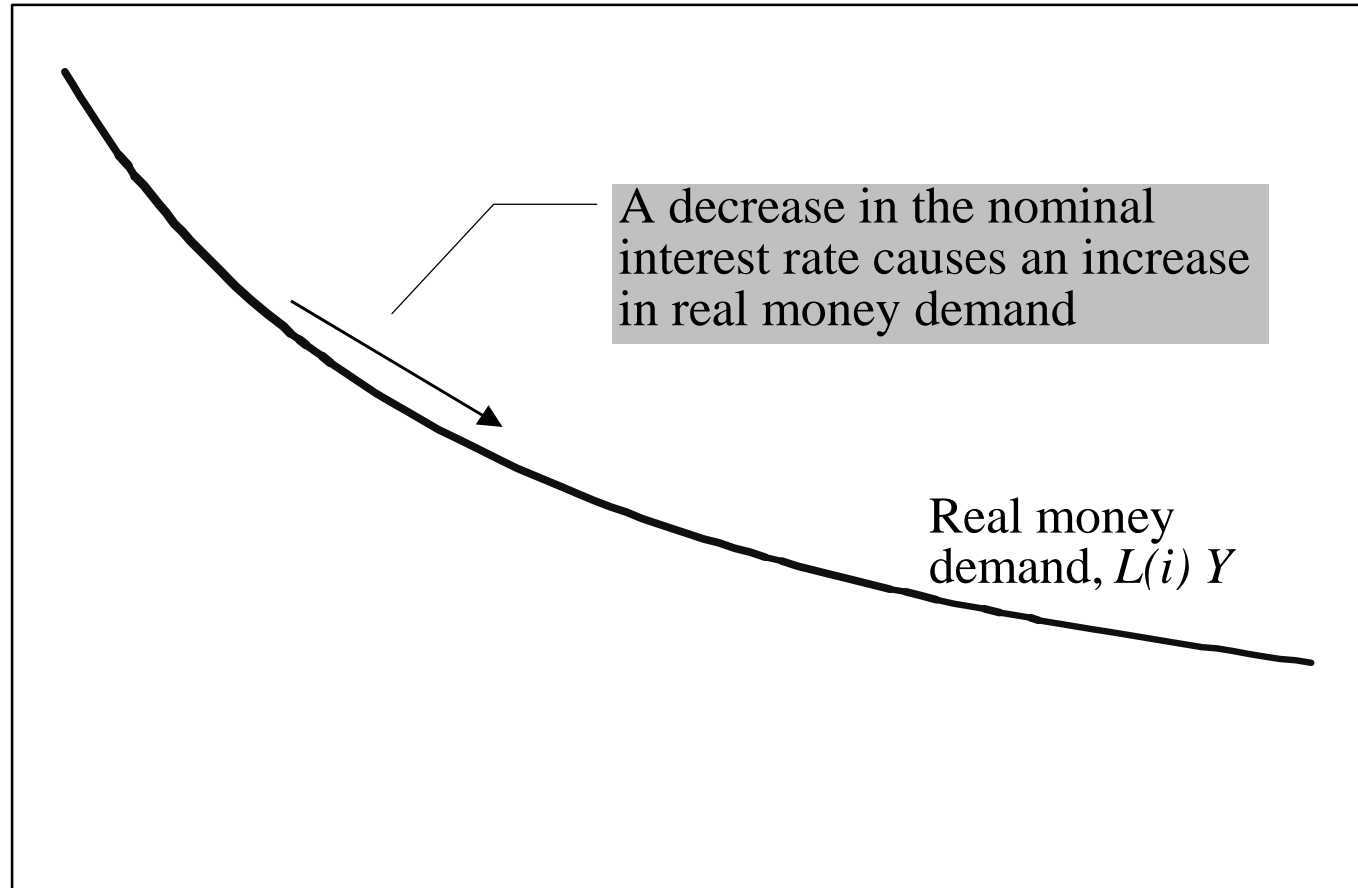
$$\underbrace{M^d}_{\text{demand for money (\$)}} = \underbrace{P \times Y}_{\text{nominal income (\$)}} \times \underbrace{L(i)}_{\text{a decreasing function}}$$

◆ Real

$$\underbrace{\frac{M^d}{P}}_{\text{demand for real money}} = \underbrace{L(i)}_{\text{a decreasing function}} \times \underbrace{Y}_{\text{real income}}$$

