ECN 105 **Fall 2000**

Handout #1

I. Differentiation of one variable.

Definition: *The process of finding derivatives is called differentiation.*
Notation: For function $y = f(x)$, *the first derivative is denoted as* $\frac{dy}{dx}$ or $f'(x)$

Summary of differentiation rules.

1.Derivative of a Constant Function:
$\frac{d}{dx}[c] = 0$, c is a constant.
Example: $y = 7, \frac{dy}{dx} = \frac{d}{dx}[7] = 0$

2.The Power Rule:
Simple Power Rule: $\frac{d}{dx}[x^n] = nx^{n-1}$
Example: $y = x^3, \frac{dy}{dx} = 3x^2$
$y = \frac{1}{x^2} = x^{-2}, \frac{dy}{dx} = (-2)x^{-3} = -2x^{-3}$
General Power Rule:
$\frac{d}{dx}[u^n] = nu^{n-1}\frac{du}{dx}$, u is a function of x

3.The Constant Multiple Rule:
$\frac{d}{dx}[cy] = c\frac{dy}{dx}$,c is a constant.
Example: $y = 7x, \frac{dy}{dx} = \frac{d}{dx}[7x] = 7\frac{dx}{dx} = 7$

4.The Sum and Difference Rules:
$\frac{d}{dx}[u \pm v] = \frac{du}{dx} \pm \frac{dv}{dx}$
Example: $y = \frac{1}{2}x + 5x, \frac{dy}{dx} = \frac{d}{dx}\left[\frac{1}{2}x\right] + \frac{d}{dx}[5x] = \frac{1}{2} + 5 = 5\frac{1}{2}$

5. Derivative of the Natural Exponential Function:
$\frac{d}{dx}[e^x] = e^x$

6.Derivative of the Exponential Function:
$\frac{d}{dx}[a^x] = a^x \ln a$

7.The Product Rule:
$\frac{d}{dx}[uv] = u\frac{dv}{dx} + v\frac{du}{dx}$
Example: $y = 2x(4 + 3x)$
$\frac{dy}{dx} = \frac{d}{dx}[2x(4 + 3x)] = 2x\frac{d}{dx}(4 + 3x) + (4 + 3x)\frac{d}{dx}(2x)$
$= 2x * 3 + (4 + 3x) * 2 = 6x + 8 + 6x = 12x + 8$

8.The Quotient Rule:
$\frac{d}{dx}[\frac{u}{v}] = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
Example: $y = \frac{x-1}{2x+3}$

$\frac{d}{dx}[\frac{x-1}{2x+3}] = \frac{(2x+3)\frac{d}{dx}[x-1]-(x-1)\frac{d}{dx}[2x+3]}{(2x+3)^2} = \frac{2x+3-(x-1)*2}{(2x+3)^2} = \frac{3+2}{(2x+3)^2} = \frac{5}{(2x+3)^2}$

9.The Chain Rule:
$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$, u is a function of x
Example: $y = (x^2+1)^3, u = x^2+1, y = u^3$

$$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx} = 3u^2\frac{du}{dx} = 3u^2(2x) = 6x(x^2+1)^2$$

10.Derivatives of a Logarithmic Function:
$\frac{d}{dx}[\ln x] = \frac{1}{x}$
$\frac{d}{dx}[\log_a x] = \frac{1}{x\ln a}$

II. Partial differentiation

Definition: If $z = f(x,y)$,*then the first partial derivatives of z with respect to x and y are the functions* $\frac{\partial z}{\partial x}$ *and* $\frac{\partial z}{\partial y}$, *defined as follows:*
$\frac{\partial z}{\partial x} = \lim \frac{f(x+\circ x,y)-f(x,y)}{\circ x}$, y is held constant
$\frac{\partial z}{\partial y} = \lim \frac{f(x,y+\circ y)-f(x,y)}{\circ y}$, x is held constant.

Notation: The first partial derivatives of $z = f(x,y)$ *are denoted by* $\frac{\partial z}{\partial x}$ *and* $\frac{\partial z}{\partial y}$or f_x and f_y

Rules for Finding Partial Derivatives of $z = f(x,y)$:
1) To find f_x , regard y as a constant and differentiate $f(x,y)$ with respect to x
2) To find f_y , regard x as a constant and differentiate $f(x,y)$ with respect to y

Example: Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ for the function $z = 3x - x^2y^2 + 2x^3y$

$$\frac{\partial z}{\partial x} = \frac{\partial}{\partial x}[3x] - \frac{\partial}{\partial x}[x^2y^2] + \frac{\partial}{\partial x}[2x^3y] = 3 - 2xy^2 + 6x^2y$$
$$\frac{\partial z}{\partial y} = \frac{\partial}{\partial y}[-x^2y^2] + \frac{\partial}{\partial y}[2x^3y] = -2x^2y + 2x^3$$

III. Total differentiation

Definition: For function $z = f(x,y)$

$$dz = \frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial y}dy$$

Example: $z = 10x^{\frac{1}{3}}y^{\frac{2}{3}}$

$$dz = 10\frac{\partial}{\partial x}[x^{\frac{1}{3}}y^{\frac{2}{3}}]dx + 10\frac{\partial}{\partial y}[x^{\frac{1}{3}}y^{\frac{2}{3}}]dy = \frac{10}{3}x^{-\frac{2}{3}}y^{\frac{2}{3}}dx + \frac{20}{3}x^{\frac{1}{3}}y^{-\frac{1}{3}}dy$$