Economics 105
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Oct.06, 1998

I. Differentiation of one variable.

. Defination: The process of finding derivatives is called differenciation.
. Notation: For function $y = f(x)$, the first derivative is denoted as $\frac{dy}{dx}$

. Summary of differentiation rules.
1.Constant Rule: $\frac{d}{dx}[c] = 0,$ c is constant.
Example: $y = 7, \frac{dy}{dx} = \frac{d}{dx}[7] = 0$
2. Constant Multiple Rule: $\frac{d}{dx}[cy] = c\frac{dy}{dx}$, c is constant.
Example: $y = 7x, \frac{dy}{dx} = \frac{d}{dx}[7x] = 7\frac{dx}{dx} = 7$
3. Sum and Difference Rule: $\frac{d}{dx}[u \pm v] = \frac{du}{dx} \pm \frac{dv}{dx}$
Example: $y = \frac{1}{2}x + 5x, \frac{dy}{dx} = \frac{d}{dx}\left[\frac{1}{2}x\right] + \frac{d}{dx}[5x] = \frac{1}{2} + 5 = 5\frac{1}{2}$
4. Product Rule: $\frac{d}{dx}[uv] = u\frac{dv}{dx} + v\frac{du}{dx}$
Example: $y = 2x(4+3x)$

$$\frac{dy}{dx} = \frac{d}{dx}[2x(4+3x)] = 2x\frac{d}{dx}(4+3x) + (4+3x)\frac{d}{dx}(2x)$$
$$= 2x * 3 + (4+3x) * 2 = 6x + 8 + 6x = 12x + 8$$

5. Quotient Rule: $\frac{d}{dx}\left[\frac{u}{v}\right] = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
Example: $y = \frac{x-1}{2x+3}$

$$\frac{d}{dx}\left[\frac{x-1}{2x+3}\right] = \frac{(2x+3)\frac{d}{dx}[x-1] - (x-1)\frac{d}{dx}[2x+3]}{(2x+3)^2} = \frac{2x+3-(x-1)*2}{(2x+3)^2}$$
$$= \frac{3+2}{(2x+3)^2} = \frac{5}{(2x+3)^2}$$

6. Power Rules:
Simple Power Rules: $\frac{d}{dx}[x^n] = nx^{n-1}$
Example: $y = x^3, \frac{dy}{dx} = 3x^2$

$$y = \frac{1}{x^2} = x^{-2}, \frac{dy}{dx} = (-2)x^{-3} = -2x^{-3}$$

General Power Rules: $\frac{d}{dx}[u^n] = nu^{n-1}\frac{du}{dx}$, u is a function of x
7. Chain Rule: $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$, u is a function of x
Example: $y = (x^2+1)^3, u = x^2+1, y = u^3$

$$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx} = 3u^2\frac{du}{dx} = 3u^2(2x) = 6x(x^2+1)^2$$

8. Derivative of logarithmic function: $\frac{d}{dx}[\ln x] = \frac{1}{x}$

II. Partial differentiation

. Defination: If $z = f(x,y)$, then the first partial derivatives of z with reapect ot x and y are the functions $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$, defined as follows:

$$\frac{\partial z}{\partial x} = \lim \frac{f(x+\circ x,y) - f(x,y)}{\circ x},$$ y is held constant

$$\frac{\partial z}{\partial y} = \lim \frac{f(x,y+\circ y) - f(x,y)}{\circ y},$$ x is held constant.

. Notation: The first partial derivatives of $z = f(x,y)$ are denoted by

$\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$

. Example: Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ for the function $z = 3x - x^2y^2 + 2x^3y$

$$\frac{\partial z}{\partial x} = \frac{\partial}{\partial x}[3x] - \frac{\partial}{\partial x}[x^2y^2] + \frac{\partial}{\partial x}[2x^3y] = 3 - 2xy^2 + 6x^2y$$

$$\frac{\partial z}{\partial y} = \frac{\partial}{\partial y}[-x^2y^2] + \frac{\partial}{\partial y}[2x^3y] = -2x^2y + 2x^3$$

III. Total differentiation

. Defination: For function $z = f(x,y)$

$$dz = \frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial y}dy$$

. Example: $z = 10x^{\frac{1}{3}}y^{\frac{2}{3}}$

$$dz = 10\frac{\partial}{\partial x}[x^{\frac{1}{3}}y^{\frac{2}{3}}]dx + 10\frac{\partial}{\partial y}[x^{\frac{1}{3}}y^{\frac{2}{3}}]dy$$

$$= \frac{10}{3}x^{-\frac{2}{3}}y^{\frac{2}{3}}dx + \frac{20}{3}x^{\frac{1}{3}}y^{-\frac{1}{3}}dy$$