Real money demand function

Nominal
interest rate, i

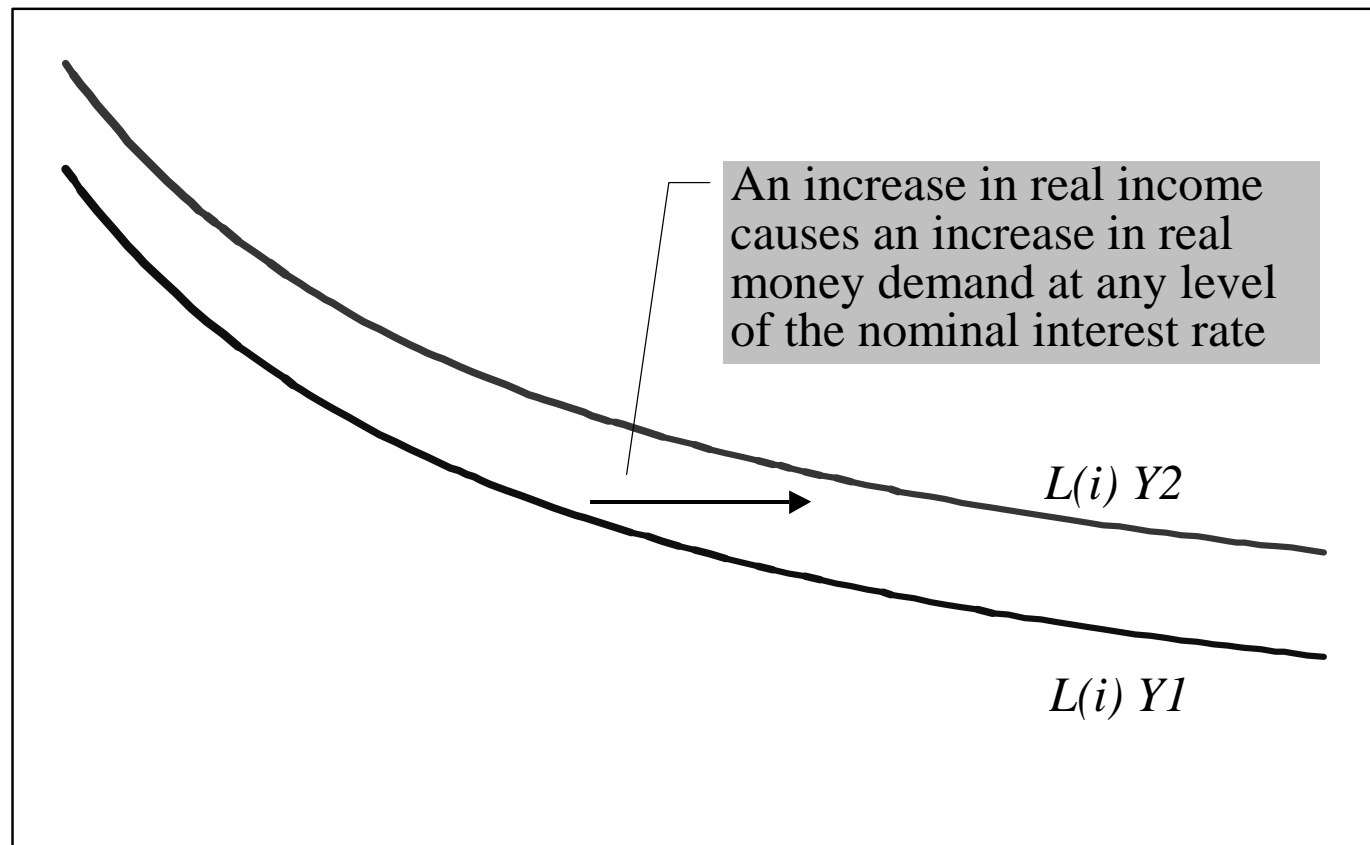


Real money
balances,

Real money demand function

A CHANGE IN REAL INCOME

Nominal
interest rate, i



Real money
balances,
 \bar{M}

Long run theory of the interest rate

- Recall: We are building a long run theory
 - ◆ We assume price flexibility
 - ◆ We assume PPP
- The addition of the term $L(i)$ is only useful if we can come up with a theory of where i comes from in the long run
- To do so we invoke the UIP condition and see what UIP implies in the long run

PPP meets UIP...

- ◆ PPP (rates of change)

$$\underbrace{\frac{\Delta E^e}{E_{\$/\epsilon,t}}}_{\text{expected rate of dollar depreciation}} = \underbrace{\pi_{US,t}^e - \pi_{E,t}^e}_{\text{expected inflation differential}}$$

- ◆ UIP (approximation) says that:

$$\underbrace{\frac{\Delta E_{\$/\epsilon}^e}{E_{\$/\epsilon}}}_{\text{expected rate of dollar depreciation}} = \underbrace{i_{\$}}_{\text{net dollar interest rate}} - \underbrace{i}_{\text{net euro interest rate}}$$

- ◆ If the left hand sides are equal, then the right hand sides must be equal too.

... implying Fisher Effect and RIP

- ◆ And together they imply the **Fisher effect**

$$\underbrace{i_{\$} - i_{\text{€}}}_{\text{nominal interest rate differential}} = \underbrace{\pi_{US}^e - \pi_E^e}_{\text{nominal inflation rate differential (expected)}}$$

- ◆ and **real interest parity (RIP)**

$$i_{\$} - \pi_{US}^e = i_{\text{€}} - \pi_E^e$$

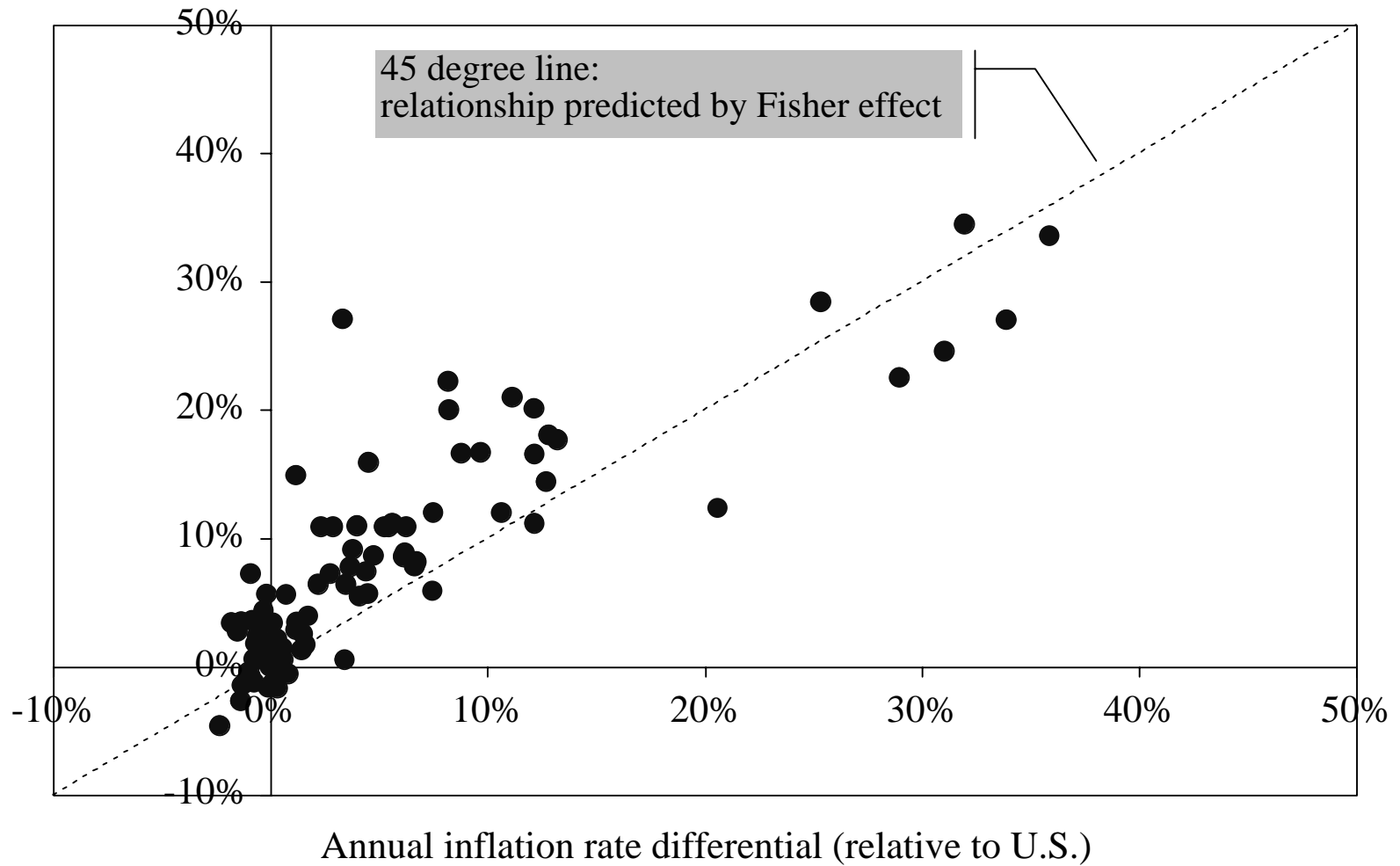
- ◆ that is, equalization of ex ante real interest rates

$$r_{US}^e = r_E^e$$

- A powerful result!
- Only true in the long run given our assumptions
- We have assumed no risk premium, but easily added

Evidence on Fisher Effect

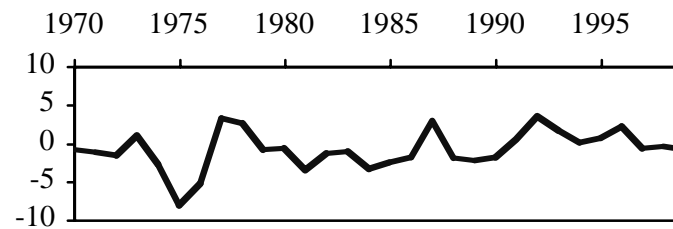
INFLATION AND INTEREST RATES



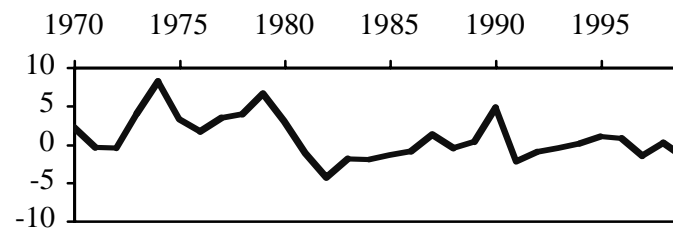
Evidence on RIP

(EX POST) REAL INTEREST RATE DIFFERENTIALS

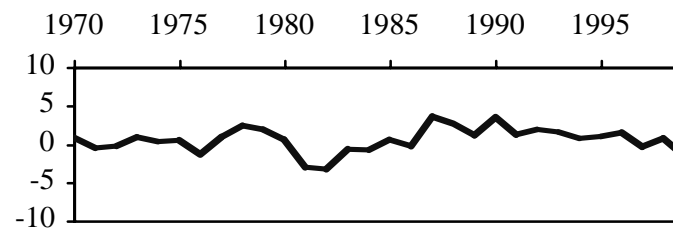
U.K.-U.S.



Germany-U.S.



France-U.S.



Summing up the standard model

- Same as the basic (quantity theory) model except that the constant L is replaced by the decreasing function $L(i)$:

$$\underbrace{E_{\$/\epsilon}}_{\text{exchange rate}} = \frac{P_{US}}{\underbrace{P_E}_{\text{ratio of price levels}}} = \frac{\left(\frac{M_{US}}{L_{US}(i_{\$})Y_{US}} \right)}{\left(\frac{M_E}{L_E(i_{\epsilon})Y_E} \right)} = \frac{(M_{US} / M_E)}{\underbrace{\left(L_{US}(i_{\$})Y_{US} / L_E(i_{\epsilon})Y_E \right)}_{\substack{\text{relative nominal money supplies} \\ \text{divided by} \\ \text{relative real money demands}}}$$

- E is still a ratio of price levels (PPP)
- P is ratio of money supply M to real money demand $L(i)Y$
 - ♦ The basic model is “good enough” if interest rates i are stable in the long run.

(Uncovered) Interest Parity (UIP):
$$i_{\$} = i_{\text{€}} + \frac{E_{\$/\text{€}}^e - E_{\$/\text{€}}}{E_{\$/\text{€}}}$$

Covered Interest Parity (CIP):
$$i_{\$} = i_{\text{€}} + \frac{F_{\$/\text{€}} - E_{\$/\text{€}}}{E_{\$/\text{€}}}$$

Real Interest Rate Parity:
$$r_{\$} = r_{DM}$$

Purchasing power parity (PPP):
$$P_{US} = E_{\$/\text{€}} P_{EU}$$

Relative PPP:
$$\Pi_{US,t} = \frac{E_{\$/\text{€}_t} - E_{\$/\text{€}_{t-1}}}{E_{\$/\text{€}_{t-1}}} + \Pi_{EU,t}$$

Monetary approach:
$$E_{\$/\text{€}} = \frac{P_{US}}{P_{EU}} = \left(\frac{M_{US}^S}{M_{EU}^S} \right) \Bigg/ \left(\frac{L(i_{\$})Y_{US}}{L(i_{\text{€}})Y_{EU}} \right)$$

Real exchange rate:
$$q_{EU/US} = E_{\$/\text{€}} \frac{P_{EU}}{P_{US}}$$